

B 822,165

PSYCHOLOGY AND LOGIC



BF
44
.K16

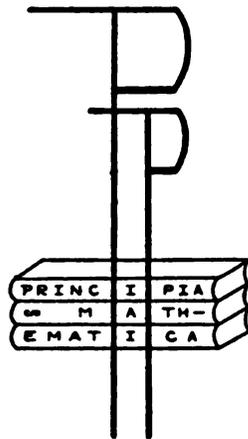
PSYCHOLOGY AND LOGIC

Psychology and Logic

Joseph Robert
J. R. KANTOR

PROFESSOR OF PSYCHOLOGY, INDIANA UNIVERSITY

VOLUME I



1945

THE PRINCIPIA PRESS
BLOOMINGTON, INDIANA

BF
44
K16
v. 1

COPYRIGHT 1945
By
THE PRINCIPIA PRESS, INC.

ARISTOTLE

*Il maestro di color
che sanno*

Whose “naive” investigations
and artless reflection in
Psychology and Logic
prove him invulnerable against
Scholastic Transformation and
Modern Dualistic Sophistication

Science of
Psychology
Hall
1-30-45
52135

transferred to 52135

CONTENTS

PREFACE	xiii
CHAPTER I	
PRINCIPLES AND POSTULATES OF SPECIFICITY LOGIC	I
Postulate I, Logic is Operational, 2. Postulate II, Logical Theory is Continuous with Practice, 4. Postulate III, Logical Operations Constitute Interbehavioral Fields, 5. Postulate IV, Logical Interbehavior Constitutes System Building. 7. Postulate V, Logical Interbehavior is Specific, 9. Postulate VI, Logic is Uniquely Related to Culture, 11. Postulate VII, Logic is Inseverably Interrelated with Psychology, 13. Postulate VIII, Logic is Distinct from Language, 16.	
CHAPTER II	
THE PSYCHANTHROPIC FOUNDATION OF LOGIC	20
The Psychological and Cultural Framework of Logic, 20. The Cultural Basis of Logic, 21. Logical History as the Origin and Variation of Logical Institutions, 22. A. Logic as a General Intellectual Institution, 23. B. Evolution of Logical Elements, Techniques, and Principles, 27. I. Terms and Definitions, 27. II. The Inductive Process, 29. III. The Postulation Principle, 31. IV. The Deductive Process, 33. V. The Proposition, 36. VI. Classification and Division, 38. VII. Laws and Canons, 41.	
CHAPTER III	
DOCTRINAL SUCCESSION IN LOGIC	45
The Greeks: Pragmatic System Building, 46. The Scholastics: Symbolic Transcendence of Nature, 47. The Renaissance: System as Technique of Discovery, 51. Vico vs. Descartes, 53. Leibniz vs. Locke, 54. The Enlightenment: Critical Limitation of Law and Fact, 56. The Romantic Movement: Dialectics of Cosmic System, 60. The Technological Revolution: Induction as Novum Organum, 63. The Contemporary Scene: Technique of System Construction, 66. 1. The Career of Logic, 67. 2. The Career of Language, 70.	
CHAPTER IV	
LOGIC IN CURRENT CULTURAL CROSS SECTION	72
Representative Trends in Contemporary Logic, 72. A. Symbolinguistic Logic, 73. Three Phases of Symbolinguistic Logic,	

76. (1) Symbolic Logic, 77. (2) Syntactic Logic, 79. (3) Semantic Logic, 83. Symbolinguism and Scientific Linguistics, 85. B. Mathematical Logic, 88. (1) Logistic Logic, 88. (2) Algebraic or Algorithmic Logic, 90. The Relation of Logic and Mathematics, 93. C. Logic as the Science of Order, 98. D. Humanistic or Nonformal Logic, 99. E. Instrumental Logic, 101. Linguistic Factors in Instrumental Logic, 105. Psychological Implications of Instrumentalism, 105. Is Logic the Theory of Inquiry? 106. Contemporary General and Specific Logic, 106.

CHAPTER V

THE INTERRELATION OF LOGIC AND PSYCHOLOGY 109

How is Logic Related to Psychology? 109. A. Logicians Accept Psychology, 112. I. Kant, 112. II. Boole, 113. III. Mill, 115. IV. Humanistic Logicians, 116. V. Instrumental Logicians, 116. VI. Other Sympathizers, 117. B. Logicians Reject Psychology, 117. I. The Essence Criterion, 119. II. The Objective or Impersonal Criterion, 121. III. The Normative and Regulative Criterion, 123. IV. The Product Criterion, 124. V. The Comprehensiveness Criterion, 124. VI. The Formality Criterion, 127. C. Logicians Neglect Psychology, 128. I. Symbolinguists, 128. II. Rationalists, 131. Coordination and Cooperation between Logic and Psychology, 133. Inadquate Psychology Obstructs Logic, 134. 1. Dualistic Psychology, 134. 2. Biological Psychology, 135. 3. Interbehavioral Psychology, 136.

CHAPTER VI

INTERBEHAVIORAL PSYCHOLOGY AND SPECIFICITY LOGIC . . 139

The Roots of Science, 139. Psychology the Science of Interbehavioral Fields, 140. Reactional Biography and Stimulus Evolution, 141. Psychology Excludes Both Minds and Sensations, 142. Scientific Obstacles to Naturalistic Psychology, 142. Science Implements Faulty Psychology, 142. Dualism Implemented by Experimentation, 144. Dualism Implemented by Constructionism, 145. Dualism Implemented by Operationism, 145. Dualism Implemented by Verbal Conceptualism, 146. The Elimination of the Mind-Body Problem, 148. The Continuum of Nature, 150. Varieties of Psychological Events, 151. Psychological and Sociological Interbehavior, 153. Interbehavioral Psychology and the Bounds of Logic, 155. Interbehavioral Psychology and the Definition of Logic, 157. Four Methods of Defining Logic, 157. (1) Definition by Subject Matter, 158. (2) Definition by Process, 158. (3) Definition by Product, 158. (4) Nonsystem-Building Definitions, 158. The Logical as the

Reasonable or the Rational, 158. The Logical as the Pervasive, General or Universal, 159. The Logical as the Conventional or Acceptable, 159. The Logical as the Theoretical, 159. The Logical as the Formal, 159. The Logical as the Necessary, 160. The Logical as the Systematic, 160. System Building and Ontology, 161. Interbehavioral Psychology and Specificity Logic, 166. 1. Cosmic and Noncosmic Systems, 166. 2. Formalistic and Contentistic Systems, 167. 3. Concrete and Abstract Methodistic Systems, 167. Authentic Specificity Logic, 168.

CHAPTER VII

THINKING AND REASONING AS LOGICAL OPERATIONS 170

Thinking, Reasoning, and System Building, 170. Thinking and Reasoning as Specific Interbehavioral Fields, 172. Intellectual Factors in Logical Situations, 173. 1. Intellectual Attitudes, 174. 2. Believing Interbehavior, 174. 3. Assuming Interbehavior, 175. 4. Hypothesizing Interbehavior, 176. Thinking as a Logical Operation, 176. Thinking and System Building, 178. Problem Solving and System Building, 178. Reasoning and Logical Operations, 180. Reasoning as Inferential Interbehavior, 180. Reasoning as Ratiocination, 181. Reasoning and Thinking, 181. Inference and Discovery, 182. Inference and Implication, 184. The Efficacy of Reasoning, 186. The Criteria of Rationality, 187. Rationalization and the Rational, 189. Interbehavioral versus Classical Inference, 189. (1) Deductive or Specializing Inference, 190. (2) Inductive or Generalizing Inference, 190. (3) Conductive or Analogistic Inference, 191. (4) Constructive or Evaluative Inference, 191. (5) Reductive or Eliminative Inference, 192. Reasoning and System Building, 192.

CHAPTER VIII

LANGUAGE AND SYMBOLS IN LOGICAL OPERATIONS 194

Language and Logic, 194. The Nature of Linguistic Events, 196. Linguistic Activities and Linguistic Things, 196. (A) Referential Linguistic Activity, 196. (B) Nonreferential Language Activity, 198. (a) Naming, 198. (b) Counting, 198. (c) Recording, 199. (d) Sign and Symbol Interbehavior, 199. (C) Linguistic Things, 200. (a) Referential Language Things, 201. (b) Remote Language Things, 201. (c) Nonreferential Language Things, 201. (d) Nonlinguistic Things, 201. Field Analysis of Language, 202. Varying Constructions Concerning Linguistics and Semiotics, 205. Language and Symbols as Operational Products, 205. Meaning in Symbols and Language, 206. The Field Principle in Linguistics, 206. Interrelation and

Confusion of Linguistic and Nonlinguistic Things and Events, 207. Confusion of Various Interbehaviors, 207. Confusion of Interbehavior with Things, 208. Confusion of Constructional Products and Things, 210. Language and Symbols in Universal and Specific Logics, 211. A. Symbols, 212. I. Symbols and Essences, 212. II. The Independence of Symbols, 213. III. The Clarity of Symbols, 215. IV. The Absoluteness of Symbols, 216. V. The Abstractness of Symbols, 217. a. Logistic Mathematics, 217. b. Formalistic Mathematics, 217. c. Constructionistic Mathematics, 218. VI. The Arbitrary Character of Symbols, 218. B. Symbol Configuration, 219. Language and Symbols as and in Logic, 221. Logical Acts and Logical Products, 222. Sentences and Propositions, 222. Sentences and Statements, 223. Sentences and Symbol Configurations, 224. Constructor and Construction, 224. Origin of Symbols and Symbol Configurations, 224. Significance and Operation of Symbols and Symbol Configurations, 225. Constructor Emphasis and System Validity, 225. Sentences and Things, 225. The Role of Language in System Building, 226. Summary, 228.

CHAPTER IX

LOGICAL OPERATIONS AND PRODUCTS

229

Logical Operations Are Constructive Acts, 229. Materials and Products of Logical Construction, 230. How Operations and Products Are Confused, 233. Range and Types of Logical Products, 236. The Semiotic of Peirce, 237. The Unifying Semiotic, 239. The Semiotics of Ducasse, 243. Typical Constructional Products, 246. Number, 246. The Variable, 249. Functions, 251. Relations, 253. Series, 255. Classes, 256. Axioms, 258. Limits, 259.

CHAPTER X

THE OPERATIONAL VERSUS THE TRADITIONAL TREATMENT OF LOGICAL PRODUCTS

261

Linguistic Factors as Logical Products, 261. A. Words and Symbols as Product Materials, 262. The Fixity of Words and Symbols, 262. The Nature of Words and Symbols, 263. Functions of Words and Symbols, 269. a. Nominalism, 269. b. Neo-Classicism, 271. c. Formalism, 272. d. Instrumentalism, 273. B. Sentences as Materials and Products of Logic, 275. Sentences as Materials of Inquiry, 275. Sentences as Judgments, 277. The Production and Product Character of Sentences, 278. a. Sentences as Syntactic Structures, 278. (1) Sentential Functions, 279. (2) Transpositional Inference, 280. (3) Sentences as Referential, 280. (4) Sentences as Calculative, 281. Sen-

tences and Propositions, 282. C. Complex Systems as Products, 284. Strata of Complex System Construction, 284. a. Local and Temporal Stratification, 285. b. Individual and Cultural Stratification, 285. c. Material and Description Stratification, 285. Varying Functions of System Products, 286. a. Algorithms and Calculi, 286. b. Syllogisms and Implications, 287. c. Mathematical Systems and Proofs, 289. (1) Mathematical Systems, 289. (2) Mathematical Proofs, 290. (3) Doctrinal Functions, 293. The Specificity of Logical Operations and Logical Products, 293.

CHAPTER XI

LOGICAL THEORY AS INTERBEHAVIORAL ANALYSIS (I) . . . 296

Logical Theory Continuous with Practice, 296. Logical Theory as Analysis of Logical Enterprises, 298. 1. The Actor or Logician, 299. 2. Work or Action, 300. 3. Materials, 300. 4. Auspices and Conditions, 300. 5. Criteria or Canons, 301. 6. Products, 301. The Logician as Source of Logical Theory, 302. A. The Logician's Aims and Aspirations, 303. I. Comprehensiveness, 303. II. Data Transcendence, 304. III. Attaining Truth and Certainty, 305. IV. Rigor, 305. V. Consistency, 305. VI. Irrefragable Demonstration, 306. VII. Specific System Building, 307. B. The Powers of the Logician, 307. I. Reason and Reasoning, 308. II. Intuition, 308. C. The Logician's Background or Orientation, 310. The Logician at Work, 312. I. Formalizing, 313. II. Symbolizing, 314. III. Totalizing and Generalizing, 315. IV. Hypostatizing Descriptions, 316. V. Tautologizing, 316. VI. Analogizing, 317. VII. Analyzing, 318. VIII. Synthesizing, 319. IX. Relative versus Absolute Techniques, 320. X. Specific versus General Techniques, 321. XI. Situational versus Autonomous Techniques, 321. XII. Progressive versus Final Techniques, 322. XIII. Temporal versus Timeless Techniques, 322.

CHAPTER XII

LOGICAL THEORY AS INTERBEHAVIORAL ANALYSIS (II) . . 324

Problem of Logical Materials, 324. Psychological Basis of Overlooking Materials, 324. Types of Logical Materials, 325. 1. Thing Materials, 325. 2. Language Materials, 325. (1) Intrinsic Verbal Materials, 326. (2) Registration Language Materials, 326. (3) Descriptive or Reference Language Materials, 326. (4) Intrinsic Symbol Materials, 327. 3. Scientific Materials, 327. 4. Relations as Materials, 328. Auspices and Conditions of Logic, 329. 1. System-Building Auspices, 329. (1) Philosophical Auspices, 329. (2) Scientific Auspices, 330. (3) Mathematical Auspices, 330. (4) Practical Auspices,

331. 2. System-Building Conditions, 331. (1) Pure and Applied Logic, 331. (2) Deliberate and Casual Logic, 332. Canons of Logic, 332. Canons and Postulates, 333. Positive and Negative Canons, 334. (A) Negative Canons, 334. (a) Canons of Nonuniversality, 334. (b) Canons of Nontranscendence, 335. (c) Canons of Canon Avoidance, 337. (d) Canons of Tradition Avoidance, 337. (B) Positive Canons, 338. (a) Canons of Identification, 338. (b) Canons of Subject Matter, 338. (c) Canons of Procedure, 339. (d) Canons of Validity, 339. (e) Canons of Simplification, 340. (f) Canons of Fittingness or Relevance, 341. Logical Products, 342.

BIBLIOGRAPHY	344
SUBJECT INDEX	353
NAME INDEX	361

PREFACE

Two basic theses underly the present work.

The first, the specificity theorem, signifies that logic is essentially concerned with specific events and not with universal and transcendent systems.

The second, the interbehavioral theorem, implies that no matter how logic is defined¹ it entails a psychological dimension which must be taken into account. Even those logicians who postulate that there are invariant relations in the universe or ultimate uniformities of nature inevitably face the investigative problem of how they are discovered. Since the various traditional psychological systems have not proved satisfactory in handling such problems and since the interbehavioral event and its product are always implied, an interbehavioral psychology is highly desirable.

The Specificity Thesis. The writer holds that the referents for the term *logic* are always individual human enterprises located in particular fields or frames of reference. Since these enterprises are constructive they comprise (1) actions or performances, (2) materials worked with, and (3) results or products.

Though writers on logic differ in their views concerning the nature of logic, and produce treatises varying widely in scope, content, and principle, they all agree that they are seeking the one true or valid system. Rarely is it suggested that no such universal system is available.²

Glance at a telling example. When Locke made his famous declaration that "God has not been so sparing to men to make them barely two-legged creatures, and left it to Aristotle to make them rational,"³ it is plain that he was merely attempting to dissociate logical action which he called reasoning from formal syllogizing. Yet Joseph⁴ accuses Locke of bringing objections against the study

¹ For example, (1) the study of the laws of thought, (2) the science of order, (3) the technique for constructing mathematical or other systems, (4) the methodology of science, (5) the art of proof or demonstration, or (6) the theory of inquiry.

² Cf. Lewis, *Alternative Systems*. Complete titles of cited articles and books are given in the Bibliography at the end of this volume.

³ *Essay*, Bk. IV, chap. 17, sec. 4.

⁴ Introduction, p. 3.

of logic, since for Joseph there is only one logic and that one the syllogistic.

To support the thesis that there are many logics and to throw some light upon the principle uniting them we propose a sharp distinction between two clearly distinguishable types of logical events. On the one hand, there are (1) the crude data consisting of (a) logical processes, acts of relating things, of inferring, of implying, and (b) the relations and relata—in short, the materials and products of systematization. On the other, there are (2) the views or theories propounded by different writers concerning which of the above processes or things are the subject matter of logic, and their resulting propositions concerning the nature of the accepted material.

By the crude or unanalyzed data of logic we understand actual *logical practice*. Most writers on logic agree that such practice consists of operations of some sort, inferring, reasoning or implying, although the operations are frequently mistaken for faculties or the products of such operations and in some cases even for the material worked upon. The latter is illustrated by the doctrine that logical subject matter comprises autonomous relations or elements to be ordered.

For our part, the crude data of logic consist of specific practices of system building. Since these logical practices have their unique scope and boundaries they differ with respect to (a) problems or purposes, (b) materials worked upon, whether concrete or abstract, and (c) results or products. It is an error to regard the materials for these system constructions as limited to propositions or sentences, or symbolic representations of relations, whether mathematical or not, since logical materials include everyday things as well.

Logical theory, it follows, constitutes the investigation and description of the specific activities of building or organizing systems. These activities may be studied as the personal behavior of individuals or as the conjoint enterprises of the members of schools, groups or traditions. The point here is that all human operations are particular modes of interbehavior of organisms and specific stimulus objects.

Traditional logical theorists have constructed statements concerning the nature of logic on the basis of their own work or that

of others (Aristotle, Lully, Mill, DeMorgan, Boole, Hamilton, Hegel) with whom they agree or disagree in whole or part. From a scientific standpoint a scale of values for logical theories can be set up on the criterion of greater relative adherence to logical practice or established theory concerning such work. Ideally, of course, logical theory should be derived from observing logical practice, though every theorist is conditioned by his interests, training, and intellectual milieu.

The Psychological Thesis. The history of logic testifies that divergences in view concerning the nature and operation of logic and the achievable results are tremendously influenced by the psychological view of the logical theorists themselves. These views are not always necessarily known to logicians, but may operate upon them by the subtle and effective agency of general cultural or specific school institutions.

The inherent logical paradox is proof enough that logicians are more influenced by psychological traditions than observations of work. Although logic is obviously a human enterprise, logicians inevitably regard it as *ultimate, universal, and transcendent*. Not only is this true of the dialectical logicians who look upon logic as an instrument to seize the universe in its totality, but also of the mathematical logicians and scientific methodologists. The symbolic or mathematical logicians seek omnipotent algorithms, whereas the methodologists strive for general techniques transcending specific problems and particular situations.

The *ultimacy* of logic is manifested by the omnipotence and finality which logicians accord proof and demonstration. Boole asks:

Shall we then err if regarding that as the true science of logic which, laying down certain elementary laws, confirmed by the very testimony of the mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes, for its practical applications, methods of perfect generality?⁵

When such ultimacy is not fortified by the testimony of the mind, resort is had to invariant objective relationships which make conclusions necessary and inevitable from premises.

Logical *universality* is undoubtedly derived from the Plotinian

⁵ *Laws of Thought*, p. 5.

logos. Despite all the variations in logical thinking since Plotinus, the idea that logical procedures are all-embracing and universally valid continues unabated. In the last analysis logic is presumed to involve the universe. Nevertheless, as we might expect, logicians sometimes do admit limited universes, however grudgingly. This was the case when DeMorgan introduced the idea of limited universes which have come to be called universes of discourse. This limitation was forced upon DeMorgan because he worked with words or names, as the following quotation indicates.

Let us take a pair of contrary names, as man and not-man. It is plain that between them they represent everything imaginable or real, in the universe. But the contraries of common language usually embrace, not the whole universe, but some one general idea.⁶

The universality or absolute coverage of logic he never intended to minimize. Though pairs of common-language contradictories have limits they exhaust a particular restricted field in microcosmic analogy to proper logical names. The latter in the form of one inclusive and one exclusive word (tree and not-tree) would cover absolutely everything. The concession here is similar to that in which DeMorgan as a formalist asserts that logical studies occupy a humble place in the total field of human knowledge.⁷ That logic should be partialled out into realms of discourse was certainly remote from DeMorgan's thinking. Logicians too numerous to mention, it may be added, would not even allow his concession on the ground that logic is never concerned with the meaning of common or specific—that is, material—terms, but only with the formal elements of sentences.

The *universality* of logic as a formal discipline is excellently illustrated by the claim that logic is one and indivisible. To the assertion that there is a non-Aristotelian logic as there is a non-Euclidean geometry the answer is returned: "What have recently been claimed to be alternative systems of logic are different systems of notation or symbolization for the same logical facts."⁸

Logical *transcendence* presents two aspects. First, logical procedures are believed to carry over indifferently from one type of

⁶ Formal Logic, p. 37.

⁷ Ibid., p. 46.

⁸ Cohen and Nagel, Introduction, p. v.

subject matter to another. As formal and general techniques logic is regarded as capable of effecting results no matter what the field of application may be. Even the scientific methodologists consider their formalized systems of procedure or investigation as capable of crossing all barriers. Secondly, logic is presumed to transcend all subject matter. Thus, logical processes are endowed with various degrees of *a priori* and infallibility. By means of sentential and symbolic structures or systems the logician supposes himself to obtain omniscience, to know what has not been observed and to attain infinities of various sorts.

Behind the unsatisfactory assumptions that logic is infallible, comprehensive, and transcendent unquestionably lies unsound psychological theory. And so while it is desirable that psychology should contribute something to both logical practice and theory, actually it has not been able to do so. Neither the various forms of traditional faculty psychology nor the current ideas that psychology is concerned with nebulous and intangible states of consciousness or the biological activities of organisms offer satisfactory co-operative techniques of handling logical problems.

Today, we submit, psychology has made such progress toward the goal of an objective science that it can be decidedly useful in illuminating some dark spots in the logical field. Since the time when Peirce stated his doubt that "anybody knows much about the operation of the mind in reasoning"⁹ there have been vast improvements in the scientific understanding of reasoning as well as of the nature of linguistic and symbolic things and acts and their interrelation with reasoning. Light can undoubtedly be thrown, for example, upon the connection between words or names, things, ideas, and biological processes. Obviously, of course, logic will no longer be regarded as a universal and transcendent system.

Current improvements in psychological theory, we hasten to add, preclude the traditional notion of the priority of psychological principles in logical study. On the contrary, the theory that psychological data are concrete interbehavioral events sets any such notion completely aside.

Psychological principles are coordinate with the biological and anthropological principles which concern the *organisms* or *persons* involved in interbehavioral events. Psychological principles are

⁹ Collected Papers, 2.184. Hereafter referred to as C.P.

coordinate likewise with the physical, chemical, and other principles which concern the nature, order, and relations of the *objects* involved in interbehavioral events. The value of interbehavioral psychology for logical investigation lies precisely in the fact that it can fit into a pattern of cooperating disciplines.

During the writing of this book I have been troubled by some overstress of points and expository repetition. This feeling has been somewhat intensified after the work was completed. I here freely make the necessary apologies. Admitting that these and other faults are inexcusable, I nevertheless hope there are extenuating circumstances. Intrinsically, the problem I have set myself, involving a survey of two distinct disciplines as well as the need to point out necessary changes in one—psychology—has resulted in a less orderly and compact exposition than a reader has a right to expect and I would have liked. Among the extraneous factors, of course, are the effects of wartime pressure upon intellectual enterprises.

I wish to acknowledge the great help rendered by H. R. Kantor throughout the writing and printing of this book. In many ways this help was much more on the order of collaboration than assistance. Since some of the material contained in Chapters 1 and 8 of this volume has already appeared in print, I want to express my appreciation to the editors of the *Journal of Philosophy* for their permission to include it in this book. The publication of this work was made possible through the generous aid of my brother, Dr. A. E. Kanter, and the Directors of the Principia Press. I take this opportunity of expressing my sincerest thanks to those who have so willingly supported my efforts.

J. R. K.

July, 1944

CHAPTER I

PRINCIPLES AND POSTULATES OF SPECIFICITY LOGIC

OF recent methodological and expository developments in logic and science nothing surpasses in merit the increasing practice of setting up postulate systems. It is a tremendous advantage to make clear at the outset of an intellectual enterprise what fundamental assumptions underlie the work. Our purpose, then, in the present chapter is to set down the postulational background of specificity logic.

Before doing so, however, it should be made plain that the nature of postulates and the procedure of constructing postulates themselves require considerable clarification. It is an error to regard the postulation process as a new or unique discovery. Intellectual workers have always been aware of the necessity to set forth the premises or assumptions upon which they base their work, even though they may have thought of themselves as simply announcing their general theories or philosophies. In the more restricted domains of science the construction of hypotheses consists essentially of a postulational technique. To hypothesize is tentatively to take something for granted in order to put it to an experimental or other controlled test.

What we take to be new in any postulation procedure is the deliberate effort to be definite and clear concerning one's investigation. This implies knowing whence principles are derived and how closely one adheres to them. Since no principles are absolute or *a priori* in any traditional sense, it follows that all postulates set up require justification both as to need and validity.

Again, in consonance with the present work postulates cannot be regarded as simply formal statements of general assumptions. Since postulates are set up for different sorts of enterprises, they necessarily vary as the enterprises vary. Naturally any set of postulates must conform to the best-established general principles of postulation, but they must also be justified in the particular enterprise for which they are formulated. In short, a postulate system for one type of logic cannot be satisfactory or valid for another type.

Furthermore, postulates are not arbitrary assumptions even in the sense that mathematical postulates are governed by nothing more than the achievement of a particular coherent system. The present increased interest in postulation may probably be traced to the growing appreciation of mathematicians that axioms are postulates or selected starting points and that mathematical systems are discretionary constructions. It must not be overlooked, however, that since mathematics is primarily concerned with systems of relations the manipulation and organization of these relations need not be controlled beyond the coherence of the system. In other sorts of logical enterprises the systems to be attained must be in harmony with the type of subject matter organized. The discretion of the scientific as compared with the mathematical systematizer is therefore limited.

And, finally, postulates are not principles set down at the beginning of an intellectual enterprise as unalterable guides to what the work shall be. Actually, postulates are derived from results of studying a particular kind of material, and thus are subject to modification as the work proceeds. Postulates, then, constitute the statement of principles valid and acceptable up to a given stage of study. The need for postulates of a specificity logic and the merit claimed for them are therefore based upon the assumption that other postulates are not found to be sufficient or satisfactory.

POSTULATE I

LOGIC IS OPERATIONAL

Logic constitutes primarily a series of operations. Superficially our linguistic conventions indicate that the referent of the term *logic* is a thing. As a type of human enterprise, however, it is obvious that logic consists essentially of logicizing activities, although as we have indicated in the Preface, the term *logic* also refers to things. In addition to the performances of individuals we must include in our study of logic the materials upon which logicians work and the products resulting from such work whether or not recorded in treatises. In view of the fact that the materials only become logically significant when logically worked upon, and that products always imply the process, we may regard the operations as primary. Especially is this true since frequently the materials of logical op-

erations are already definite products of former intellectual operations.

Logic, however, is only primarily or essentially a series of operations. To limit logic to actions alone (acts of inference, acts of implications), even when such actions are always regarded as operations upon things, results in an overemphasis of one factor in the logical enterprise. The consequence may well be a serious misinterpretation of our field of investigation.

Emphasis upon the operational character of logic obviates two undesirable conditions. First, it helps to set aside the notion that logic is concerned with impersonal existences or realms of reason, for example, selected systems of eternal Platonic Ideas. Unfortunately, the fact that some logical work does consist of organizing independently existing relations has led to confusing such operations with impersonal or cosmic reasoning. Actually, the objective relations existing independently of persons, such as the weight ratios of chemical elements, constitute specific empirical events. To discover and organize such relations is in no sense to deal with cosmic reason or reasoning, even if we minimize the necessary human factors on the plea that they are incidental activities of becoming aware of them.

In the second place, an operational emphasis prevents confusing logical products with autonomous existents. Like so many other forms of behavior logical operations result in products, whether recorded or unrecorded propositions, syllogisms, symbols, orders, arrangements, or treatises. Whether we regard these products as actions (attitudes, beliefs, or utterances) or as things (recorded calculations or treatises) they owe their existence, character, and value to the preceding operations; thus the operations are as primary in importance as they are in time. The same principle applies to logical rules and principles no less than to syllogisms or calculi of various sorts which are also products of logical operations.

Another advantage of stressing the operational character of logical processes is that it enables us to take account of the contribution of nonlogical actions and things to the development of logic. At least it helps to attain a proper understanding and evaluation of both logical work and its products.

Again, the operational view precludes the notion that logic is

essentially an organon or instrument—a fixed pattern or process for achieving truth or conducting inquiry. This instrument conception is objectionable whether logic is regarded as (1) a tool for practical purposes, (2) a general organon for regulating thought or reason, or (3) a tool for transcending concrete experience.

That logic is operation must be admitted even by those who believe that by logical processes they can attain cosmic formulae or achieve union with a logos, for such processes whether considered inferential or intuitional are concrete actions or operations. Moreover, it is clear that all metaphysical ontologists simply create their cosmoses by verbal tools out of verbal materials. It is easy to see that when logicians like Hegel, Bradley, and Bosanquet assert that the whole of reality is the ultimate subject of all judgments they are actually reducing reality to what they call judgments.

POSTULATE II

LOGICAL THEORY IS CONTINUOUS WITH PRACTICE

Granted that logic constitutes a series of operations, are these operations (1) acts of reasoning, inferring or implying or (2) the construction of rules for thinking and reasoning? Logic is widely if not universally regarded as the general theory of inference, of mathematics or of inquiry, and is therefore looked upon as different from logical practice. This distinction is implied not only in the classic differentiation between formal and empirical logic or between logic and scientific methodology, but also in the attempts to separate technical inferential and calculative behavior from the more intimate and personal activities of the same genus. We find two serious objections here. First, such notions tend to conceal if not overlook the fact that all logic consists of the operations of persons with a variety of rules, and secondly, there is discernible here the undesirable tendency to staticize and reify logic, thus making it autonomic and transcending.

The operational or interbehavioral view of logic at once removes the barrier between logical theory and logical practice. Logical theory is distinguished from logical practice merely on the basis that the former constitutes the examination or *study* of operations, whereas the latter consists of those *operations* themselves. We wish to stress the fact that study operations, whether theorizing, thinking, or reasoning, are all, of course, fundamentally operations.

There is consequently no generic difference between these two alleged types of logic. Note, too, that logic as a series of organizational operations is not merely an intellectual enterprise, when intellectual is regarded as different from various sorts of concrete manipulations. Intellectual behavior is likewise the interbehavior of individuals with objects, howsoever different in detail the operations may be.

We propose, therefore, a pragmatic operational continuum. At the lower boundary this continuum includes the acts of an individual organizing materials, say, for assembling a machine, or arranging letters to form words—which acts we may regard as basic or primary operations. Next in the series are the activities of the observer of the operations of the first grade. These also constitute performances or operational behavior. Then we have the activities of the individual who criticizes the observational processes of the second-level worker and this, too, is likewise the performance of operations. Palpable and important as the differences are between the various operational levels of this continuum, by virtue of particular subject matter and gradation of skill we may still say that there is no difference in principle between the practical organization of objects and the most technical or abstract logical procedures.

It must be pointed out that the series is strictly limited in range and does not involve any objectionable regress. We are exclusively concerned with the occurrence of specific events involving activities of individuals in contact with manipulatory objects, whether the latter are crude, as in the first level, or highly refined and subtle materials (abstract relations) as in the work of the mathematical logician. At most, we might perhaps add as the last of the series the behavior of the student of the abstract products of logical theories.

POSTULATE III

LOGICAL OPERATIONS CONSTITUTE INTERBEHAVIORAL FIELDS

Since logical activities or the logician's performances are inevitably interrelated with materials worked upon, they constitute only partial factors of operational events. Moreover, since the materials operated upon determine the logician's operations, the materials must be regarded as likewise performing operations coordinate with those of the logician. Logical enterprises, therefore, constitute interbehavioral fields.

Interbehavioral fields comprise the locus of a mutual and interdependent set of actions. Neither the fact that the logician is concerned with (1) objects or things or (2) abstract relations detracts from the conception of an interbehavioral field. Even if such objects exist only as arbitrary constructions they are derived from and substituted for by natural objects. Accordingly, all logical operations constitute, in principle, concrete, observable, natural events. Above all, the interbehavioral fact stands in the way of any belief in detached psychic processes as a basis for general and transcending reasoning.

Furthermore, every particular interbehavior is set in a special reference frame. As a result any specific type of logic, whether formal, empirical, symbolic, or methodological, is not only conditioned by the type of material dealt with, but also by the general conditions under which the work is done. These conditions include the unique problems and life circumstances of the particular logicians involved. And since life circumstances of logicians are essentially cultural, logic is definitely a psychanthropic occurrence.

One fundamental implication here is that there is no logical process or product which is not localized in a definite spatiotemporal frame of reference. The logical domain can never transcend specific actions and things. This contradicts directly the view of Jevons' who argues that not to allow for such transcendence is to set impediments to pure logical reasoning. When Jevons writes:

Material existences must exist in space, no doubt, but intellectual existences may be neither in space nor out of space; they may have no relation to space at all, just as space itself has no relation to time. For all that I can see, then, there may be intellectual existences to which both time and space are nullities.

he assumes that logic is concerned with "ideas," completely unrelated to anything but verbal assertion. Again when Jevons relies upon Hume's maxim "that an object may exist, and yet be nowhere,"¹ his doctrine becomes identical with that which reduces everything to fleeting states of consciousness. What the proponents of this doctrine overlook is that this domain of consciousness is itself a product of an interbehavior of a man or set of men with ob-

¹ Principles, p. 269.

² Treatise, Bk. I, part 4, sec. 5.

served activities and doctrines derived from such observational behavior.

The interbehavioral-field postulate implies that we may interbehave with (think and talk about) the objects of our own creation. Russell⁸ remarks upon the difficulties attendant upon the proposition "the round square does not exist," since it is natural to ask what is it that does not exist, with the implication that there is a round square object, though it has the odd property of not existing. Such objects Meinong had taught must be in the realm of Being. Clearly this realm of Being is only a verbal product which is confused with things that we can interbehave with not merely by verbal but by contactual manipulation. When infinities, infinitesimals, classes, groups and other mathematical objects are under discussion, this trivial illustration attains serious and important dimensions.

In no sense, however, is logical interbehavior to be interpreted as formal modes of action inevitably evolved out of biological processes, possibly as a means of achieving biological or social adjustment. Whenever such a notion is seriously taken it meets with the obvious objection that a general motive or purpose is imposed upon and made to limit logical operations. It is a necessary requirement of the interbehavioral principle that particular logical operations must be accepted as actually occurring events. Obviously, such events always have and always reveal an historical background. But this background must not uncritically be regarded as an absolutely determining factor in any present situation. After all, persons interbehave with immediately present objects and conditions. Hence the exigencies of these conditions make possible and often result in new forms of operations and new sorts of logical rules and procedures. Certainly in some cases logical operations may be correctly described as biological and/or social adjustments, but the correctness can be determined in specific instances only.

POSTULATE IV

LOGICAL INTERBEHAVIOR CONSTITUTES SYSTEM BUILDING

If there is one generic characteristic of logical operations or interbehavior it is that such work consists of system building. Logical operations have evolved as techniques for achieving systems as

⁸ Principles, p. xi.

products. But despite this general descriptive character of logical interbehavior, system building must be regarded as a specific type of event. To be logical, interbehavior need not involve any unique form of material, any particular manipulations or criteria of organization. Certainly logic cannot be limited to the interrelation of symbols or symbol operations as in mathematics, or the arrangement of propositions according to some plan of necessary inference, whether syllogistic or synthetic deduction. Indeed, as is forecast in the popular references to the logic of archaeological construction or the logic of curriculum making, any form of systematization of actions, objects, or events constitutes logical work. We have already suggested the inclusion within logical boundaries of overt manipulations of various sorts—for example, the assembling of mechanical systems.

With the multiplication of geometries and arithmetics even mathematicians as essential system builders have been forced to give up the notion of logic as a unique form of organization of abstract materials with incontestable criteria. True enough, mathematicians still hope to attain general analyses which shall incorporate all the essential features of mathematics. But even if such an ideal could ever be attained it would only illustrate one form of system building.

The exemplary logical stress of validity does not characterize a principle uniquely pertaining to logic. On the contrary, it merely emphasizes a phase of any system-building process. In the obvious absence of transcendent criteria, validities can only be established upon conformities based upon selected premises. Such validities vary in detail with the character of the problems encountered and the kind of material worked upon. Prior to an inspection of a system no determination of validity is possible. A general statement, however, may be ventured—namely, no type of validity criterion, whether noncontradiction, social need, completeness or symmetry, can claim to be anything but a single feature of specific system building.

There are several corollaries to this postulate. In the first place, logic is no more a philosophical than a scientific, a technological or a commercial discipline. In no sense does it have any exclusive and ultimate subject matter, technique, or products. Whatever classificatory scheme it is fitted into must be regarded as an arbitrary one

for a particular purpose based upon observed system-building operations.

In the second place, system building is construction. Logical work always begins in particular situations with given materials. All creation, therefore, reduces to manipulation, analysis, dissection, generalization, and substitution. All system products are only relatively novel, representing the limited transformation or received (observed) raw materials or extrapolations from them. Not even the most original inferential or problem-solving activities constitute more than a calculative ordering or arrangement of factors rooted in the objects or conditions stimulating the constructional enterprise.

POSTULATE V LOGICAL INTERBEHAVIOR IS SPECIFIC

Granting that logic consists of definite system-building interbehavior with actual materials, it follows that logical work constitutes specific and particularized enterprises. This specificity applies equally to process, field, and result or product. In brief, there is no significant logic that covers all events on the basis that whatever exists or may exist possesses common, pervasive, general characters or properties. Nor is there any general procedure for effectively dealing with any kind of material in the sense perhaps of organizing the relations which propositions sustain to each other.

Specificity logic makes plain the futility of seeking to achieve comprehensiveness, necessity, and finality in local (particular fields) or general (cosmic) investigation. When the logician is cosmically or metaphysically inclined, his activities are presumed to apply to or achieve results concerning *a* or *the* universe. The localistic logician, on the other hand, deals with more specific problems, such as the methodology of science or the foundations of mathematics. Whether the logician regards himself as concerned with local or general problems he attempts to lift both himself and his work out of his psychological and anthropological frames of reference.

Illustrations are numerous—for example, the attempts of scientists to expand their formulae to include the universe, and of mathematicians to reach out toward infinities. In every case, however, it is clear that such constructions consist of the work of per-

sons acting upon their own responsibility and according to their own lights. Universes and totalities are mere verbal constructions, while linguistic and reflective behavior can only be valid procedures in operating with specific situations.

When we insist that logical work constitutes specific system building, this is not to deny the value of general principles, which, indeed, are implied in the very system-building process. No reflection and certainly no science can either be initiated or precede without generalization or simplification, but the achieved universals are merely organizations of results obtained by direct and indirect contacts with particular things. The comprehensiveness, necessity, and finality of such organizations are in every case functions of specific situations.

So inherent is ultimacy in the thinking of logicians that even those who theoretically accept the specificity principle violate it in practice. It is true, those who give up cosmological systems still insist upon some absolutistic form of logical procedure, whether it be induction, deduction, syntacticism, a single interpretation of probability, or some general pattern of inquiry. All of these activities and many others constitute particular operations suitable for special purposes or phases of a large number of system-building procedures. It is hardly a mitigation of the universalizing tendency generously to adopt several of these processes. An objective and unbiased observation of logical work forces upon us the conclusion that there are many types of logical situations. The kind of problem we have and the kind of material we deal with determine our methods and results.

Nor does the specificity postulate exclude any system from the system-building domain. It leaves room for the organization of the most abstruse tautological structures. What it does insist upon, however, is that each system be restricted to its proper bounds. For example, it warns us against any one system to cover all. It demands the recognition that scientific methodology is merely a name for many different problems, events, and techniques. Similarly, it claims that mathematics consists of different branches requiring varying postulates and techniques. Not only do current mathematical systems differ from Aristotelian or traditional formal logic, but they involve series of unique system constructions with enormous variations between them.

To believe that there is only one logic or that logic is a universal system is simply blinding oneself to the actual facts of reasoning and other logical operations. Nor is it helpful to satisfy ourselves with pairs of logics such as logic (pure) and methodology (applied) or formal logic and the logic of truth. Nothing less is adequate than to take account of the innumerable system-building operations performed under the adopted restrictions of validating criteria.

Specificity logic suggests that logicians may advantageously abandon such notions as that geometry is the prototype of all rational procedure or that the *formality* and *generality* of mathematical analysis are the veritable signs on the high road of reason. Again, we must renounce the doctrine that simplification is the one sovereign procedure in reflection or logic and accord to any known technique its demonstrated merit in the construction of logical systems. As we have indicated, there are no absolute elements or factors for logic, though there are numerous principles of organization in any logical enterprise.

At the basis of all totalitarian and ultimate systems is the confusion of a construction with the product of that construction or the improper identification of a referent with a reference, a symbol with something which it stands for. Because our descriptions of logical work are constructed out of sentences or symbols we tend to translate all system-building interbehavior into organizations of propositions or equations. In this way generalized intellectual systems are substituted for the actual manipulations of things in organizational procedures. Furthermore, since constructions can be erected upon prior constructions, there is grave danger of accepting such systems as more independent than they really are. A totality so derived is inevitably the groundwork started from; that is, a system of some sort is adopted and then the assumption is made that by some logical procedure it is discovered.

POSTULATE VI

LOGIC IS UNIQUELY RELATED TO CULTURE

Since logic consists of actual interbehavior, it sustains unique relations with the human situations in which it occurs. In the first place, as a human enterprise, logic cannot escape certain cultural influences. For example, the totalitarian and transcending tendencies

of logic reflect theological and cosmological components of our civilization. Similarly, pragmatic and instrumental logic mirrors recent industrial and technological phases. As the history of logic indicates, both the gross and subtle changes of the civilizational milieu of logicians have given particular turn and complexion to their logical views. The same conditions have influenced the rise and decline of schools or traditions. Here we have an inevitable circumstance which can only be set aside by arbitrary verbal assertion.

On the other hand, logical interbehavior is never absolutely determined by cultural conditions. Culture is so complex and so diversified as to allow a fair measure of independence for particular logicians. Consider the opposition between Hamilton and Mill among British logicians and the conflicts between various school members in the American scene. Beneath and beyond all conventions is the world of nature which gives the possibility at least of progressively peeling off the accumulated layers of trend and tradition. It makes possible a freedom from beliefs and theories concerning natural and cultural events, including logical theory, with a resulting unencumbered orientation toward these events themselves.

The relation between a logician's work and the environing scene is, therefore, one of delicate balance. The worker assuredly is somewhat fashioned by the intellectual climate in which he lives, but, on the other hand, so great are the variations in that climate that specific workers are able to give their activities individual point and direction. Thus logical systems vary in their postulates, subject matter, particular techniques, and end results. Such variations depend upon an inclination toward mathematical, social or human, or natural-science aspects of a single cultural complex. It is for this reason, for instance, that we do not regard it necessary that the present postulates conform to the independence criterion of mathematical postulation.

Just as specificity logic is not inimical to principles, so it is not opposed to positive results or the certainty and probability which actually can be the realizable goal of the logician. Nor is the specificity conception averse to a relative infallibility, if only the worker is sensitive to particular materials, times, and places. Cultural influences need not be a bar to the definiteness and validity

of logical systems, provided we keep close to the interbehavioral situation. Particular tautological systems may be absolute, while in scientific circumstances systems may be complete and final to the moment of their construction. In the scientific field the ability to observe the worker's contact with objects and events affords a unique advantage for testing logical construction.

POSTULATE VII

LOGIC IS INSEVERABLY INTERRELATED WITH PSYCHOLOGY

If logic is the field of system-building operations, obviously the psychological factors involved cannot be overlooked nor even minimized. That psychology and logic are inseverably interrelated, however, is a proposition independent of any particular view of logic.

Certainly all naturalistic logicians—those who in favor of specific intellectual orientations reject absolute organons and canons as a basis for omniscience—appreciate the importance of psychology for logic. But even they are not sufficiently imbued with ideas of relativity and specificity to observe that while psychology is subject to rules of logic, logic is likewise subject to psychological laws.

The history of logic demonstrates not only that logical theory varies with varying notions of psychology, but also that most of the difficulties of logic are owing to inept psychological notions. This point may be illustrated by the case of Russell. In a Platonic mood he wrote:

Throughout logic and mathematics, the existence of the human mind or any other mind is totally irrelevant; mental processes are studied by means of logic, but the subject-matter of logic does not presuppose mental processes and would be equally true if there were no mental processes. It is true that in that case we should not know logic; but our knowledge must not be confounded with the truths which we know.⁴

Upon the psychological premises revealed in this quotation the conclusion is only to be expected. Russell himself, however, has seen the consequences of transcending the limits of human interbehavior, and has pointed out the wreckage in the wake of those who believe that by means of abstractions and implications they can

⁴ Discussion, p. 812; Principles, p. 4.

validly extrapolate beyond any actual data. We need only be reminded of the transfinite paradoxes and ineptitudes checking the progress of those attempting to proceed with infinite numbers of steps.

Accordingly, Russell has utterly repudiated this kind of logic, and espouses the extreme that "logic is dependent upon psychology."⁶ The conversion apparently followed the recognition that logicians could not escape the meanings of words or symbols, as indicated by the following:

The problem of meaning is one which seems to me to have been unduly neglected by logicians; it was this problem which first led me, about twenty years ago, to abandon the anti-psychological opinions in which I had previously believed. The problem is a difficult one to state rightly, because we are concerned with the relation between words and things that are not words.⁷

Unfortunately, however, this change of heart does not involve an abjuration of unlimited abstractionism. Russell's resort to psychology, then, amounts only to a swing of the pendulum away from medieval realism to nominalism.

Logical constants, therefore, if we are able to say anything definite about them, must be treated as part of language, not as part of what the language speaks about. In this way, logic becomes much more linguistic than I believed it to be at the time when I wrote the "Principles."⁸

Nevertheless, Russell himself realizes how unsatisfactory this linguistic or nominalistic view is, since he declares: "How far it is possible to go in the direction of nominalism remains, to my mind, an unsolved question." To adopt this linguistic position would be to accept the ideas of the formalists of whom Russell says:

They are like a watchmaker who is so absorbed in making his watches look pretty that he has forgotten their purpose in telling time, and has therefore omitted to insert any works.⁹

⁶ Cf. Symposium, p. 43.

⁷ *Ibid.*

⁸ Principles, Introduction, p. xi.

⁹ *Ibid.*, p. vi.

Howsoever unsatisfactory the situation still remains, it is fatal to neglect the place of psychology in logic.

Clearly, logic is not psychology; each has its own distinct field of investigation. Obviously, if logic is not psychology, neither is it non- nor anti-psychological. Upon the well-established principle that no absolute barriers mark off any discipline from another, it is wise for logicians to accept whatever positive aid psychology offers in constructing logical systems. Naturally the alliance suggested presupposes a different kind of psychology from the historical approximations. Both the traditional submergence of logic in psychology—as among the sensationists, for example, Mill and his successors—as well as the rejection of all contact between logic and psychology, as by the formalists and realists, are based upon outworn psychological notions.

Not that psychology alone has important bearings for logic. Since logicians are cultural organisms, biology and anthropology likewise weigh heavily among the related disciplines, though a fair case may be made for a greater intimacy between psychology and logic than between logic and the other two. So far, however, we have stressed the side of the logical individual. Because logical interbehavior, whether regarded as reasoning or system building, involves materials or objects to work upon, one may well argue that the success of the work depends as much upon the objects as upon the operations—if, indeed, the two can be so far separated. Let us grant, then, that there really is no closer connection between logic and psychology than between logic and physics, chemistry, geology, or even nonscientific areas of events.

Logicians generally fail to realize that like physics, chemistry, mathematics, etc., psychology also has become a natural science. At any rate, an objective psychology is in course of evolution, which is neither concerned with psychic states nor with biological functions, but with the observable and experimental behavior of organisms with stimulus objects in definite natural and human fields.⁹ It is only such a naturalistic psychology that can aid logicians in building significant and valid systems and achieving their realizable goals of certainty and probability.

The significance of the present postulate may be illustrated both

⁹ Cf. Chap. 5.

by a negative and a positive example. As to the former, objective psychology can dispose of the age-old problem of universals which still plagues logical workers.

(a) Realists stress stimulus objects, though they endow them and their qualities with a substantial existence far removed from actual human interbehavior. This extrapolating process is especially clear when these objects are constructions or logical products, as in mathematics.

(b) Conceptualists are inclined so far toward responses as to absorb stimulus objects and to make responses themselves mentalistic. Universals for the conceptualist, then, are concepts and ideas with a tenuous if any connection with natural objects.

(c) It is characteristic of mentalistic logicians that whatever they cannot assimilate to sensations they put into words or names. Hence arises the nominalistic tradition which reduces universals to words, thus missing the whole natural fact of constructional behavior based upon actual contacts with stimulus objects, including mathematical relations.

The positive contribution of psychology consists of nothing less than aiding logicians to observe that every variety of system building involves specific interbehaviors of persons with things. In detail, this amounts to revealing such facts as: (1) the difference between acts of repeating mathematical operations and the verbal symbolic construction of infinite domains, (2) the limits of verbal or symbolic abstraction of qualities and relations, and (3) in general the nature of meaning, knowledge, and assertions as acts, and their relations to the natural or cultural things interbehaved with.

POSTULATE VIII

LOGIC IS DISTINCT FROM LANGUAGE

Specificity logic as compared with traditional generalistic logic postulates an inverse connection between logic and language. This means essentially that logic is not exclusively nor even primarily concerned with terms, symbols, sentences, or any other linguistic things. Contrary to logical tradition, for the most part symbols, sentences, or statements are only means for referring to logical operations or for recording the results of system building. This is, to be sure, not a negligible relationship, and it is precisely because

system building is so closely connected with language that we must scrupulously guard against confusing the two.

The significance of this postulate is clarified by considering DeMorgan's¹⁰ assertion that formal logic is concerned with names, that is, terms and not with either the *ideas* or *things* to which the names belong. "The reality of logic is the examination of the use of *is* and *is not*." But we need not stop with this extreme view. We object just as much to the notion that the subject matter of logic is the meaning or implication of sentences, a notion which connects logic so closely with grammar. We need not be impressed with the fact stated by Cohen and Nagel¹¹ that "it is not always easy to draw a sharp line between the grammatical and the logical writings of philosophers like Aristotle, Duns Scotus and C. S. Peirce."

In point here also is the following statement of Russell. "The study of grammar, in my opinion, is capable of throwing far more light on philosophical questions than is commonly supposed by philosophers."¹²

And again, "I believe, partly by means of the study of syntax, we can arrive at considerable knowledge concerning the structure of the world."¹³

We should not deny that many systems can be built out of equations or sentences or even symbols, words, and letters, but this is not to overlook that the system-building process, and not the materials built with, characterizes logic.

That linguistic facts are closely related to logical behavior and products follows from the fact that language is an integral feature of every complex human situation. Note that language comprises both activities and things. Thing language consists of characters of various sorts (words, signs, propositions, equations) which serve as records and reminders. Activity language constitutes interbehavior which may include processes of recording natural or human events. Now both of these types of happenings may be the raw materials for, as well as the records of, system building. Yet, if serious errors are to be avoided, system-building interbehavior

¹⁰ Formal Logic, p. 42 f.

¹¹ Introduction, p. 17.

¹² Principles, p. 42.

¹³ Inquiry, p. 438.

must be kept sharply distinct from linguistic items, whether (1) acts (referential or symbolic operations) or (2) things (records, items, and carriers of civilization). Mathematics, for example, may be differentiated into (1) system-building operations—say, the development of an algebra, (2) the activities of referring to certain postulates or relations, and (3) the equations and treatise embodying the results.

Many historical logical difficulties can be traced to the misconception and misuse of language. For instance, the fundamental technique employed by thinkers to lift themselves out of concrete inter-behavioral fields is, of course, the manipulation of words or symbols. Similarly, not only are symbols and propositions regarded as stable objects, as they surely are in some of their dimensions, but also thought to lend reality and permanence to what they refer to or stand for.

There is a transparent continuity in the logos conception which begins with the identification of God or the universe with the word and the various procedures of achieving totality, permanence, and finality by the use of words or symbols. This perennial preoccupation of logicians with words is not merely a reflection of the inevitable place of verbal behavior in carrying out inquiries, but a rationalistic notion that "languages are the best mirrors of the human mind," as Leibniz¹⁴ declared.

An interesting recent version of the logos idea is found in Dewey's statement that all culture and all that culture involves is a product of language, since language is the only means of retaining and transmitting to subsequent generations acquired skills, acquired information, and acquired habits.¹⁵ Concerning language he says:

Language includes also not only gestures, but rites, ceremonies, monuments, and the products of industrial and fine arts . . . a tool or machine, for example, . . . is also a mode of language. For it "says" something, to those who understand it.¹⁶

Not only must we not identify logic and language, but if language is to help and not hinder our logical work we must develop

¹⁴ *New Essays*, p. 368.

¹⁵ *Logic*, p. 56.

¹⁶ *Ib. id.*, p. 46.

adequate notions about language. Simply because some statements are meaningless we need not set up such verbontological conclusions as "logic has come to be recognized as being essentially the same as the theory of the formal structure of languages";¹⁷ or believing that all of science may be reduced to language or symbolism.¹⁸ Such verbontological views may be traced back to Condillac's "une science bien traitée n'est qu'une langue bien faite,"¹⁹ and still further to Locke, Hobbes, and the Scholastics. It is views of this nature which lie at the basis of illegitimate finalities and generalities.

In concluding the summary of specificity postulates we claim that they not only are in line with actual logical work but also set no improper limits to such work. To recognize that logic is a distinctly human enterprise operating in specific human situations is no deprivation, but rather an aid in carrying out legitimate logical purposes. It results in keeping in touch with actual conditions and in dealing scientifically with language and thought. What we lose is merely the delusion of infinite powers; what we gain is the realization that certain motives and conventions influence our selection and organization of factors in particular fields of work.

Although we have not clung too closely to the convention that postulates should be stated in positive rather than in negative terms we add a few sentences to indicate what logic is not.

Logic is neither science nor the theory of inquiry or investigation. The notion that logic discovers and applies rules for scientific work is objectionable on at least two grounds. First, such a view is based upon the idea that system and order, in other words, abstractions and rules, are more important than the materials and actual investigation. Actually contacts with events may require new and as yet completely unknown techniques. Secondly, the particular processes of investigation with their errors and missteps are minimized. This view is also fatally reminiscent of formalistic and deductive ideas of science. There is a science of logic as well as a logic of science.

¹⁷ Helmer, *Languages*.

¹⁸ "The scientific fact is only the crude fact translated into a convenient language." Poincaré, *Foundations*, p. 33.

¹⁹ *Oeuvres*, vol. XXIII, p. 7.

CHAPTER II

THE PSYCHANTHROPIC FOUNDATION OF LOGIC

THE PSYCHOLOGICAL AND CULTURAL FRAMEWORK OF LOGIC

LOGIC is the work of man. Naturally, then, the activities comprising this work along with the resulting products inevitably reflect his psychological and cultural frames of reference. We have only to glance at the doctrinal history of logic to have demonstrated that particular logical tenets and systems are conditioned by the ideology of the thinkers involved.

What a logician believes systems to be, and his notion of how they are built up depend, for instance, upon his knowledge and understanding of the nature of mentality. Is reasoning for him a power or force necessarily presupposed by all existing things and the processes of representing them? Does he, like the absolutist, consider mind as the universe aware of itself, in the sense of identifying mental phenomena with an all-pervading *Nous* or *World Reason*? Or, like the instrumentalist, does he regard mind as a distinctly human form of action in a particular and concrete world of events? Is there for the logician an objective world or only mind in which all cosmic events merely play parts, as in perennial and universal dreaming? Further, does the logician think about actual events and things, including himself, or does thinking in itself exhaust the whole of being or existence?

Similarly, we may ask what view the logician holds concerning language and symbols. Are words the bearers or correlates of psychic processes called thoughts? Or is language referential behavior and the transcription of such references; or are words identical with things? Again, are symbols signs of absolute reals somehow pictured or represented, or are these symbols products of actions with a referential (descriptive) or representational (constructional) connection with things whether natural or culturally created? Such are only a few of the views which the varying changes of human circumstances have introduced into the historical records.

To know the influences operating in the formation of any type of logic is not only to understand that logic but also its intimate

relations to other disciplines. Does the logician, for instance, derive his views from the mathematical disciplines, the biological sciences or from the fields of rhetoric, law, argument, and debate? Incidentally, too, the form in which the logic is set up, whether verbal or symbolic, is conditioned by psychanthropic influences.

The logician perhaps more than other intellectual workers, with the possible exception of the mathematician, has unwittingly assumed that he operates in more or less complete isolation from human circumstances. Such an assumption is decidedly erroneous. Nothing is more certain than that differences in logical doctrine are ascribable to the influence of social, political, historical, and specific scientific conditions. To neglect these points is to risk the danger of embracing a theory developed under particular cultural circumstances, only to regard it as the one universal and sufficient doctrine.

THE CULTURAL BASIS OF LOGIC

If we are effectively to examine logic as it arises out of its cultural matrix and see how changes in the latter correspond to differences in the former we must briefly examine in what culture consists.

For our purpose we take culture to be civilization—in other words, the activities and institutions of a set of people, whether a society, nation or larger ethnic unit. Institutions we understand as the objects which are partially or wholly transformed natural things and those human artifacts which are constructed to facilitate human enterprises. The latter may be forms of action (customs, habits) with or without direct relation to natural events. In sum, then, culture consists of the activities of groups of individuals and the constructions arising from those actions.

Institutions, though comprising behavior or behavioral products, when formulated and embodied exist beyond the lives of individuals. Moreover, they serve as models and bases for the culturalization of newcomers to the society which harbors them. In this way they continue a more or less autonomic existence, even though their variations make them into mere links in a chain of tradition.

Logic, whether practice (logicizing) or systems (logical products), like social organizations, religions, laws, languages, etc., must be counted among such institutions. Logical institutions, just as all others, have arisen out of certain activities with resulting products.

Throughout their development, which, when Western logic is considered, we may date from the time of the Greeks, they have been added to and modified in addition to being preserved and cherished.

We have postulated that the perennial and essential feature of logic which justifies a common institutional name is simply that of system building. Logic is organizational behavior tending toward intellectual or other kinds of systems. This system-building process is the nucleus of all logical institutions. The outer layers, with their detailed differences which may have longer or shorter lives, consist of views about logic, logical treatises, conventions and techniques of various sorts, or variations of material which are organized into logical systems.

Obviously, then, since we are dealing with what are fundamentally products of human behavior, logical institutions may be studied in their variations from one cultural period to another or in the civilizational cross sections of a particular time.¹ The period-to-period institutional variations are illustrated by the popular notion that in all periods logicians can be divided into varying types of Platonists or Aristotelians. That is to say, some logicians favor forms and abstractions; others incline toward the concrete or material elements. The cross-sectional variants of logic are exemplified by the differing views prevailing among a congress of logicians or by a set of treatises.

LOGICAL HISTORY AS THE ORIGIN AND VARIATION OF LOGICAL INSTITUTIONS

Our study of logic as a psychanthropic product will proceed along three distinct but interrelated lines. First, we shall consider the historical rise of logical study as the development of a particular type of intellectual institution. The result of this development is conventional logic as evolved under the auspices of Western European culture. Secondly, we shall investigate the development of technical processes such as universal definition, concept formation, categorization, and inductive and deductive inference. The origin of these processes we study as particular features of logic as a general discipline. Our third line of inquiry, which will occupy us in the following chapter, deals with types of doctrines

¹ The current cross section of logic we shall consider in chap. 4.

concerning the nature and function of logic. These different doctrines, such as the Aristotelian formal logic or the Herschel-Mill methodology of science, imply times and places. Also, they correspond to differing theories concerning (1) psychology in general, (2) reasoning and thought, and (3) language and symbolism.

Among mortals it is axiomatic that there is no enterprise without risk. Whoever attempts to trace out the evolutionary sweep of any type of cultural institution is necessarily bound to overlook if not to be ignorant of a good portion of the particular details of his material. All he has to guide him are the surviving records, and these unfortunately provide only indications and trends. What actually happened in the full, rich ebb and flow of events he can only surmise; especially must he miss what particular individuals have wrought. What the records reveal are the doctrines which by dint of circumstance become established and propagated at the expense of sometimes better and sometimes poorer specimens.²

A. *Logic as a General Intellectual Institution*

The history of logic, in common with all histories, constitutes a conjoint construction by a set of historians. These historians by their selective and descriptive labors influence each other and thus cooperate in producing a product presumed to represent the development of a thing or set of events. Such products are, of course, embodied in a set of texts. Logicians conspire (we use this word advisedly) to take the Aristotelian writings not merely as the first great system of logic, but as evidence for the discovery of logic itself. Recall here Kant's famous assertion³ that since Aristotle logic has not had to retrace a single step, nor up to Kant's day made any step in advance, so that to all appearances it could be considered as completed and perfect.

Actually, of course, such systems constitute accumulations of ideas, views, and doctrines which through formulation become embodied and fixed. A similar, definitely traceable embodiment is the formulation and codification of geometry in the Euclidean Elements. It seems a traditionally hallowed procedure to take

² The writer here takes issue with Kant's (Jäsche, Immanuel Kant's *Logik*, Einleitung, sec. IX) opinion that "only that survives which has an intrinsic value" through the sifting of time.

³ Critique, Preface.

these codifications and systems simply as existing without regard to the labor and trials that went into their making. For this reason history is sometimes described as the discipline which both rejects and forgets. But we must keep in mind not only the labors of those who have effectively made these products, but also the stirring events which constituted the matrix of their work.

When we differentiate between the history of logic as the summarized and depersonalized depiction of the development of logical activities and products and the actual performances and results achieved, we cannot escape the fact that the latter concern the work of particular logicians. We are able, then, to take account of the way logicians have behaved in particular situations, how their motives and purposes have become embodied in their products. As already indicated, our present inquiry concerns the cultural and other influences which constitute not merely the *mise en scène* but likewise the *raison d'être* of any logical performance. Though we cannot do other than abide by the selective and eliminative character of history we may project a series of problems that will throw light on the nature of logic as an intellectual institution.

A legitimate question is whether logic is exclusively or primarily a Greek construction. Scholars are not lacking who consider the possibility of an Oriental contribution.⁴ It is no less legitimate to suggest that possibly we are missing information concerning the nature of logic when we ignore developments of system construction before the conventional Greek origins and in other places at the same time. As to pre-Greek origins we may be obliged to search in other than conventional propositions, say, in early mathematical, legal or other fields. Archaeologists are making such headway in uncovering evidences of cultures in the Near and Far East, long antedating the Greeks, that we may certainly expect information from such sources. The early dating of other institutions such as law, social organization, and art certainly encourages such expectations.

In what sense is Aristotle⁵ justified in claiming to have founded a new science, even allowing for some beginnings by others? And what are the relative merits of the views that Aristotle is the

⁴ Cf. Ueberweg, *System*.

⁵ *De Sophistica Elenchus*, 183 ff.

father of logic or only a compiler and systematizer?⁶ Here also arises the question how far such a problem can be arbitrated by a consideration of the cultural needs and opportunities that could give rise to logical institutions. Is it possible that Western European writers are accustomed to trace not only logic but most of our other institutions to the Greeks simply because we are the inheritors of a Greek legacy? Of course, Shelley exaggerated when he declared "we are Greeks; our laws, our religion, our art, have their roots in Greece," but he was not wholly wrong. Now it is incumbent upon us not merely to discover what we do and do not owe to the Greeks, but, still more, not to call logic that which we inherited as Greek institutions, without considering where the Greeks obtained them or what other cultures, including our own, contributed to our logical theory and practice. In view of what we are learning of other ancient civilizations we must become cautious of regarding Greece as exclusively "the mother country of the mind."

Whether or not our study of logical origins begins with the Greeks we must search for and discover the concrete human problems stimulating the need for terms and their organization into valid systems. Logic as a discipline arose from the observation of what individuals did when they attained to satisfactory or valid conclusions. For example, Aristotle set up a series of rules which he thought essential for achieving results in argument, discussion, and science. A trenchant observation is that it mattered not to Aristotle what materials were dealt with. The fundamental thing was organization and system, bringing things together, so that their relationships could be observed. In this sense the Aristotelian syllogism must be looked upon as a schema for organization, bringing terms together through the middle. It is for this reason that Aristotle regarded the various modes of the syllogism simply as variations of the first, which was the perfect and complete one. Notice that Aristotle's description of the syllogism as "discourse in which certain things being posited, something else than what is posited necessarily follows from them," clearly indicates this systematizing and organizing feature. That this system operates in argument, debate, or scientific demonstration was a matter of indifference to him.

⁶ Enriques, *Historic Development*, p. 4.

⁷ *Analytica Priora*, I, 1, 24b, 18; *Topica*, I, 1, 100a, 25.

It is interesting to note Joseph's⁸ criticism that Aristotle's definition or description of the syllogism is too wide. Joseph wants to confine a syllogism to an argument in which the terms must be subject and predicate. In other words, he wants to stress more the linguistic factors. This criticism is, of course, based upon a very different type of cultural period from that of Aristotle's. Joseph is concerned with a notion of thought and its relation to language, a notion completely foreign to Aristotle. One justification, however, is that perhaps when logic is to be dealt with as an institutional product the sentential or subject-predicate form of description is worth stressing.

In the meantime the establishment of logical institutions, in other words, the setting up of a logic as an impedimentum of culture, must be made in linguistic terms, in terms of statements which are themselves traditionally constructed out of subjects and predicates. This really is saying that if an institution is established the only way it can be referred to is by means of language. However, we stress the fact that the tenor of Aristotle's discussions are inter-behavioral in that he was simply discussing the procedures of persons in specific situations. In this case the specific situation might be called general in the sense of a systematization.

The followers of Aristotle, especially the logicians of the Middle Ages, made absolute what was originally set down merely as a schema for regulating particular sorts of conduct. What was in Aristotle a set of treatises or perhaps even only lecture notes dealing with various phases of language (de Categoriae, de Interpretatione), inference, syllogism, probable knowledge, argument, etc., became codified into the Organon. Probably those who regard the Organon as a propaedeutic to the various sciences or intellectual activity in general are more correct than those who consider it to be the fundamental technique of reasoning. Even the propaedeutic view is undoubtedly somewhat overdrawn, inasmuch as we must not mistake the Greek search for principles as statements of absolute and totalitarian systems.

Logic as Organon later came to be the description of the faculty of reason. The descriptive terms employed to formulate the nature of the Organon became the representatives of the reasoning faculty, which gradually shifted in description from the operation of

⁸ Introduction, p. 249.

world reason to the inevitable reasoning power of individuals. An interesting modification of the Aristotelian concept of the faculty took place. Whereas for Aristotle faculty meant simply department of inquiry,⁹ it later was transmuted into a power of mind or intelligence. Logic then gradually evolved into the science of thought or reason, and since it was so closely connected with the ultimate powers of reasoning, logic became *Scientia Scientiarum*.

B. *Evolution of Logical Elements, Techniques, and Principles*

Just as the general institutional character of logic became established by a cultural fixation (linguistic and otherwise) of activities (arguments, systematizations, classifications, etc.), and of persons in particular situations, so a similar institutionalization resulted in the fixation of particular phases or items of logic. The latter constitute the separate elements, processes, techniques, and principles making up the total logical complex. As the history of logic indicates, the construction of these separate phases of logic proceeded upon the basis of operations upon logic as an object. These operations consisted of reflecting upon it, defining it, analyzing and improving it. Thus the historian of logic concerned with the origin and development of these particular phases as they are conventionally formulated must inquire into the cultural background of their development.

Instances of the elements of logic and their nature are obtainable from an examination of the treatises comprising the Aristotelian *Organon*. For example, the two *Analytics* treat of the syllogism; the *Prior Analytics*, as a generalized process, in other words, as pure or formal, with consistency as a criterion. The *Posterior Analytics* is concerned not only with consistency but with truth as well. The *De Categoriae* is presumed to deal primarily with terms, while *De Interpretatione* is concerned primarily with sentences and propositions. The *Topica* and *Sophisticis Elenchis* are regarded as studies of the correct application of sentences and are presumed to handle the probable and the inductive basis of logic or in general Dialectic as against Analytic.

I. *Terms and Definitions*. Terms as special logical items are traced back to Aristotle's *De Categoriae*, although prior lists of categories are discoverable in the Platonic dialogues, especially the

⁹ *Analytica Priora*, I, 1.

Sophist 251, particularly 251d and the Theaetetus 193. In a unique sense, then, terms are considered as the atoms of logic and have become the basis for the universals and abstractions required to obtain principles of certainty amidst the flux of actual events.

In the Greek, especially the Aristotelian situation, it is clear that terms are indifferently simple, everyday linguistic elements, the basis of grammatical analysis, scientific names and categories in the sense of the analysis of things. The innumerable attempts to fixate these factors into the logics of the succeeding centuries are obviously subject to the criticism that what in the Aristotelian period were simply trial and error products of comparatively elementary reflections are transmuted into most sophisticated features of formalized systems. In succeeding ages these terms or categories have been made into concepts as a consequence of the efforts to relate terms and symbols to ideas or perhaps identify them. Such an evolution of terms in logical writings can be readily understood as mutations reflected by the changing cultural scene.

Aristotle¹⁰ attributes to Socrates the earliest definite attempt to seek for definitions, though Democritus touched upon the subject in the field of physics. Because the historians of logic are universalists and absolutists they look upon this search for and the setting up of definitions as a process of establishing certainty. Since such certainty as well as logic itself is essentially concerned with forms this process of definition is regarded as differentiating the formal and material features of events. Only by this distinction of formal and material is it believed that principles can be discovered and employed.

The problem of the origin of definition as a process is also connected with the problem of the One and the Many. In a sense Socrates is treated as answering Antisthenes and others of his ilk. Whereas Antisthenes agreed that predication is impossible and that every truly existing thing could only be defined through itself, Socrates aimed at the discovery of the abiding and essential character possessed in common by any number of things.

Depending, however, upon the cultural framework of these historians, and in a sense all logicians are historians, they have differed in their notions of the nature of both form and matter. Sometimes both form and matter were aspects of things; very fre-

¹⁰ *Metaphysics*, 1078b.

quently the forms were mental or reasoning principles imposed upon the content of things, and often both the forms and material content were mental.

From our standpoint this distinction consists of a product of interbehavior with things. Formal and material are abstracted characteristics of things or events, such that their likenesses and differences can be indicated and compared with those of other objects and events. It is a definite type of activity which hypostatizes these likenesses and differences into principles regarded as operative in events or which can be used to control such happenings. This hypostatization results in the development of such constructions as the empirical and the *a priori*.

Forms and the *a priori* whether taken as formalizations of activity or the essences of things have been inseparably connected with relations between events. As such they have been regarded as basic to principles, anticipative or prophetic of events. In this connection arose the whole tradition of universals, ultimate forms or categories, which have played so large a part in traditional logic.

Granting this historical item as valid, what actually must be recorded here is a form of behavior, an occupation designed for intercommunicating, understanding, and agreeing upon certain things and events. Obviously, this activity is performed as linguistic behavior. It is entirely proper to say that we have here the origin of the logical interest in terms and their sentential combinations, but we must not overlook the fundamental kernel of this development—namely, the activities of individuals in carrying out certain purposes. In other words, we must consider the relationship between the conversation items of the protagonists in the Dialogues and in consequence the relationship between terms and the events which they originally referred to or summed up and described.

II. *The Inductive Process*. Aristotle as one of our first historians of science attributes to Socrates the discovery of the inductive process.¹¹ This is undoubtedly one of those historical attributions that by selection and recording fixates into a single, stable, and unified structure what may well have been different sorts of behavioral events. The most one might say is that probably Socrates through the Platonic treatment of him may be regarded as the

¹¹ *Metaphysics*, 1078b.

formulator of the technique of arriving at general notions or principles useful in those dialectical enterprises in which stable definitions are required or desirable. We may then closely connect the name of Socrates with the process of inductive definition.

As a matter of fact, Aristotle himself treated a number of distinctive behavioral items under this general heading. At times, of course, induction refers to the simple achievement of terms, definitions or universals.¹⁸ Joseph,¹⁹ however, points out that in this connection the term *epagoge* does not occur.

Again, induction is the partial or propaedeutic process antedating all development of deductive reasoning.²⁰ Here induction is an inferential process by which universal propositions are attained by complete enumeration. This form of perfect induction can be thrown into the form of a syllogism by means of which the first and immediate premise can be established.

Still another form of induction is that of an observational procedure quite at variance with deduction in which the individual through contact with particular instances is led to appreciate the universal,²¹ or the universal appears or is induced by the evidence of particular instances.²²

In the form of induction there is a complete departure from the formal handling of premises and conclusions. Here first premises or principles are obtained by behavior variously described as insight or intuition. If logic is taken to be a single enterprise it must be accorded two divisions. Besides the syllogistic interrelation of premises and conclusions, which is the analytic division, there is another phase which inquires into the origin of the premises. This is the dialectic division, its primary aim being to discover valid bases for the insight and judgment of the logician. But is it necessary to regard logic as one enterprise?

Assuredly, this diversity of description of the inductive process and the sharp diremption between Analytic and Dialectic have been a source of considerable unhappiness on the part of logicians who take absolutistic and universalistic positions. Although since

¹⁸ *Analytica Posteriora*, II, 13, 7ff.

¹⁹ Introduction, p. 379.

²⁰ *Analytica Priora*, II, 24, 62b, 15ff.

²¹ *Analytica Posteriora*, I, 1, 71a, 21ff; 18, 81b, 16f.

²² *Topica* I, 18, 108b, 11.

Aristotle's time the existence of inductive and deductive logic should have made for openmindedness on this point, we still find that writers do not take a simple inspective view of Aristotelian induction or, for that matter, logic in general. Certainly it is clear that Aristotle simply discusses the various activities of persons in argument and debate, their search for scientific principles and organization of propositions. It is perhaps more than unfair (because it impedes our understanding) to impose a fixity and an in-harmonious unity upon the Aristotelian writings because of the later development of a universalistic tradition.

Fundamentally, then, induction whether under that name or the name of dialectic, was simply a process of passing from particulars to generals. This process took many forms and was not even confined to propositions. As we have indicated, in the Aristotelian writings the process had to do with proving points against opponents in debate, achieving scientific principles, and developing definitions, etc. It is an interesting cultural development that the Renaissance logicians who elevated induction into a basic logical or methodological process, presumably as a complete contrast to the analytical or formal logic of Aristotle, did not themselves agree upon the nature of the inductive process. Surely the implication that it is a single type of logical procedure, general enough to be employed in all sorts of situations and in connection with all sorts of things, is not demonstrable.

III. *The Postulation Principle.* Of great importance in the study of the origin of logic is the development of the postulation principle. This principle as the process of constructing or adopting assumptions either in argument, demonstration, or as a necessary feature of system building we have already touched upon in the previous chapter. Despite the importance of postulation conventional historians of logic almost entirely neglect the subject. For the most part, they assume that logic is *one* thing—namely, the formalized, demonstrative or reasoning process represented by syllogistic logic. As a result they ignore the system-building procedure of the mathematicians as well as the inductive phases of Greek dialectics.

Indeed, the whole subject of assumptions, hypotheses, and postulates as it appears in Greek logic receives no attention, though it is impossible to overlook the fact that these items were certainly

considered important by the Greeks who studied them. It must not be forgotten that Greek logic developed in close intimacy with mathematics and especially with geometry. It is obvious, of course, that the systematic origin of geometry could only be based upon postulation as a basic feature. Here we may be reminded of the five postulates which Euclid incorporated in his *Elements*. He regarded it as necessary for his system not only to set down certain definitions and to acknowledge certain axioms, but also to suggest that five constructional processes be allowed.

When we examine Euclid's five postulates it is plain that they are not exactly alike.

1. It shall be possible to draw a straight line joining any two points.
2. A terminated straight line may be extended without limit in either direction.
3. It shall be possible to draw a circle with given center and through a given point.
4. All right angles are equal.
5. If two straight lines in a plane meet another straight line in the plane so that the sum of the interior angles on the same side of the latter straight line is less than two right angles, then the two straight lines will meet on that side of the latter straight line.

For instance, the fourth postulate may be taken as a definition or simple device concerning how to draw figures. The fifth obviously is freighted with considerably more than simple constructional assumption. Here is justification for the assertion that the Euclidean system is neither a system of mensuration nor a geometry, but a mixture of the two.¹¹

The discrepancies found in Aristotle's various references to postulates and his differences from Euclid both indicate the importance that postulational principles assumed among Greek thinkers. That these principles did not carry equal importance until the 19th century is explained by the absolutistic character which logic took on after the Greek period.

Furthermore, such criticisms as we now find of the Greek notion of postulation¹² similarly indicate the significance of postulation

¹¹ Cf. Jeffries, *Scientific Inference*, p. 108.

¹² Enriques, *Historic Development*, p. 20 ff.

among the Greek logicians. Inevitably, of course, their ideas were different from those now possible. Enriques is quite right in pointing out that the Greek conception of definitions, axioms, and postulates no longer corresponds to the hypothetico-deductive form of modern mathematical science. Though he does grant that the constructional feature of the latter is implicit in Greek logic the differences indicate the great changes in the cultural conditions of the two periods. On the other hand, it is a mistake to assume that the postulational theories of the modern mathematical period are the exclusive and most correct theories. Certainly it is not uncommon today to transform constructive processes into absolute elements. True enough, the Greeks were absolutistic in their views and could only conceive of one geometric system, one, moreover, closely connected with intuitive ideas of space. But the very fact that they were so bounded by concrete events made possible a notion of general system building in logic with the consequence that logic was not made into a single universal type of system.

IV. *The Deductive Process.* Formal logicians credit Aristotle with the first establishment of the theory of the syllogism. Even the first use of the term *syllogism* in its inferential sense is attributed to him.¹⁹ This credit is presumed to be much more fundamental than appears on the surface, for the syllogism is regarded by such logicians as the *fons et origo* of all reasoning and hence of all logic. The syllogism has become the model of the deductive process, a process which itself has become the basis of logical inference. This inferential process is supposed to yield certainty of knowledge and reflection, and even those who connect deduction with induction or make induction prior to deduction really consider deduction as more fundamental. Thus it has been the fate of the syllogistic or deductive process to be made the keystone of all the conceptions of logic as universal and absolute processes. Yet originally deduction was simply the organizational principle whereby rather rigid relationships between accepted propositions could be organized. With Aristotle the syllogism was in no sense absolutistic. It was a technique of discourse the elements of which (sentences) might be entirely linguistic or refer to commonsense objects. Certainly the definitions of reasoning and the syllogism²⁰

¹⁹ Ross, Aristotle, p. 32, also Kapp, Greek Foundations, p. 70.

²⁰ *Analytica Priora*, I, 1, 24b, 18; *Topica*, I, 1, 100a, 25.

suggest that Aristotle is dealing with particular activities of individuals in connection with all sorts of objects and events. Deduction for him, therefore, may be regarded as verbal as well as rational.²¹

Considerable importance must be attached to the criticisms of Aristotle's treatment of the syllogism and deduction. Joseph²² asserts that his definition of the syllogism is too wide and too close to its etymological significance according to which it includes every argument in which from two truths a third is inferred. Joseph would restrict a syllogism to an argument in which from the given relation of two terms as subject and predicate to the same third term a necessary relation between the original subject-predicate term would follow. He would exclude relations of quantity, velocity, etc.

Ross,²³ on the other hand, criticizes the Aristotelian formulation of the syllogism because it was not developed with an eye to scientific procedure. According to him Aristotle did not even pay attention to mathematics, the one exact science. In other words, Ross thinks Aristotle adhered too closely to subject-predicate relations inferred from their relation to a third term. In opposition to Joseph, Ross asserts that Aristotle should have made the syllogism cover relations of equality, direction, and other mathematical inferences.

Obviously Aristotle can be read differently. Each scholar who places a high premium upon the logical value of the syllogism or deductive process can find ample justification for the lack of perfection in Aristotle's account. But why do these scholars overlook not only the casual trial and error and commonsense conditions of Aristotle's work but also the different cultural conditions under which that work was done and which were so different from their own? To us it is clear that the critics approach Aristotle's problems from a standpoint and a period when Aristotle's doctrines were formalized and absolutized beyond anything possible for an Athenian scholar.

To look upon Aristotelian logic as absolutistic one would be obliged to take into account a number of cultural developments with which Aristotle was not conversant. In the first place, there

²¹ Cf. Kapp, *Greek Foundations*, chap. 4.

²² Introduction, p. 249.

²³ Aristotle, p. 32.

was the specialization of deduction and inference in general as parts of a particular form of system building—namely, the mathematico-geometric. Secondly, words and linguistic products in general were hypostatized to make them identical with what was regarded as the essence of things. We refer, of course, to the Post-Aristotelian development of spiritualism and its linguistic representation.

After inference and the deductive method became integrated with geometry, a fixity and an omnipotence were ascribed to the syllogism. Of the work of Euclid it is said:

Definitions, axioms, and conclusions are joined together link by link as into a chain, firm and inflexible, of binding force, but also cold and hard, repellant to a productive mind and affording no room for independent activity.²⁴

With the medievalists who departed further and further from contact with objects and events in a scientific way and who therefore clung closer and closer to verbal materials, the syllogism became most abstractive, and reasoning fundamentally the inter-relationship of terms. The progressive elaboration of formalistic techniques made the terms of syllogisms so abstractive that deductive procedures could be regarded as absolute. Thus reasoning became simply the handling of terms. By description and hypostatization the syllogism developed into a rigid instrument for absolutely relating elements of systems far removed from any concrete activity with things. This notion of the syllogism in various modifications has greatly influenced formal logicians even of the present day.

Notice, however, that the notion of the syllogism as an absolutistic technique in logic contrasts markedly with its earlier role as an operational technique for building systems out of any material, linguistic or otherwise. Today with the development of the hypothetico-deductive conception of logic and mathematics the notion of the syllogism has been greatly modified, and the process of deduction in general has been given a very different treatment. Deduction has been enlarged to include inferential processes, not strictly tied up with terms. More, deduction has been made the basis for the inductive procedures of science.²⁵ Still, there is no fundamental abatement of the tremendous influence of the syllogistic or deduc-

²⁴ Brill, *Antrittsrede*, p. 198.

²⁵ Enriques, *Historic Development*, p. 238.

tive process. Nor is there a general recognition that the deductive process is a specialized technique for system building.

V. *The Proposition.*²⁶ In the matter of propositional doctrine Aristotle suffers the injustice of being at the same time regarded as the creator of a basic universal logic and as producing a faulty or at least incomplete system. On the other hand, Aristotle is looked upon as the formulator of propositions considered as the basic unit of what is essentially logic. The invention of the proposition is celebrated because it is the basis of all inference and implication. Aristotle is credited with the appreciation that the proposition is the fundamental instrument of interrelating terms, that is, the subject, predicate, and copula. In this sense the proposition is fundamentally the basis or the expression of judgment and syllogistic reasoning. At the same time, Aristotle is criticized for making the judgment too particularistic and flexible and also for connecting too closely the judgment with the special threefold system of the syllogism.

If Aristotle requires defense one could point out that his critics do not distinguish between Aristotle's own doctrines and those which became connected with them in subsequent cultural periods. For example, he is misinterpreted as making the proposition an outward expression of a personal judgment. The important thing, however, is not to assume that Aristotle needs defending, but, in the interest of understanding the origin of the doctrine of propositions, to study the evolution of logic as a cultural event and trace out the development of constructions concerning the proposition.

Beginning with the fact that the proposition is an item in the system of logical institutions we may follow its evolution in terms of the varying cultural matrices which gave it its diverse character at different times. On this basis we must look upon the Aristotelian doctrine of the proposition as amorphous, unstable, and incomplete. Perhaps it may best be regarded as an early formalization of linguistic references to make them stand out for inspection and use. When Aristotle declared that "spoken words are the symbols of thoughts and written words are the symbols of spoken words,"²⁷ he was simply referring to the elementary facts of personal actions

²⁶ In this historical sketch we do not regard it necessary to distinguish carefully between sentences and propositions.

²⁷ *De Interpretatione*, 16a.

and their conventional means of recording. Furthermore, he was merely asserting that these records could be used to go back to ideas and intentions and forward to argument and demonstration. On the whole, the important use to which the formalization of intentions and statements of these intentions can be put is to organize propositions as premises and conclusions in argument, demonstration, and science. If we do not slight the inductive features of Aristotelian logic we may regard the organization of formalized propositions or sentences into syllogisms as the apex of demonstration. In no sense, however, must we overlook that Aristotle's doctrine of propositions is concerned with the everyday relations of belief and ideas with speech and with a naive and objective doctrine of language.

Evidences for the rather indifferent and more or less chaotic treatment of propositions are found in the Aristotelian discussion of the grammatical aspects of sentences. As a matter of fact, if we regard the *De Interpretatione* as part of the *Organon* we notice that it is devoted as much to grammar and grammatical principles as to problems of reasoning, inference, etc. To point this out is no serious criticism but merely a distinction between Aristotelian developments of logical institutions and later fully formed ones. It is not a defect of Aristotle that he was a man of his own time.

In Aristotle's period thinking and reasoning were still regarded as objective activities of the individual. Thus it is no error to interpret Aristotelian propositions as simply statements of fact, assertions of science, and references to attitudes, thinking, and reasoning. The symbolic character of language, implying psychic processes and supernatural objects as the thoughts symbolized, was still gestating in the womb of time to be born only after the lapse of centuries.

Under the post-Aristotelians, sentences or propositions became the correlates or symbols of psychic things. Following the fusion of Greek thought with various Oriental trends logical sentences or propositions dealt either with pure verbal items or evolved into processes for achieving absoluteness and certainty in thinking. When Aristotelian writings penetrated far enough into the period of the Scholastics, the Aristotelian sentences and terms became the instrumentalities for the quarrels between the Nominalists, Realists, and Conceptualists. On the whole, all these discussions

centered around verbal constructions either (1) constituting the material dealt with, (2) representing existent or nonexistent (subsistential) things or (3) symbolizing psychic things (concepts) somehow connected with observable things. These quarrels, of course, were the final outcome of the various debates of the Church Fathers concerning theological problems.

The further development of sentences as items among logical institutions proceeded along several definite lines. In the first place, sentences were further abstracted and in general there was a progression from words to symbols. This development is traced back to Leibniz who performed this abstractional process by identifying propositions with mathematical statements. One fundamental result here is that reasoning which was connected with contentful language organized into syllogisms was dispensed with. Perhaps it is a distinctive feature of this evolution that the presumptive certainties of syllogistic logic were set aside in favor of the inclusion of probabilities. In a sense this change may be symbolized by the Leibnizian addition of the principle of sufficient reason to the fundamental laws of identity, contradiction, and excluded middle.

With the rebirth of formal logic in the 19th century, the symbolic character of sentences became more and more closely identified with logical equations, so that gradually the original contentful true-false criteria of sentences or assertions were replaced by their implicational character. The true-false criteria were transformed to give specific qualitative differences to propositions in a formal system.

A further development of the proposition doctrine was the propositional function which allowed full play to mathematical symbols, such that propositions were presumed to be constructed of variables instead of definite and constant terms.

VI. *Classification and Division.* Despite the relatively scanty treatment accorded classification and division in logical treatises these subjects are of fundamental importance for a thoroughgoing study of logical origins. An investigation of these processes not only indicates the changes they have sustained under varying cultural conditions, but also throws considerable light on the nature of logic itself.

As specific logical institutions classification and division like some other nonsyllogistic processes can be definitely traced back

to Aristotle's predecessors. Recall Aristotle's vigorous attack upon the Platonic exposition of the problem of division. His objections to the Platonic process of dichotomy indicate definitely that classification and division do not proceed merely on the basis of abstract form, but get down to the actual characteristics of particular things. It is significant to notice, too, that the subject of classification and division is treated by Aristotle much more fully in the *De Partibus Animalum* than in the more definite logical treatises. In this domain of logical elements we can see a satisfactory demonstration that for Aristotle the logical processes were not exclusively related to what are now regarded as formal logic, but also to language, rhetoric, and science in the sense of dealing with things.

It is probably not incorrect to say that in the earliest development of logical institutions classification and division definitely show the relationship of actual things to logical forms and, in general, to logical products. To trace these subjects back to their origins indicates how descriptions and general linguistic forms are derived from the referential function of language as interbehavioral processes.

If it is agreed that Aristotle's treatment of classification and division cleaves to the elementary activities of system building whether the materials be linguistic events, things, or abstract classes or groupings, then there is clearly visible a definite trend in another direction among his successors. This is well illustrated by the large place dichotomy assumes in the Porphyrian doctrines, for example, in the famous tree. As the history of logic teaches, the doctrine of classification and division becomes more and more concerned with words without regard to things and certainly with wide divergence from operations with things.

Under the auspices of a scientific culture, classification and division take on a variety of patterns. The old formalistic tradition maintains itself in a special compartment called Formal Logic, but with the development of science, classification and division concerned with the early stages of investigation take on a new character. It is in this sense that Jevons²⁸ declares that the value of classification is coextensive with the value of science and general reasoning. In this period classification and division again become interrelated with actual things. Under the present auspices formal

²⁸ Principles, p. 674.

logical processes are connected with what are traditionally called physical and metaphysical divisions. Actually, these two processes paralleling formal classification and division assume the role of analysis. As such they are subsumed under general though not formal logical processes on the basis that they really do not involve actual manipulation as in chemical analysis or biological dissection. For example, when in physical analysis a watch is divided into case, hands, face, and works, or when in metaphysical division the size, accuracy, color, and durability are analyzed, the activity consists of conceptual or thought processes in the conventional sense.

With the more recent developments of Formal into Symbolic Logic under the impulse of mathematical studies, classification and division acquire another set of characteristics. Now these processes are presumed to apply to abstract objects and products and do not necessarily involve the activities of handling them. The materials dealt with are classes or compartments which may be occupied or empty. According to Venn¹⁹ symbolic classification and division arise directly out of the classical process. They are bifurcate, dealing with nothing but formal contradictions, but the great improvement is that every alternative that the given terms will admit are taken account of. In his words:

Suppose we start with a class S , and that we are concerned with three attributes which shall serve as the bases of division, viz. X , Y , and Z . We divide S into A and not- X . We then proceed to subdivide both of these by the introduction of Y , thus obtaining four classes. Introduce the third attribute Z , and make the same division again, and we get eight resultant classes. And this we might continue doing with any number of such dividing attributes.²⁰

Aside from completeness and accuracy of method what is further claimed is that symbolic division and classification by their formality avoid the objection that one may be making classes, though the objects or materials dealt with do not possess such class traits, while, on the other hand, they offer a generally more powerful pair of processes. Jevons' protestation that he is dealing with logic and not mathematics would no doubt be regarded by mathe-

¹⁹ Empirical Logic, chap. 12, 13

²⁰ *Ibid.*, p. 319.

matical logicians as an unnecessary and even debilitating assertion.

The process of classification and division whether in the Aristotelian or Symbolic tradition in a unique way marks them as partial institutions in the general field of system building. Because these processes are less complicated than the syllogistic they better display the methods of organization of words or things into systems. This is true even when both of these processes are taken to be the reverse of system building. Even if they are regarded as analytic they still show, starting from the top of a completed system, how the binding materials or procedures are employed in its construction. Certainly, classification, beginning with individuals to be grouped into *infima* species classes, then into *genera* and finally to *summum genus*, indicates the basis and process for selecting materials to construct a system.

VII. *Laws and Canons*. The procedures followed in the development of the "laws" and "canons" of reasoning or logic are similar to those employed by Greek thinkers to create a Logic or Organon out of various modes of action in the fields of rhetoric, inference, grammar, and general system building. This procedure can be easily observed by following the details during the construction of the canons and laws presumed to govern and regulate reasoning.

Roughly the procedure begins with the fixation of actions by linguistic description. In the early descriptive stages the verbal or linguistic formulation still maintains a close connection with what is done—actual events. Later, through the discussion of the correctness of such descriptions more and more rigid formulation is achieved, so that finally what were simply references to procedure become rules or laws for that procedure. The basic principle is increased generalizing with accompanying formalization, such that various normative principles become instituted in linguistic organization. When this stage is reached there is little or no connection between actual inferential or authentic logical forms of interbehavior and the rules or laws. The whole set of logical institutions recedes further and further from actual events.

The complex formalizing procedure is excellently illustrated in the development of the Aristotelian logical tradition. For example, in division or analysis various commonsense procedures become established as definite rules simply by linguistic formulation.

We may cite here the following rules: (a) a division must be based upon some criterion or *fundamental distinction*, (b) the division must be exhaustive or include all the elements, and (c) the elements analyzed or divided off must be unlike each other. Clearly these rules are simply commonsense suggestions to be followed if a simple organization of materials or elements is to be accomplished. Analysis or division, then, becomes a technical procedure. Through the description of the process one may devitalize or desubstantiate the material divided or analyzed. The description or formulation of the abstracted process, then, can be made into prescriptive rules or principles.

Because the evolution of logical institutions is similar to that observed in other departments of culture we may compare the present situation with that obtaining in the domain of law in which actions that become common or culturally habitual are described or perhaps pointed to because a noticeable departure occurs or for some other reason. A neighbor moves a boundary stone which is objected to. The result is the assertion that boundary stones are not customarily moved and the ensuing formulation of a principle to that effect. Later the spoken or written reference to the custom becomes prescriptive and thus behavior is transformed into law.

The rules of the syllogism similarly exemplify the process of evolution of logical institutions. When we set before us the syllogistic rules as follows we see that they consist only of descriptions of what one does in connecting things or sentences in this particular kind of system.

1. Every syllogism has three and only three terms.
2. Every syllogism contains three, and only three propositions.
3. The middle term must be distributed once at least and must not be ambiguous.
4. No term must be distributed in the conclusion which was not distributed in one of the premises.
5. From negative premises nothing can be inferred.
6. If one premise be negative, the conclusion must be negative, and vice versa, to prove a negative conclusion one of the premises must be negative.
7. From two particular premises no conclusion can be drawn.
8. If one premise be particular, the conclusion must be particular.

The practical character of syllogistic rules is indicated by the recognition that the last two rules are corollaries. Also, as Keynes²¹ points out, the first two are not rules for the validity of an argument but rather serve to define the syllogism as a form of argument. Fundamentally this writer would reduce the rules to two.

A further formalization of the rules of syllogistic reasoning leads to the *Dictum de Omni et Nullo* and still more the principle of *Nota Notae*. At the peak of the pyramid of rules stand the so-called laws or canons of thought. These are regarded as the ultimate principles, the application of which governs the interrelationship of major, minor, and middle terms in syllogisms.

Since we plan to devote an entire later chapter to the study of the nature and origin of the laws of thought as the generalized principles of logic or system building it will suffice here merely to anticipate some of the outstanding points. Especially we are interested in suggesting that the conceptual variations concerning these principles or laws constitute evidence of the constructional process which is an integral feature of all system building. Originally, these laws were regarded as laws of things; later they became necessary laws determining how we must think or reason. This development marks a transition from the unsophisticated procedure of Greek system building to the highly sophisticated technique of medieval thought. In both instances the evolution of these general laws constitutes the definite construction of rules or boundaries which are abstracted and generalized from system-building behavior.

As a further illustration of this formalization procedure in logic we may cite the development of axioms. In building a mathematical system, for example, that of Euclid's *Elements*, axioms in the form of common notions are set up as governing principles. Originally these are for the most part simply formulated features of organizing items, which become incorporated in a technical system, and later in conventional usage and description achieve the status of fundamental and inescapable rules governing system building.

Still another example of law and principle evolution is the employment of the rule of contradiction in developing geometric systems. This rule has suggested the technique of *reductio ad ab-*

²¹ Keynes, pp. 287-294.

surdum as Euclid has so lavishly exploited it in the *Elements*. Whatever may have been Euclid's motive in using this technique or the corresponding *reductio ad impossibile* in general logic, the entire procedure in mathematics²² may certainly be regarded as a simple device for effectively developing a particular kind of intellectual system.

The laws or rules of scientific methodology or inductive logic follow the same principles of linguistic description or formalization and generalization. All the various canons of induction constitute more or less artificial verbal constructions which maintain some sort of tenuous connection with actual procedures in observation and investigation of natural events.

²² Cf. Enriques, *Historic Development*, p. 7.

CHAPTER III

DOCTRINAL SUCCESSION IN LOGIC

HISTORY or genesis offers no more than an index to the character of present events, since the latter comprise novel and unique factors. Certainly the persistence of a descriptive designation such as the term *logic* should not mislead us concerning the differences in the objects and activities subsumed under that name. On the other hand, we cannot overlook the continuity in human behavior and institutions nor the powerful influence of cultural traditions upon the logical doctrine of any period. Precisely because the study of logical history manifests these continuities and variations we can observe in the career of logic its general (institutional) and specific (operational) character.

To pass in brief review some characteristic successions of logical doctrine reveals distinct variations in logical work, technique, and materials operated upon. Inevitably such a review demonstrates that the variations in (a) theory and practice of logicians, (b) logical institutions, and (c) systems constructed correspond to varying cultural circumstances. Furthermore, such a psychanthropic approach to logic plainly indicates that the above differences are based upon underlying (1) *psychological*, (2) *linguistic*, and (3) general *scientific* and *philosophical* doctrines.

Concerning psychology, does the logician believe that reasoning consists of an objective intuition of cosmic intelligibility or the carrying out of particular processes for specific systematic purposes? The issue stands out boldly when we examine it in the related mathematical domain. Does mathematics consist of the analysis of absolute and inevitable relations of things or does it constitute the development of hypothetico-deductive systems?

Since logic inevitably involves reference and description we shall inquire whether language is simply accepted as things or processes or whether a definite inquiry into the nature of language is made. When such an inquiry is instigated is it concluded that language consists of expressions of psychic states (ideas, thoughts), simply symbols standing for things, or merely contents that can be indifferently related in various ways? Again, when symbols instead

of words are employed, do they have autonomous values or do they derive their values or significance from the particular systems in which they are placed and used?

As to the scientific and metaphysical factors in logical-doctrine succession, are the logicians absolutists or relativists? Do they incline more toward ontology or theory of knowledge? Or, being disposed to turn away from metaphysics altogether toward science, do they then make logic into methodology or the study of inquiry and investigation?

We should hardly expect the linguistic, psychological, and scientific factors to stand out plainly or appear in clear-cut isolation from each other. Frequently they are only to be analyzed out with difficulty and then are thoroughly interconnected. Nevertheless, the study and analysis of logical systems in different periods will reveal the presence and character of these factors as well as their particular impacts upon logical doctrine.

THE GREEKS: PRAGMATIC SYSTEM BUILDING

The modern condemnation of Aristotle's sentential demonstrations reveals a glorious tribute to Greek logic. As Enriques' depreciatingly remarks, the Greeks developed their logic on the basis of a naive realism which regards thought as a copy of the direct envisagement of concrete events. In other words, Aristotelian logic attempts to achieve an elementary form of certainty and stability through the organization of statements. Possibly when Greek thinking becomes differentiated from its medieval incrustation we may discover that the worst fault we can find with it, which, culturally considered, is no fault at all, is that the Greeks did not have so many civilizational details as we now have. Otherwise, Greek logic consists of an operational technique for arriving at certain definite conclusions. At any rate, in such comparatively simple logical activities, the logical enterprise clearly consists in the pragmatic construction of systems.

Aristotelian logic comports with an objective psychology according to which psychological events are exclusively actions of organisms. The psychology of Aristotle allows for no processes unbounded by spatiotemporal frames of reference. Though devoid of essential details, Aristotelian logic anticipates by many a century

¹ *Historic Development*, p. 25.

a behavioristic assimilation of thinking with language and consequently establishes (1) an extreme intimacy between inferences and (2) relations between statements or propositions.

Turning to the linguistic factor, Aristotelian language certainly constitutes a naive happening. Language in this period has not yet taken on the sophisticated subtlety which allows for the construction and imaginative expansion of things and properties; it still remains a straightforward reference to things. It would have been impossible, therefore, at this time to have erected gigantic systems of commentaries, each succeeding step of which departed further and further from the original things referred to.

So far as general science is concerned, this is the period when knowledge comprised investigation of observed things and when nature was still regarded as the sum total of concrete objects. It is not necessary to review all the salient characteristics of Greek science, with its horror of the infinite and of interminable regression. We have also referred to the variations in logical system depending upon particular materials dealt with, whether rhetorical, dialectical or scientific.

It is appropriate here to point out the danger of injecting modern views into ancient writings. Indeed, for this very reason the succession of doctrines must be studied in their own cultural settings. To disregard this plan entails two risks. First, the almost certain misinterpretation of a historical type of logic and, secondly, improperly fortifying one's own conception of a universal logic. An example of such untoward analysis is apparent in Enriques' criticism of Aristotelian logic without regard to the centuries of cultural development between the Greek period and our own. Enriques criticizes Aristotle because he did not adopt a rationalistic view based upon centuries of mathematical development. In addition, Enriques involves himself in the questionable assumptions that there is only one logic and that this logic is exclusively the system-building procedure of mathematical rationalism.

THE SCHOLASTICS: SYMBOLIC TRANSCENDENCE OF NATURE

Even though the immediate followers of Aristotle began to treat the various Aristotelian treatises as units of a single discipline, the Organon, the definite institutionalization and formalization of logic were achieved under cultural auspices different from those

under which Aristotle and his immediate successors lived. Logic as a powerful instrument of formal and abstract reasoning became established under the fostering influence which made Roman civilization the paragon of organization and perennity. Aristotelian logic became Logic, to remain unchanged only after the hesitant, multiple, and debatable doctrine became replaced by a rigid, unified, and authoritative system of syllogisms.

That *Formal Logic* endured so long is not surprising. Because logical processes became abstracted from actual events they could live on in the form of verbally articulate systems, as the bare skeleton of thought without the risk sustained by living things. Not that changes were lacking. A long time was required for the perfection of this logical instrument. The historical development of formalization includes the introduction of Aristotle's *De Categoriae* and *De Interpretatione* by Boethius in the Roman period, and later from the twelfth century onwards the advent of the *Posterior* and *Prior Analytics* through the Arabian contacts with scholars of the western world. After formal logic became thoroughly institutionalized, however, it served effectively as a mirror with which to reflect the views of those who looked into it.

To examine the particular conditions out of which the formalizing and universalizing of logic arose is to appreciate the dependence of logical principles upon the human circumstances engendering them. We may briefly pass over the beginnings of the crystallization of logical processes among the Peripatetics, since they confined themselves to the subsumptive work of classificatory science, as in the case of Theophrastus.

Fundamentally, the universalization of logic began under the auspices of the organizing trend of the Roman unification of the political world. The completion of this process, on the other hand, may be connected with the transformation of Greek and Roman Polytheism to post-Pagan Monotheism. Though the word *logic* may have been derived from either a Peripatetic or Stoic variation of the Heraclitean *logos*, a term for objective reason or order, logic as the name of institutionalized processes owes its origin to Neo-Platonic mystic sources.

The transformation of Aristotelian into Scholastic logic began in a decidedly spiritualistic culture. Recall that the translocations of persons with the successive downfall of the Greek and Roman

empires resulted in a new type of civilization. The objective individualism of the classic age gave way to the subjective personalism of the age of faith. The individuality of persons as men and members of a national and temporal *polis* was replaced by the unique participation of an imagined personal essence in an eternal spiritualistic commonwealth. Naturally these changes became reflected in a modified intellectual situation. It was Neo-Platonism with its essential mixing of Eastern and Greek thought which resulted in the basic theologies of the Patristic and Scholastic thinkers.

The new logic as contrasted with Greek, pragmatic system building was impelled by a different motivation. No longer was logic a process of organizing and systematizing arguments or scientific materials in a particular natural world. Neither could man attain security or peace through political action or operations upon concrete events. Logic was now completely divorced from natural science, a process which began in Alexandrian-Hellenistic times. Its aim and goal became the attainment of personal salvation through the ecstatic speculative union with the One which lies beyond reason and beyond earthly existence. Logic became, then, formal and authoritarian. By means of its abstract and symbolic techniques man could achieve his desires and needs. Through verbal argument he could protest his denial of nature by transforming nature into a pale and deceiving image of spiritual life.

The final establishment of formal logic was attained when the Patristic objections to Greek learning and the classical world gave place to the Scholastic acceptance and assimilation of the Aristotelian corpus. With the advent of new cultural conditions, for example, the development of travel, industry, and trade, and the renewed contacts of man and nature, the formal institutions of logic could be used to implement a new world view—namely, a recognition of the omnipotent power of symbols to achieve various ultimate goals.

In this manner logic became the basis for disputation and the exercise of refining verbal materials. Greek dialectics, which was concerned with the probable and with matters of fact, became fused with the demonstrative to make an absolutistic and rigid formal system: The activities of logicians were carried on with abstractions of abstractions rather than with particular problems having a definite factual locus. Thus originated the innumerable forms and

essences which dominated the thinking of Europe between the Greeks and the Renaissance.

The nominalistic-realistic controversy without doubt exemplifies the power that words or symbols came to have in the logical domain. While it is undoubtedly true that this controversy is a reflection of the individual's struggle for social and political independence, the battle between those who make universals into names and those who do not indicates the importance of words in the intellectual realm. Such a situation is in distinct contrast to that in which contact with actual events is stressed. Certainly the earlier part of this period is still dominated by the Augustinian emphasis of insight and reason. Recall that Augustine opposed *ratio scientiae*, the method of science concerned with the rational apprehension of the temporal, in favor of *ratio sapientiae*, the intellectual apprehension of the eternal.²

Turn now to the detailed psychological background of Scholastic, formal logic. With the depreciation of worldly interests and human behavior, the emphasis is upon the theological character of the soul, its freedom and indestructibility. It is only natural that in such a long period as we are including in this section there were innumerable changes in doctrine. Unable even to hint at many of them we must suggest that from the early Neo-Platonic idea of mind as one phase of the various emanations of the One, a series of steps carry the psychology of the period to the Scholastics, who regarded mind as a rational power interrelated with the Deity, but extending to a series of faculties comprehending the facts of nature.

Nomina sunt numina. The proverb that names are supernatural forces adequately suggests the linguistic conditions of this period. In an age when contacts with nature were tenuous and acquaintance with a divine totality the final issue of all knowledge and learning we may expect language and even words to have a fundamental character of their own. Words, especially names, were magically identified with essences. Certainly the divine name was regarded as part of the divine essence with all the potency that essence implies.³ The mighty events that revolved around the Homocousian-Homocousian axis is a matter of common knowledge. What could be accomplished with words was excellently illustrated by the struggle

² De Trinitate, XII, 15, 25.

³ Angus, Religious Quests, p. 144.

of Arius with the problem of the noneternity of the Second Person of the Trinity. Not being able to agree that "there was a *time* when He was not" since time had to be created and He was before time and all things were created, he simply affirms that "there was when He was not." Through Him God made all the worlds or ages.

Scholarship and wisdom in this period were the processes whereby the essences resident in words and symbols could be extracted. In our terms these words and symbols were made to yield whatever the various traditions of this culture demanded or required according to the lights of the interpreters. Logic at this time was primarily an enterprise of developing a rational—that is, age-serving or age-satisfying—verbal system. Taking the Scholastic period as a whole it was one of Terms, of Sentences, of Texts, the period when the Scriptures, the written words, whether those of the sacred books or of the resurrected and transformed Aristotelian treatises, were cherished without much regard to their referents. Not yet is language referential, when it takes on the function of description and interpretations based upon description.

Imbedded in this era is a notion of reasoning as an all-powerful and universal capacity, a capacity without limits because of the identification of man with some form of totality. Fundamentally this results in endowing man's reasoning ability with an autonomy which creates the things reasoned about. Through the manipulation of abstract symbols all sorts of infinites and supreme powers can be handled. There is no hesitancy, no humility in the face of actual events. While the Scholastics would certainly repudiate the earlier formulations of their own way of thinking, as represented, for example by Tertullian and others who made reasoning inferior to faith, the Scholastic view of reasoning is undoubtedly of a piece with the earlier phase. Not until the next period is there even a general trend toward limiting the activity of reasoning to natural events, though in that period too the reasoning about natural occurrences serves to transform them to fit reasoning processes.

THE RENAISSANCE: SYSTEM AS TECHNIQUE OF DISCOVERY

In the present period the distinctive cultural matrix for logic may be characterized as the tremendous development of contacts with things. As an age of discovery the Renaissance period, espe-

cially its culmination in the 17th century, is marked by an extraordinary development of science and technology. It is but necessary to mention that here was the dawn of our own era with its furious and successful conquest of nature. This may well be called the age of exploration, of great invention, the development not only of enormous knowledge concerning the construction of the astronomical cosmos, but also of all sorts of machines* which led to the famous Baconian cry that knowledge is power.

By contrast with the Scholastic period of culture the present age emphasizes actual events because of the accumulation of materials and techniques. Things natural and constructed force themselves upon the thinking and practice of mankind as over against the previous era in which man was primarily occupied with himself and his destiny. With the tremendous changes in the life of Europeans of this time there is a definite turning toward the conquest of things.

Logic and other intellectual pursuits now revolve about technique, method, ways of discovering and manipulating things. Typical are Bacon's diatribes against words, propositions, syllogisms, and logic itself, which stand between the thinker and natural things. Howsoever little Bacon himself participated in actual interactions with things, howsoever small his understanding of experimentation, even that of his own time, his criticisms of deduction and simple enumeration as induction make him a representative of the age.

To what small extent the so-called inductive logic of Bacon and his fellow discoverers interfered with the deductive tradition is a matter of historical record. Moreover, the perpetuation of rationalistic doctrine even by such genuine observers and experimenters as Kepler, Galileo, and their contemporaries through their assimilation and elaboration of mathematics argues for no break in the logical tradition. Still, the new ways of thinking engendered in this period cannot be minimized.

Ultimately the new Organon simply took its place beside the old. The new logic became methodology of science, whereas classical logic maintained its place as before. Neither in this nor in any later period does logic lose its ultimate and universal character. So far as the arguments for the Baconian inductive logic go, we cannot overlook that they continue an attack against sterile deduction made

* Wolf, History.

by Agricola, Valla, Vivés, and others in the interest of classical dialectic.⁵ With respect to such scientists as Copernicus, Kepler, and Galileo it is questionable whether their progress was altogether prompted by the things they studied or whether they were also influenced by Pythagorean number metaphysics.⁶

Though we characterize the Renaissance era as a single unique period, that culture is no longer so homogeneous as it earlier appeared. Populations are tremendously multiplied and the unifying factor of ultimate destiny and purpose has ceased to dominate man's activities. Many nations are now established with their own local habitat and particularized cultural variations. Even logicians themselves are no longer of one profession; all these cultural tendencies are inevitably reflected in their attitudes and formulations.

An outstanding difference between logicians concerns the balance they maintain between their adherence to tradition and inclination toward things. Those thinkers who favor the older tradition stress the organizer and his products, while others place greater emphasis upon the things organized. On the one hand, there is the rationalistic attitude in which the emphasis is upon the power of the mind to discover laws, fortified by mathematical traditions. In the mind inhere powers great enough to attain to the ultimate relationships which result in the discovery not only of things but their ultimate nature. On the other hand is the so-called empirical view, more or less strengthened by humanistic tendencies, which stresses things and their influence upon the knowing mind.

1. *Vico vs. Descartes*. The contrast may be excellently illustrated by the opposed views of Descartes and Vico. Croce⁷ tells us that Vico distinguished himself among the opponents of Descartes in that, unlike others who attacked the great thinker on the ground of his danger to religion, Vico fought Cartesianism because it was inimical to all knowledge which could not be reduced to geometrical deduction. Fairly early in the Cartesian era Vico fervently attacked this philosophy because it attached no value or importance to history (since founded upon human testimony), to observations of nature (when not subject to mathematical interpretation), or to practical wisdom (because based upon an empirical knowledge of

⁵ Windelband, *History*, part 4, chap. 1.

⁶ Cf. Burt, *Metaphysical Foundations*; Strong, *Procedures*.

⁷ *Philosophy*, p. 2.

human nature). As an exponent of the investigative attitude, Vico regarded himself as leaning upon Galileo, the experimentalist, and Bacon, the empiric, and was therefore opposed to the intuitive search for clearness and distinctness.

Notwithstanding that Vico no less than others of his time was steeped in medievalism, he symbolized the objection to reducing human events to the abstractions of number and size or minimizing their value when they cannot be so reduced. It is no small merit of Vico to have recognized that the division of the *Naturwissenschaften* (the natural or exact) from the *Geisteswissenschaften* (the social or human sciences) is based upon nothing more substantial than the Idols of the Theatre. Even a brief excursion into our intellectual history makes plain that such a hypotactic separation of the human from the natural sciences is not founded upon necessities of observation, but upon the fact that we are in a genuine sense the inheritors of medieval ideology.

2. *Leibniz vs. Locke.* Perhaps an even better illustration of the contrast we are suggesting can be found in the divergence of views between Leibniz, the mathematician, and Locke, the physician. The former, impressed by the absoluteness of the deductive systematization of geometry and its extension to algebraic or analytic systems, insisted upon the power of the intellect to arrive at absoluteness and completeness, even though this meant moving toward the possible rather than toward the existent. Locke, on the other hand, believed that the great achievements of Newton were not derived from mathematical powers of the intellect, but rather that these powers had to be supplemented by results of observation and experiment.

Beneath the divergences found in these two types of thinking there is nevertheless a fundamental agreement with respect to the psychological factor. Both the rationalists and the empiricists are mentalists; in consequence, for Locke the contents of things are reduced to mental states, so that their existence is precarious. Objects lose their content in the nebulosity of states of consciousness. The great difference between the two types of thought thus becomes minimized.

As in the former period, the linguistic aspect of Renaissance discovery technique may be summed up in the well established rule that certainty and absoluteness can be achieved through the use of

language or terms. Leibniz regarded terms as the symbols for an ultimate reality. In this sense he becomes the father of symbol-linguism, which may be traced back to his Universal Characteristic and Rational Calculus. Under the influence of the Leibnizian Characteristic, syntactic thinking stressed the importance of proper and workable symbols. Leibniz assumed that if thinkers could only organize a system of symbols with unambiguous one-one correspondence with the essence of things, they would have a key to the nature of reality.⁸ As he makes plain in the fragment on a Universal Characteristic, the fundamental symbols consist of numbers:

Unsere Charakteristik aber wird alle Fragen insgesamt auf Zahlen reduzieren und so eine Art von Statik darstellen vermöge deren die Vernunftgründe gewogen werden können.⁹

Such symbols afford logical demonstrability in place of the psychological clearness and distinctness of Descartes.

Today, this view can be translated as suggesting that the variables in a propositional function or an equation must have proper values. But admittedly, if our present-day emphasis upon proper symbolization goes back to Leibniz and perhaps even to Plato, there is a premium placed upon abstractions which somehow reach out toward or constitute things or essences. For both Plato and Leibniz were not beyond believing that the numbers 1 and 2 in the ratio $\frac{1}{2}$ signify the musical octave or are the musical octave.¹⁰

The rational calculus derives from the principle of necessary geometrical inference. The marriage of logic (thought) and mathematics (numbers) presumably makes for accuracy and certainty because it gives rise to a system of formal counters which can be manipulated according to accepted rules and yield inevitable results. Modern rationalism reflects the modified hope of achieving independent systems of equations and perhaps probabilities guaranteeing an intuition of reality. The novelty of current symboling is centered in the enlarged powers which pure reason attains by the use of sentences. Not only are mathematical equations equated with logical propositions, but both of these are further equated with scientific description. Thus the emphasis of sentences affords a

⁸ Cf. Dialog.

⁹ Zur allgemeinen Charakteristik.

¹⁰ Ibid.

basis for all sorts of syntax constructions which in themselves unite truth and reality.

Locke as an opponent of this ultimate rationalistic employment of language turns to what has traditionally been called the nominalistic procedure. For him language merely gives names of things and not the ultimate essences. It is this type of tradition that developed into syntactic logic.

THE ENLIGHTENMENT: CRITICAL LIMITATION OF LAW AND FACT

When in our brief sketch of the logical succession we reach the Kantian period we may no longer limit ourselves to large-scale societal and political culture as influences upon logical development. By the 18th century the growth and complication of culture are such that there emerges a plurality of scientific and intellectual problems and products which themselves constitute broad aspects of the culture of the time. Now and henceforth it is possible to study logic against a rich, general intellectual background. The differences in logical doctrine arise from the variant contacts of logicians with particular sciences and other technical intellectual pursuits. From this time all treatises reveal familiarity with and interest in the mathematical, religious, scientific, philosophical, historical or other developments of the learned world.

Thus the Kantian period may be characterized as the age of natural law. Not only is astronomy well established, but physics also. Kant himself mentions Galileo, Torricelli, Stahl, and others and declares that "In our age natural philosophy is in the most flourishing condition, and among the students of nature there are some great names, for example, Newton."¹ In general, Kantian logic reflects the confidence that this age places in the discovery of natural law, in mathematical formulation. "All nature," says Kant, "is nothing but a combination of phenomena which follows rules, and nowhere is there any irregularity."² Kantian logic may then be expected to follow two general leads. First, it is projected against a scientific background—the natural or mathematical sciences, to be specific—for only so much is scientific as can be subjected to mathematical treatment. Secondly, rule or law is emphasized. Kantian

¹ Jäsche, *Kant's Logic*, p. 33.

² *Ibid.*, p. 1.

logic constitutes primarily a criticism of Knowledge, the burden of which is to show how certainty may be achieved despite the necessity for concrete and observational processes in carrying on scientific investigation.

Notwithstanding the attempt to connect logic with natural science, in the sense of indicating what limits science places upon reason or showing the power of reason in science, logic is still under the domination of metaphysics. Fundamentally, the critical period is simply a variation of the large cultural tradition dominated by theology. Indeed, it has been pointed out that the tremendous emphasis of natural law in the 18th century was simply a translation of theological principles into the form of mathematical equations. Basically, logicians as well as natural scientists are interested in the ultimate nature of man and his relative position in the world of nature and the world of grace.

In view of this situation Kantian logic is essentially metaphysical, as evidenced by Kant's revival of the Dialectic phase of logic. In the *Analytic* he considers the powers and limits of the understanding in determining the character of scientific knowledge. Here the stress is on the thorough and final establishment of knowledge as necessary because of the dependence of things upon the forms of the mind. In the *Dialectic*, on the other hand, another form of cognition altogether is at work. Reason is not a fitting of categories to sensory contents. There is no external material to articulate with ideas. Here knowledge has to be set aside in order to make room for belief. The apex of Kantian philosophy is simply to differentiate the two realms and indicate the respective forms and limits of each.

The varying estimates of Kant illustrate how historians in choosing and emphasizing certain evidence make him, on the one hand, into an enormous initiator of new directions in thought, and, on the other, primarily and essentially a reconciler of established views. An excellent case has been made to show that Kant's fundamental aim, in which he was highly successful, was to unite the rationalists of the Leibniz-Wolff Continental tradition with the British metaphysicians of the Locke-Hume tradition. An easy and undoubtedly veracious interpretation of the Kantian philosophy is to consider it as the synthesis of Continental rational forms and British empirical content. One may well sum up Kantian epistemology as a successful synthesis of categories and sensations. His famous proposition about

the blindness of perceptions and the emptiness of concepts signifies the necessity for this synthesis.

With respect to logic Kant is likewise regarded from two opposite angles. On the one hand, he is looked upon as a revolutionist and, on the other, merely as a slight contributor. Those who hold the latter view quite properly point out that his treatment of technical logic was meagre. Aside from his early essay on the *Mistaken Subtlety of the Syllogistic Figures* and the lecture notes on traditional logic collected by Jäsche he left nothing on this subject. Also this view certainly is supported by his esteem of the perfect Aristotelian logic.

Nevertheless, the innovation view seems better founded. Kantian logic, it is true, is bounded by the traditional faith in the stability, uniqueness, and formal character of logic. Yet it is not improper to interpret the *Critique of Pure Reason* as itself a logical treatise.¹² Not alone in its form but in its fundamental content it is moreover a genuinely new type of logical treatise. The innovation consists in bringing logic into contact with the work and achievement of the concrete sciences. As a treatise on Logic, unique in modern thought, the *Critique of Pure Reason* under the auspices of Newtonian science undertakes to show not only the processes of reasoning, but also its limits and the methods of transcending them.

It is a tactical error, therefore, to treat Kant simply as a critic of medieval reason in disregard of his role of attempting to connect logic with the developments of modern Newtonian science. This very attempt accounts for the weakness as well as the strength of Kantian logic, for the science of this time was essentially rational mechanics, since all science then was thoroughly integrated with Euclidean geometry.

That the *Critique of Pure Reason* is not looked upon as logic is owing to the problems with which it deals. Kant himself thought of it as epistemology and metaphysics, though primarily the former. As we have seen, it seeks to establish the certainty and limits of knowledge. Logic traditionally is regarded as exclusively concerned with the pure forms of thought. But it was nevertheless Kant's idea that forms must be supplied with content. Of course, the two can be analytically separated. Certainly there is vacillation in Kant's thinking. On the one hand, he believes that all knowledge

¹² Windelband, *Principles*, p. 7.

must be derived from actual contact with things. But logic can transcend contact with particular objects which is the work of the special sciences. The special sciences can enlarge knowledge; logic can only criticize and correct it. Logic has to do with the necessary laws of thought or understanding, not with the empirical knowledge derived from the character of things. The divergence in Kant's views is to be accounted for by the fact that the science and logic of his time were both deeply set in the matrix of the spiritualistic culture from which they have not yet freed themselves.

In view of its time it was inevitable that logic should be formalistic and cosmic. According to Kant's conception:

Logic is a rational science, not according to its matter but only according to its form; it is an a priori science of the necessary laws of thought, but not in regard to particular objects but to all objects in general; it is thus a science of the correct use of the understanding and of reason in general, but not subjectively regarded, that is, not from the point of view of empirical (psychological) principles which establish how the understanding actually thinks, but objectively regarded, that is, from the point of view of a priori principles which determine how it ought to think.¹⁴

For Kant mind and reason are not only fundamentally psychic but transcendent. With respect to the nature of mind Kant does not in any sense depart from the traditional spiritualistic conception developed in the Alexandrian period. In line with this notion, sensations for him are ultimate transpatial phenomena based upon unknowable noumena, whereas the categories are also fundamental aspects of a transcendental unity which at the other pole likewise stems from a transpatial cosmic origin. The entire process of constructing phenomena bespeaks this traditional view.

From our present perspectival angle we can see the imperfections of the Kantian logic, which is based upon the aesthetic intuitions of absolute and separable space and time. Kant's cultural milieu has not yet entered upon the developments of science and mathematics which include non-Euclidean geometry and which lead to a lesser confidence in the completeness, fixity, and inclusiveness of rational systems. The Kantian ideas of mathematics, based as they are upon the fundamental intuitions of space and time, could hardly antici-

¹⁴ *Logik*, p. 16.

pate even the view that logic consists of hypothetico-deductive system building, much less that such mathematical systems are not the exclusive processes and products of logic. The Kantian logical system, therefore, as a product constitutes a rational formulation of all that is and can be, whereas logical processes are simply descriptions of how this cosmic system is analyzed, even though this analysis presupposes the prior existence of the system.

THE ROMANTIC MOVEMENT: DIALECTICS OF COSMIC SYSTEM

Were we able to transport ourselves back to the human circumstances under which Hegelian logic was constructed we should undoubtedly observe with the men of the time a strikingly new and stirring departure in logical doctrine. Not only did Hegelian logic mark a complete break from the formal logic of the conventional tradition, but it made logic for the first time into a comprehensive system embracing all things and all thoughts.

From our present point of vantage, however, the novelty in Hegelian logic merges with a number of tendencies current at the time. For example, we might regard the Hegelian system as simply an enlargement of the dialectic portion of the Kantian *Critique*, an enlargement which gave it its unique character. By criticizing the Kantian veneration of science and Kant's deduction of the categories, Hegel elaborated a system of principles which absorbed all nature and made it one with the object of what he regarded as man's highest aspiration, namely, the Absolute or God. In opposition to Kant, Hegel declares: "The real infinite far from being a mere transcendence of the finite, always involves the absorption of the finite into its own fuller nature."¹⁸

Hegelian logic, then, may be viewed as a unit variant of the logical continuum. Moreover, it is essentially sensitive to the events which it mirrors. Above all, it is a verbal formulation of human events and may be looked upon as an index to the social and political happenings centering about the French revolution. It is a commonplace to consider Hegelian logic as stressing historical fact as over against Kantian logic which is so intimately integrated with the abstractions of natural science. When we consider that Hegel is a mouthpiece for the Romantic stirrings in Europe and is constantly emphasizing the interrelationship of opposing elements in social

¹⁸ Wallace, *Logic*, p. 93.

life we can understand his totalitarianism and repeated attempts to make logic the instrument for organizing a harmonious unity.

To keep in mind the cultural matrix of Hegelian thought is not only to understand its emphasis upon union of opposites and the dialectic suppression of opposing ideas, but also to see clearly the basis of his objections to the ideas of his immediate predecessors. To a romantic the Kantian logic must have appeared arbitrary and formal. Though Kant revived the dialectic stage of logic he sundered it sharply from the analytic and made the former subordinate. Moreover, he made his analytic stage directly dependent upon the aesthetic factors which were forcefully stressed. Such a procedure appeared improper to those who observed carefully the developments of the time. Even at this period the claims of history, of national aspiration and the interest in Folk psychology were making themselves felt. Developments in biology, philology, electricity, and other scientific materials aside from mechanics were becoming more than mute whispers. The limitation of logic within the boundaries of Euclidean geometry and Newtonian mechanics became more and more anomalous, even to Kant. This fact accounts for the *Dialectic* and the three *Critiques*. Nor were Rousseau and his ideas without influence upon him; indeed, it is to this source that we may trace Kant's development of feelings as ultimate psychological data along with cognition and volition.

The historian of culture may well regard it as inevitable that the chorus of post-Kantians would insist that the fanciful distinction between appearances and things in themselves must be *aufgehoben*. So far as logic is concerned it must be harmonizable and totalitarian. The cry was that logic is not concerned with the application of a limited number of categories to the sensation content of the mind. Logic is frankly all-embracing and complete. The fundamental thing is to find a law by which one state of mind or judgement attains to others, perhaps first its opposite, until by means of a dialectical process the entire cosmos is stormed, conquered, and pacified.

Doubtless the appeal that Hegel's logic made to his contemporaries was its fullness, its absorbing power, and the expansiveness it allowed its adherents. The massive system the *Dialectic* made possible could take in all logics as well as everything else. Though Hegel had little use for traditional formal logic he still could

absorb it. Even if compared to his *Dialectic*, syllogistic logic was of little moment, Hegel yet declared that the many species of syllogism were well worth discovering, even though the value of the project was only equal to that of discovering sixty odd species of parrots and one hundred thirty seven species of veronica.¹⁶

Because Hegelian logic stems from a uniting of historical theory and theology it naturally has very little to do with actual things. Despite the assumption that it is concrete, replete with the fullness of life and therefore not formal and abstract, it is concerned only with things as *mind*. The whole dialectic interplay with being which turns into nothing, finally to be fused as becoming, is simply an employment of metaphor to establish the three-phase process of thought, namely, thesis, antithesis, synthesis. In the final analysis, dialectic is only the law of the unfoldment of spirit. It exhibits the power of the mind or Idea to make itself one with reality. In this process we see the significance of the concrete and how what appears at first as a plausible objection to formalism becomes an elusive verbal contention. Hegelian logic turns out to be a modern version of Plotinian spiritualism—modern because it does not merely derive all from the One, but presumes to show how it happens. The Hegelian pantheism is panlogicism, an elaborate pseudoanalysis of how the existence of any part or phase of the Absolute by war and contradiction inevitably displays its identity with its opposite until it achieves its place in the whole. The modernity of Hegelian dialectics lies in its verbal stress of movement, change, and in its assertion of concreteness. As Royce¹⁷ says, Hegel's Absolute is a man of war. "The dust and blood of ages of humanity's spiritual life are upon him; he comes before us pierced and wounded, but triumphant."

Although Hegelian logic is remote and even fantastic, it is not really outside the tradition of logic in its attempt rationally to assimilate all the essential contents of the world. Its uniqueness arises entirely from the kind of cultural events which it aims to portray. Instead of reflecting a neat and tidy universe of natural events in a stable social system it attempts to harmonize the upsurge and rebellion of individuals and masses of people, and the competing and clashing of states. Thus, it seeks to attain a complex, seething, all-inclusive totality.

¹⁶ W. von Hart, vol. II, p. 329.

¹⁷ Spirit, chap. 7.

Romantic dialecticism admirably illustrates the process of making logic comprehensive, absolute, and ultimate. This process is, of course, the facile use of words. There are several component techniques. For one thing, words are used to refer to anything the system calls for, such as harmonization of opposites and the attainment of absolute, concrete universals. In the second place, words are employed to represent or analogize various happenings. A nation dominated by another becomes, in words, no nation, though it once was a nation and may perhaps by a war of liberation again reestablish itself as a strong and powerful unit.

Such words as spirit, reason, and absolute are simply metaphors. The same may be said of words presumed to stand for actual psychological and nonpsychological processes, since genuine cognitive processes or objects find no place in the dialectic system. Hegelian dialectics in no sense deals with what persons actually do in carrying on their inferences, judgments, and problem solvings nor with the things presumed to supply the stimuli for such actions. Still, it is not to be denied that Hegelian logic is a mirror of its time and that it does describe though by the remotest of analogies some of the events of the period. Certainly we must agree that Hegel has developed a system. Whether or not one wishes to call it satisfactory logic is another matter; the objector's criticism simply reflects a different social milieu. The interesting thing, however, for the critical student of the logical succession is that the Hegelian technique of operation is similar to that found in other constructions. The purpose may be different and the materials the opposite of those employed in the Romantic dialectic, but the basic system-building process is present in all types of constructions.

THE TECHNOLOGICAL REVOLUTION: INDUCTION AS NOVUM ORGANUM

There is no better nor more direct way to introduce the present type of logic and its period than to assert that its exemplar is Mill's logic, localizable in the midst of the age of steam and the developments of the industrial revolution. Technical inductive Organonism is concerned with a type of logical thinking that emerges from a cultural complex in which technology has reached a very high stage of development. So far as the period is concerned it is an age of developing political liberalism dependent upon the growth and

self-assertion of the middle class, an age of social responsibility dictated by the upsurge of obligation for the newly developed populations of factory workers, an age of expanding material prosperity and commerce, with its fermentation of technical problems of local and international distribution (economic and transportation process) and governmental controls. Science, as a result, takes on a special form. Through the extreme development of technology the experimental method becomes so prominent a feature of culture that its techniques and procedures stimulate careful observation and analysis.

When we reach the period of Mill's logic we should no longer expect distinctive changes in the elements of the cultural pattern. Instead we may only look for effects resulting from a cumulative stabilization and harmonization of the elements in the pattern. Upon this point, however, we find opposing views depending upon whether the writer is a historian or chronicler.

The cultural historian finds that the period in question represents an outcome of contacts with things developed since the Crusades. For him the Crusades reflect a condition of European civilization in which the surging movements of the population culminate in the mixing of cultural traits, interchange of industrial products, and the intellectual and commercial cross fertilization of nations. The needs of industry and commerce, modes of travel and transportation resulted in developing tools, technology, methods, and procedures. All these changes, correlated with many others, produced the new Organonicism. In turn, this period became the raw material for a later development representing a still further accumulation of cultural elements of the same general type. The various periods, then, are articulated and continuous.

The chronicler of this period, which we roughly localize in the 19th century, is, on the other hand, not so much impressed by the continuity of events as by striking circumstances of the moment. He therefore stresses the period's novelty as well as that of the doctrines developed. Accordingly, he does not regard Bacon and the Baconians as authentic inductionists, and mitigates Bacon's originality and the value of his method. But what about Bacon's attack upon Aristotle? That, too, is minimized by the proponents of Mill's logic on the ground that Bacon was a hostile critic of experi-

mentation, whereas Aristotle was at least a careful observer of nature.

Turning to Mill's logic it must be granted that it possesses admirable features deserving acclaim not only from Mill's contemporaries but from current logicians as well. In the first place, it is definitely based upon investigative scientific procedures. For Mill the logic of science is the universal logic, and therefore applicable to all inquires in which man can engage, besides being a test for all inferential conclusions.¹⁸ Not only is Mill's logic designed to foster contact with nature but also to embrace the Galilean tradition of experimental science. In connection with the latter, Mill leans heavily on the methodological suggestions of men intimate with science, such as Herschel and Whewell. Again, Mill's logic contains a distinctively humanistic factor traceable to Comte's influence. This factor suggests the articulation of Mill's logic with actual events and a departure from the notion of logic as pure abstractionism.

Another favorable characteristic of Mill's logic is its consistent emphasis of the content as over against the forms which have been traditional in logic since the Middle Ages. From the standpoint of later developments, unfortunately, the constant preoccupation of the logician with content results finally in a preoccupation with sensations.

In spite of all these factors, which apparently stress logic as the work of man and his coming into contact with and organizing things, Mill's inductive logic turns out to be an organon, a single instrument for carrying on all the fundamental intellectual processes. Induction is thus a general operation of mind by which general conclusions are reached, a process of inference which proceeds from the known to the unknown.¹⁹

Like the organon of the post-Aristotelians who departed from Aristotle's own systems of system making, the logic of Mill also perpetuates the comprehensive and absolutistic character of logic. With the multiplication of cultural factors and the development of various sciences, inductive logic is, of course, a new organon, but its essential characteristics are the same. Logic is not merely an

¹⁸ Mill, Bk. III, chap. 1.

¹⁹ Bk. III, chap. 2.

organon or instrument for discovery, but also for proof. Both discovery and proof depend upon certain limited and fixed techniques. Though Mill may include the logic of practice, and hope, as did Vico, Bacon, and Locke before him, to extend his organon to social as well as natural events, the whole enterprise, after all, results in a single, comprehensive system with fixed boundaries. The uniformity of nature is not alone a fact which leads to system and organization; it turns out to be also an absolute limit to such processes.

Underlying this totality and inflexibility is the traditional notion of spiritistic mind. For Mill, of course, the mind consists of states of consciousness which constitute the ingredients of things. The subject, then, of psychology, says Mill, "is the uniformities of succession, the laws, whether ultimate or derivative, according to which one mental state succeeds another; is caused by, or at the least, is caused to follow, another."⁹⁰

Inductive logic, consequently, like all the others in the succession does not actually deal with things. The logical organization of objects and events depends upon laws of mind typified by the proposition that if ideas have once been together they have a tendency to be together at a later time. There is no room here for actual system building. Moreover, induction is made into an absolutistic procedure standing at the opposite pole from deduction.

Technological organonism is essentially metaphysical and takes on a nominalistic pattern. Universals and general principles are regarded as verbal or propositional. From the standpoint of logic as specific enterprises of organizing units into systems, Mill's logic simply emphasizes the sensation particulars as metaphysically determined substances. Language for this system constitutes abstractions from the commonsense references to things and not organizations of propositions employed to record (1) the results of contacts with objects and events in everyday experience and (2) the investigative results in scientific research.

THE CONTEMPORARY SCENE: TECHNIQUE OF SYSTEM CONSTRUCTION

An impressive implication of our study of the doctrinal succession in logic is that there has been a constant attempt on the part of

⁹⁰ Bk. 6, chap. 4, sec. 3.

logicians to achieve proper criteria for system making. On the other hand, the principle of system making has not been properly recognized by logicians, or, perhaps better put, the process of system making has always been confused with the need to develop a particular type of system. Today, the work of logic brings this system-making principle to a focus. Yet, as the career of logic indicates, the notion is still prevalent that there is only one type of system. For example, in current logic, systematization is fundamentally confined to the organization of relations. Hence the flourishing of the symbolic and mathematic logics. A large segment of the logical domain today is dominated by this notion that mathematical logic *is* logic.

To be sure, the notion that logic has to be the systematization of certain kinds of material or that it must follow the rules of a certain discipline has arisen because of particular cultural circumstances in each case. If it is true that the contemporary scene brings to the surface the principle of system making, that fact may have come about because of the mathematical dominance of our intellectual culture. The question whether or not this is the case can be illuminated by a careful consideration of the nature of the intellectual processes in general as well as of language and symbols, a task we undertake in Chapter 7.

1. *The Career of Logic.* Our brief survey of doctrinal succession has uncovered certain definite trends in logic's career. Beginning with the Aristotelian tentative formulations we have an elementary and commonsense technique for organizing systems in any field. Logic itself has not yet become thoroughly doctrinized into a discipline. The logician is still a nonprofessional intellectual worker bringing together whatever material he may happen to be interested in, primarily by way of classification. He is, therefore, not technically bent on creating a single, comprehensive system. In a sense, we may say the logician, that is, the elementary system builder, is not a theorist of system building nor does he have universally applicable principles for the purpose.

With the Scholastics, system building becomes a technical process. In this period the necessity to achieve a cosmic system forces to the front the problem of developing an all-inclusive, harmonious, and noncontradictory formula. Thus arises the demand for a theory of how this systematization process should proceed. Of

course, there is only one technique in which this can be accomplished and that is by putting together a system of noncontradictory sentences.

Though this achieved crystallization of logic as a universal process of obtaining certainty and fixity is never more relinquished, the changing conditions of culture bring about modifications. For example, in the Renaissance period the reconciliation of man and nature gives logic a methodological turn. Logic becomes an instrument for organizing the natural cosmos, which is typically represented by astronomical objects and events. Logical systems take on the character of determining procedures. These methods on the Continent are regarded as a sieve to separate out clear and distinct ideas from those that are obscure and confused or as a tool for the discovery of necessary conclusions based upon mathematical relations as Platonic or medieval Reals.

Among the British, who are somewhat more inclined toward concrete materials, the qualities rather than the forms of things assume importance. Logic here becomes a search for methods of procedure. Induction or the building up of systems begins to vie with the simple subsumptive procedures or the mere handling of analytic systems.

After Galileo and the development of physics, which employed the power of Euclidean geometry, logic becomes critical. The fundamental problem, as with Kant, is the guarantee of the *a priori* by the fact that knowledge depends upon the categories or the forms of mentality as well as qualities which are evanescent by themselves. Here are the beginnings of the appreciation that logic has to do with development of self-consistent systems.

With the rise of the Romantic period and the growing sensitivity of man toward the changing conditions of human life, thinkers begin to take account of the complexities of human events. To gather these all up into one comprehensive system logicians break down the limiting bounds of analysis and rediscover the possibilities resident in the dialectic form of procedure. In this period system making becomes rampant and the old elements of logic become retranslated in order to hold together all qualities and all values.

With the new era inaugurated by the technological revolution in the 19th century and the sense of power thereby achieved logicians reinterpret scientific methods as definite processes for building

up tremendous logical systems. Methodology of science becomes a deliberate instrument for establishing the uniformities of nature.

The observer contemplating the contemporary scene and casting his glance over logic's career may well ask himself what of all that has emerged has vanished and what is left. The career of logic provides its own answer. From the earliest developments of logic, cultural institutions centering about system building arise and maintain themselves with great variations in function and material worked upon. So far as function is concerned, we may cite arbiter of proof, organ of discovery, criticism to achieve certainty, method of investigation, et cetera. The materials worked upon include propositions, states of sensory consciousness, concepts, and the cosmos. Throughout its entire doctrinal history, however, logic is consistently a process of system building, whether or not known to its practitioners. Probably the outstanding result of studying logic's career is the logicians' full recognition of the authentic function of their tools and their work.

Among the primary cultural influences discernible in the current logical scene are the rise and great spread of evolutionary doctrine. The extensive developments in the biological sciences culminating in the doctrine of evolution and fundamental interrelationship of animal organisms, as also the overwhelming evidences for the place of man among the other animals and in nature in general, have injected new emphases into logic. For one thing, the naturalistic aspect is stressed. Moreover, the notions of the authenticity of change and the lack of fixity in events in general as well as in species and animals have brought to a focus the place of man in cultural origins and variations.

Logic becomes in a unique sense empirical. The certainty and stability which are still sought are regarded as consequences of thinking or reasoning. It is hardly to be questioned that the evolutionary influence has had a powerful impact upon postulation theory and certainly has emphasized the utilitarian character of logic.

What is still lacking in the current scene and what we suggest will lead to a desirable improvement in logical doctrine is the appreciation that logic is a concrete, system-building procedure that must be divorced as much as possible from undiscerned cultural influences. Specifically, logic will profit from its freedom

from domination by traditional metaphysics, and may thus look forward to being the latest of the disciplines to separate itself from philosophy. To become fully aware that his unique function is system building will allow the logician to deal untrammelled with the specific materials he is interested in organizing. On this ground logic becomes a generalized discipline constructed on the basis of logical enterprises. As such it is dependent and can only exist upon the basis of logics. How many? No one can say. The problem of ascertaining this number is identical with the problem of discovering the number of any type of enterprise or product. Those who wish to know this number must keep alive to the common and unique factors in such enterprises, the problems of the system builders, the variations in materials, the influences upon the workers, and the similarities and dissimilarities of the products.

Undoubtedly, it will be asked how such a program can be carried out. It is precisely at this point where the need of an objective psychology enters and, further, an adequate interpretation of language.

The career of logic demonstrates that all authentic logical doctrines are based upon a mentalistic psychology. Throughout its career logic has had to contend with the conception of mind as substance, power or states basic to thought and reasoning. Only on this basis could logic be regarded as transcending concrete events and specific problems and in general as built upon a plan of *priorities*, generality, and fixity of method or procedure. With such a psychology underlying it, logic almost necessarily was immune to the place of events or things in the determination of system building or to the influence of culture in providing criteria for types of system building. It is psychistic logic which is responsible for ideas of ultimate and perennial thought or reasoning.

2. *The Career of Language.* Logic's career is likewise informing concerning the place of language in logical evolution. Among the Greeks in the formative period language still consists of common-sense referential things and acts. Words constitute elements of references to things, while categories are still analytic derivations from sentences as units of reference to actual events. Sentences have not yet attained the technical status of propositions in a formal domain of doctrine. The distinction made between sentences and propositions on the truth-falsity criterion²¹ is based upon the direct

²¹ De Interpretatione 3, 17A, 3.

referential character of elementary language and not formal logical sentences.

It is only with the Patristics and Scholastics that words become symbols—autonomic terms which in the beginning substitute for actual things and events and later become factors in autistic assertion and argument. In this stage of development words and symbols need have nothing whatever to do with actual things and events.

With the assimilation of mathematical factors in the logical thinking of the Renaissance, language in the form of symbols assumes greater prominence. True enough, verbal and symbolic logic pursue parallel courses, the one leading toward scientific methodology and the other continuing traditional logic, but the appreciation of the role of linguistic or representational processes grows apace. Words and symbols both become increasingly recognized as elements of system building, though the nature and interrelations of systems are not fully appreciated.

In the Rationalistic logics of Kant and Hegel language functions in such a manner that mathematical symbols and verbal categories point in different directions. Mathematical symbols are stressed as formal, whereas verbal things, as categories, are presumed to be close to objects. Categories include content and may even constitute materials, qualities, and relations.

In the so-called empirical logic of Bacon, Locke, Herschel, and Mill localized in a different cultural setting language becomes a deliberate process for achieving certainty. The nominal linguistic technique, howsoever inadequate it turns out to be, still presupposes a definite appreciation of the role of language in logical enterprises.

Having so far only considered the current logical scene as an outgrowth or development from the doctrinal succession we turn now to an examination of some of its details.

CHAPTER IV

LOGIC IN CURRENT CULTURAL CROSS SECTION

REPRESENTATIVE TRENDS IN CONTEMPORARY LOGIC

IN the logical as in every other department of culture the current scene is teeming with the accumulated products of former generations. In addition, the current logical domain is replete with numerous achievements of contemporary logical workers. It is also not unimportant to consider the influences of new logical centers upon the multiplication of logical theories. All these cultural cross currents inevitably lead to variations and complications in the details of logical doctrines and systems.

Among contemporary logicians some are primarily influenced by scientific pursuits, some by general metaphysical interests, whereas others are impressed by mathematical developments or incline toward linguistic problems. Refining our analyses still more we discover varying doctrinal influences concerning the nature of science, mathematics, language, and symbolism, as well as the central psychological processes involved in all of them. These variations are, of course, functions of the relations the workers bear to the classic philosophical tradition.

Effectively to examine the contemporary logical situation it is expedient to confine ourselves to a comparatively small number of representative trends. For our present purposes, therefore, we consider only four of the most outstanding—namely, (a) the symbolinguistic, (b) the mathematical, (c) the instrumental, and (d) the humanistic movements. As the names suggest, these trends are primarily influenced respectively by (a) developments in the appreciation of the nature and role of language and symbolism in intellectual work, (b) the impact of mathematical development upon logic, (c) the ever-increasing claims of industrial and scientific methods and techniques in modern civilization, and finally, (d) the stress of personal and human elements in logical theory.

Obviously the newest trends are only relatively new. They simply represent variations in traditional doctrine influenced by current cultural conditions. It is disastrous for a student of the

current logical scene not to take account of the continuity in logical development since Greek times. For example, along with the newer instrumental and mathematical logics, formal logic whether of the genuinely Aristotelian or medieval types is still with us. Naturally the original structure of formal logic has undergone numerous modifications owing to the enormous technological and commercial developments of our present culture.

These continuities in material (terms, propositions, syllogisms) and form of treatment (for example, emphasis of deduction) are only examples. We must further keep before us the fact that the logical enterprise itself is a cultural institution. Remaining with us, then, are the perennial notions of logic as the general or total systematization of things or thoughts, as the essential way or process of thinking or reasoning, and as the basis for certainty of knowledge or proof. Indeed, the belief in logic as the rock of ages or the way of achieving such a rock has never perished from the earth.

To stress the continuities, however, is in no sense to minimize the novelties in viewpoint, procedure, and product. To understand logic it is important to study the precise details of what particular logicians do with respect to their problems, interests, and attitudes. Moreover, we must examine the technical, social, and political background of their work, for it is this work and its products which constitute the very warp and woof of logic. Following this procedure, the student of the current scene comes to close grips with what he studies. This fact is of value to the logician. Not only can he observe the influence of the environing frame of reference upon himself and the logic he espouses, but he can also discern the threads connecting the principles he uses with their origins in the continuum of history.

A. *Symbolinguistic Logic*

Whether logic is regarded as a study of the ultimate operations of thought, the enterprise of discovering the fundamental relations of things, or the description of the inevitable methods of science, it is, of course, considered as universalistic in each case. Accordingly, it is formalistic and simplifying. This means it ultimately must come to classes and relations. Now there is no other way for logicians to operate except by means of representations of these ultimates and totals. Hence the inevitable place of language and

symbols in logic. For two reasons. First, such activities must be representational. To deal with anything except by immediate manipulation, even in the case of communicating about things, involves representational behavior. This goes back to the necessary referential character of language.¹ Secondly, it is obvious that the handling of complicated happenings, even the multiplication of simple things, is facilitated by the effective use of representations—that is, symbols.

Symbolinguistic logic, whether or not looked upon as derived from mathematics, represents the flowering of symbology. The achievement of current symbolic logic is a new and effective treatment of symbols. Obviously, since the handling of symbols cannot proceed without regard to their representational function, present logical achievement must be credited with the development of all sorts of new ideas concerning relations and their implications. Symbolic logic is therefore not merely the study and development of symbols, but also concerns the results to be obtained from handling symbols.

Just what is included in symbolinguistic logic is a question not easily answered. Thus, Lewis, introducing a treatise on symbolic logic,² writes:

The study with which we are concerned in this book has not yet acquired any single and well-understood name. It is called “mathematical logic” as often as “symbolic logic” and the designations “exact logic,” “formal logic,” and “logistic” are also used.

In his *Survey of Symbolic Logic* he also mentions algebra of logic, calculus of logic, algorithmic logic. He concludes that since the use of symbols has characterized logic from the beginning the new logic merely extends this use in ways required by or conducive to clarity and precision, and hence the subject matter of symbolic logic is merely logic—the principles which govern the validity of inference. Couturat who earlier faced this question preferred the term algorithmic logic or, better still, logistic. This writer likewise adopted the notion that logic is “the normative science of the formal laws of correct thinking.”³

¹ See chap. 8.

² Lewis and Langford, *Symbolic Logic*, p. 3.

³ *Principles*, p. 136.

Obviously, then, the logicians classed as symbolinguistic have not been able to separate clearly the large number of factors involved in the specific trend which they represent. On the whole, the symbolinguistic movement reflects an increasing awareness of the nature of symbolic and linguistic events in system-building enterprises. To a great extent logicians realize that much of the improvement in logical thinking depends upon correlated developments in understanding the constructional processes involving linguistic and symbolic materials. Not only symbolic logicians, however, have become interested in the contribution that an understanding of language can make to the handling of logical problems. The instrumentalistic logicians are likewise increasingly sensitive to the importance of symbolism in logic. For this reason we have here a distinct departure from the situation in which language has always constituted an important item in reflective enterprises, to a stage at which problems of terminology and definition are steadily occupying a prominent place in scientific and logical work. Today such important movements as symbolic logic and syntactic logic and their interconnections with logical positivism and semantic analysis signalize the novel and unique position which linguistic facts are beginning to occupy.

As a reminder of the great prestige which language has always enjoyed among thinkers, recall the ample discussions devoted to the nature of language and its place in thought since Plato. Consider how much reflection has been expended on language problems by Aristotle, Roscellinus, Hobbes, Leibniz, Locke, Bentham, Mill, Mauthner, Poincaré, etc. From the earliest days of dialectics the belief has been tenacious that grammar is inseverably connected with logic and science. In recent times this view has been frequently celebrated in such book titles as the *Grammar of Science*, *Grammar of Politics*, *Grammar of Ornament*, *Grammar of Banking*, etc.

Why this perennial interest in language or dialectics? Does it merely indicate that with changing circumstances new formulations concerning the invariable connection of language with logic and science become necessary? Or rather, does it show a failure to achieve a satisfactory notion of language and its relation to thought? The writer hazards that the latter is true. An examination of the linguistic views underlying the prominent current movements mentioned reveals not only the essential difficulties of

present-day linguistic theory, but also paves the way for a reformulation of the nature of language itself and its place in logic and science.

Three Phases of Symbolinguistic Logic. Though, as we have indicated, symbolinguistic logic constitutes a number of phases which interpenetrate and fuse, it is still possible to differentiate three distinctive aspects upon two bases. The first basis is the sharp traditional division of language study into semantics, syntax, and morphology, in addition to phonology. Semantics stresses the significance of words, symbols or elements, whereas syntax is presumed to be the study of the conjunction of elements to form sentences or propositions. Morphology, in turn, is delimited by the interest in the structure or organization and variations in words or symbols. Phonology for the most part is regarded as the study of vocabulary and the component sound or sight units of which words or symbols are constructed. Certainly the earlier developments of symbolic logic were intimately concerned with either the morphological, phonetic or visual aspects of symbols.

The second or genetic basis for distinguishing certain aspects of symbolinguistic logic concerns the origin of the particular aspects themselves. For example, the symbolic type of logic originally constituted a movement the main end of which was the improvement of logical symbols. This movement was influenced first by the attempt to secure greater generalization for the terms and operations of the traditional Aristotelian logic and, secondly, by the observation that mathematics had profited materially from the development of constantly improved symbols. In the same sense the syntactic aspect of logic originated as a criticism of the improper connection of propositions with unsatisfactory metaphysical content. Finally, semantic logic arose as a result of the too drastic formalization of logic by confining it to syntax without regard to content or referents. The logicians who stress the semantic trends aim at nothing less than the creation of a "unified conceptual apparatus which would supply a common basis for the whole of human knowledge."

In taking these three closely related symbolic, syntactic, and semantic aspects as typical of current symbolinguistic logic we neither overlook the lack of unanimity among the expositors of

* Tarski, Introduction, p. xi.

each nor the frequent shifts and changes in doctrine. Since we are more interested in symbolinguistic problems than controversies within or between schools, and inasmuch as there is an essential equivalence in the underlying linguistic views, we may treat the three aspects as distinct phases, thus securing the advantage of referring to illustrative cases without too much emphasis on the particular exposition.

(1) *Symbolic Logic*. If it is possible to isolate symbolic logic as a distinct type or phase of logic we can at best assign to it only a minor value. In other words, we cannot find in it a unique feature altogether distinguished from all logical enterprises or products. In a genuine sense symbolic logic constitutes only the beginnings of some other types of logic and certainly the mathematical. We may then regard it as a stage in the development of formal logic. This is true whether we think of symbolic logic as introducing symbols, either the mathematical plus, minus, times, and division, or as initiating the recognition of a common symbol-building basis in traditional formal logic and mathematics.

Again, in the sense that symbolic logic is concerned with stylistic or form factors and not the content of terms, propositions, and syllogisms it is only a stage in the development of logic and not a unique type of system or product. From this angle symbolic logic may be regarded as the study of representational processes and techniques in system building though one must be aware of the danger of separating these processes too far from the materials and constructions of systems.

In the third place, if, as is really the case, the development of new symbols implies actually new techniques and new systems, symbolic logic takes a minor place in logical development. This is true because symbolic logic, as represented, for example, by Venn's treatise,⁵ merely deals with the simplest components of modern formal logic. Here the application of algebraic notation and even the distinctions between symbols for classes and for operations merely apply to the most elementary processes of system building, namely, very elementary ordering of classes.

Though symbolic logic may be treated as a secondary feature of modern logical development it cannot be lightly set aside. For it is obvious that the emphasis upon forms or symbols arose from

⁵ *Symbolic Logic*.

necessary and important system-building problems. It is impossible to deny the utility of marks, diagrams, and schemata of various sorts in reference, criticism, and construction work in logic. To minimize the symbol and diagram-designing efforts of Leibniz, Euler, Lambert, De Morgan, Boole, and others is akin to the criticism of the symbolic development in mathematics. If it is feasible to regard logic as system building these aids cannot be condemned as mere externals. Nor is it to be overlooked either that such objects may themselves be the materials for important systems.

As Venn⁶ suggests, historically symbolic logic takes its departure from an attempt to enlarge the scope of common or Aristotelian logic. Improvements within the Aristotelian tradition are urged on the ground that the terms of the logic should not be confined to subject-predicate interpretation or representation but to classes and interrelations of classes. Even this does not go far enough. What is needed is a complement of compartments which are or are not filled by propositions containing certain terms.

Lewis⁷ points out that the development of symbolic logic had to go beyond symbols for terms, as these were already well developed in the old traditional logic, for example, *a, b, c*, and *s, m*, and *p*.⁸ What was required was a progression from the use of symbols for terms to the use of symbols for propositions and even more for the relations between them. Lewis indicates that already in Leibniz we find an insistence upon symbols on the ground that symbols are common to all workers, and that compact and appropriate ideograms should be substituted for the phonograms of ordinary language.

Basic to the appraisal of symbolic logic is the fact that probably one of the greatest influences upon its development was the interest of mathematicians in making exact and precise the propositions of traditional formal logic. This influence is effectively revealed in the picture Venn⁹ presents of the early objections to

⁶ Symbolic Logic, Introduction.

⁷ Lewis and Langford, Symbolic Logic, chap. 1.

⁸ "The canonical forms of the Aristotelian syllogism are really symbolical, only the symbols are less perfect of their kind than those of mathematics" (Boole, Mathematical Analysis, p. 11.).

⁹ Symbolic Logic, Introduction.

symbolization and generalization in logic on the ground that an unnecessary and undesirable mixture of mathematics and logic was being attempted. Venn himself takes the position that the symbolism simply furnishes the advantage of greater and greater generalization.

Our characterization of symbolic logic as a minor phase of modern logic is based upon the premises of the conventional view that there is only one logic and that the symbolic stage represents a development, perhaps a generalization, of that logic. When, however, we take the specificity view and regard logic simply as a process of system building irrespective of the number and type of systems, we might accord symbolic logic a higher rating. It is a distinct development to turn from purely verbal propositions, no matter what their references are presumed to be, to symbols and symbol combinations and processes, whether or not they are mathematical. The fundamental point here is that symbolic logic illustrates a new and different type of logic, as well as the general processes of system building.

The assumption that symbolic logic simply supplies a new kind of language for logic we regard as fallacious. Actually, the development from verbal description of relations to the symbolic representation of classes or any other sort of system material constitutes a distinct advance in the system-building procedure, in that it clarifies this procedure, whether the materials of construction are presumed to be merely discovered or developed and organized.

Symbolic logic, then, we conclude, constitutes a specific form of logic with its own processes and subject matter yielding unique and important products. Insofar as it conforms to these descriptions it is really of greater importance than its supporters believe, even if they wrongly assume that symbolic logic is the universal and correct logic.

(2) *Syntactic Logic*. There are two primary avenues by which syntactic logic has entered into current logical thinking, one through the interest in satisfactory questions, the other through problems concerned with mathematical foundations.

As to the former, Wittgenstein has popularized the views that the so-called sentences of metaphysics and ethics are pseudosen-

tences and that philosophy or logic of science consists of a critique of language and a logical clarification of thought.¹⁰ Wittgenstein and the Logical Positivists, therefore, desiring to banish all the otiose and pseudoproblems of traditional philosophy, turned to the linguistic problem of differentiating between questions that have sense or not. This form of approach to scientific or philosophic thinking, which interrelates language and other activities, Kant has traced back to the ancients who declared that to ask and answer improper questions is like milking a he-goat and collecting the results in a sieve.¹¹

The syntactic interest in proper questions is seconded by the quest for a sound mathematical foundation. Hilbert¹² especially has persistently striven for a system of sheer formal marks which can be manipulated analogously to a series of abstract numbers. Pure mathematics would then consist of a formal symbol structure completely dominated by the criterion of consistency which would lead to a formal theory of language or logical syntax. By means of such a formal structure of symbols and proper questions the symbolinguists believe they can arrive at valid scientific results.

To illustrate how syntactic systems can solve problems Carnap considers the opposition between the view of Whitehead and Russell that "numbers are classes of classes" and that of Peano and Hilbert that "numbers are primitive objects, independent elements." There are two steps in the solution: first, the two theories must be translated into the formal mode of grammar: (a) Numerical expressions are class expressions of the second order and (b) Numerical expressions are not class expressions, but elementary expressions. The second step is to connect each with a specific language system, L_1 and L_2 . The result is: "In L_1 numerical expressions are elementary expressions" and "In L_2 numerical expressions are class expressions of the second order." Carnap goes on to say: "Now these assertions are compatible with each other and both are true; the controversy has ceased to exist."¹³ How simple! But this technique is not merely applied to questions of definition and of the nature of number, but to all scientific work.

¹⁰ Tractatus, 4.0031 and 4.112.

¹¹ Inaugural Dissertation, par. 27, and Critique of Pure Reason, p. 47.

¹² Grundlagen, and Hilbert and Ackermann, Grundzüge.

¹³ Philosophy, p. 26f.

By asserting that the language of physics is the primary language of all reflection the syntacticists believe they can similarly resolve any scientific question. Science, like diplomacy, becomes then the use of the right language.

An essentially logical development of the syntactic view is to regard deduction or inference as purely a process of transformation of sentences.¹⁴ This is based on the assumption that logic is concerned with sentences. Accordingly, given an appropriate rule, so that *pirots* is a plural substantive, *karulize* a third person plural verb and *elatically* an adverb, we can then have a sentence, "*Pirots karulize elatically.*" This sentence with the sentence "*A is a pirot*" can become the basis for deducing the sentence, *A karulizes elatically.*" A fundamental feature of this syntax or general language according to Carnap is that it is completely formal and devoid of meaning either in elements or relations.

The moral support for making logic into syntax and for asserting that "the *logic of science* (logical methodology) is nothing else than the *syntax of the language of science*"¹⁵ is its foundation in the formalistic theory of mathematics. Syntactic logic is a symbolinguistic counterpart of the Hilbertian treatment of mathematics as a strict calculus—that is, a system of rules for manipulating mathematical formulae. The aim is simply to achieve a noncontradictory system.

Should one regard this emphasis of formality and abstractionism as remote from human affairs and scientific thought, there are two retorts. The first is founded on the assumption that thinking rests on the three pillars of consistency, adequacy, and simplicity, which amounts to saying that only by a formal deductive method can you have any worth-while thinking. The second sets up the bogey of mysticism. If you don't adopt this type of thinking you seek a mystic transcendence of rational thought.¹⁶

Symbolinguists habitually overlook that, whether in the guise of numerology or some other form, symbology has always been the core of mysticism. We are not thinking merely of infinitesimals and infinities, but likewise of numerous forces and final causes obtained by sublimating mathematical processes. Even the most

¹⁴ Cf. Carnap, *Logical Syntax*.

¹⁵ *Ibid.*, p. 7.

¹⁶ See, for example, Rosinger, *Material truth*.

precise probability calculations and trend-line constructions have not been proof against mystical cosmic pronouncements. Instead of regarding trend lines as records of past events or as samplings of current events they are frequently looked upon as mystic forces. The history of science is replete with instances in which numbers are substituted for objects which are then transformed into mystic essences. The magical absolutism and totality of numbers have strengthened the appeal of so-called primary qualities, as though the height of a person were more real than his complexion.

In all this we find a fundamental analogy in the contrast between the rationalism and experimentalism of the Renaissance period. The rationalists built upon the foundations of Euclidean geometry—that is, fundamental deductive processes, which, when analyzed, consist of operations within a system whose boundaries are assumed. Probably no rationalist can be accused of such extreme dogmatism and arbitrariness as to set up an autistic system wholly unrelated to occurring events. Even the arch rationalist Leibniz cast aspersions upon the numerological Cabalists. Furthermore, despite their ultra abstractionism, the rationalists, building upon Euclidean deduction, did include commonsense facts of space which “intuitively” entered into their premises. Nevertheless, all rationalists carry their formalism to an extreme which sets them off very definitely from the experimentalists.

The experimentalists, on the other hand, who were, of course, also rationalists, stressed operations upon events. Whatever systems they built up presumably maintained some contact with those events. When the historian of science compares the Galileans with the Cartesians, the balance in favor of the former is obviously not weighted by numbers, but by their preoccupation with things. The experimentalists are important not because they describe their results in number symbols, rather because they derive their numbers from interbehavior with events. If the experimentalists may be distinguished as inductionists, it is because the symbols they used were checkable in terms of observation. The symbols of the deductionists, on the other hand, are transparent enough for the light of truth to shine through them. Their propositions are in Scholastic terminology evident *ex terminis*. Symbols, whether words, mathematical signs, or what not, have powers of their own with which

they could only have been endowed by medieval theological speculation.

(3) *Semantic Logic*. Several considerations suggest the inevitable development of semantic logic. In the first place, since modern logic is intimately interrelated with language, it is to be expected that logicians complete the circle by going (a) from morphological structure (sounds and shapes), (b) through syntax organization, (c) to semantics.

Secondly, logicians are bound to inquire into the nature and function of their elements. Even though symbolinguistic logicians were originally inspired by mathematical developments they could not confine themselves to certain terms whether those of non-mathematical or mathematical formal logic. They had to become alive to the fact that what terms were used, and how, involved something beyond the shapes or symbol characters. The germs of this type of transcendence were already contained in the mathematical conception of variables which lies close to the heart of semantic logic. System making could not proceed far without the realization that terms were not fixed and self-contained entities whose "meaning" and "significance" were predetermined. Such realization could remain dormant as long as one dealt with limited mathematical systems. But when various types of systems are studied, then the values accruing to the terms become problems. In other words, logicians had to move toward the operational conception that terms must be deliberately evolved according to the functions they are to have in particular systems and situations.

Thirdly, semantic logic is motivated by a wholesome dissatisfaction with the syntactic form of symbolinguism. In syntactic logic relations are confined simply to the terms or elements in the language. As a matter of fact, a syntactic language is presumed to consist of nothing but a series of elements with rules for their relational transposition. An obvious conclusion is that such formations and transformations of elements are not only extremely limited in scope but sterile. The result could have been easily foreseen. To have significance an organization of terms cannot be self-contained. Even a so-called abstract mathematical system is regulated by, or at least attains, a significant status only within a particular mathematical situation. Hence arose the distinction between using and

mentioning symbols. Metalogics not only are necessary enlargements upon systems of formal elements but also indicate the necessity of moving on from syntactics to semantics.

Fourthly, logicians seem to be forced to go beyond simple systems of elements or syntactic language, to reach referents, even though such referents are restricted within the formal limits of true and false categories. Sentences then become semantic as in the work of Tarski, Carnap, and others. Semantic logic, whether or not path breaking,¹⁷ is designed to serve as a criterion for the truth of sentences. The technique is to indicate that the sentence, "the set of objects Σ satisfies the axioms of Euclidean geometry" is not merely a remark about language but a significant assertion about Σ .

The role of linguistic motives and their value in logical situations are indicated in the definition Tarski gives of Semantics.

. . . the whole of speculations that concern themselves with those concepts in which, roughly speaking, are expressed certain correlations between the expressions of a language and the objects and facts indicated by them.¹⁸

The sentence "The expression 'the conqueror of Jena' symbolizes Napoleon Bonaparte" is an example of a semantic structure.

As Ushenko¹⁹ points out, the semantic factor goes only so far as to relate metalogical and logical sentences. The evidence for this is found in Tarski's contextual definition of truth.

. . . we will recognize as correct all such sentences as: *the proposition "it is snowing" is true if and only if it is snowing*; the proposition "*the world war will begin in 1936*" is true if and only if *the world war will begin in 1936*; in general every sentence of the form: *the proposition x is true if and only if p* , where " p " is to be replaced by any proposition of the language investigated and " x " by any proper name of the proposition, whereby the proper name belongs to the province of the metalanguage.²⁰

Ushenko writes as an intuitionist, that is, as one who requires the mediation of concepts between words and things. On that ground he is therefore disposed to deny that semantic logic is logic.

¹⁷ Hofstadter, On semantic problems.

¹⁸ Grundlegung, quotation from Ushenko, Problems, pp. 120-1.

¹⁹ Problems, p. 120 ff.

²⁰ Grundlegung, quotation from Ushenko, Problems, pp. 121-2.

This attitude simply supplants the essential criterion of logic as system building with another criterion.

Although from the detached standpoint of pure system building one conventional system is as good as another, still the semantic version of symbolinguism is potentially more important than the syntactic. This evaluation is made on the assumption that a philosophical logician would shirk from confining himself to the sheer organization of sentences, and therefore it is an advantage to stress the connection of sentences with things referred to. If we grant this point we must next ask whether semantic systems are purely linguistic. Clearly, the enterprise of ascertaining whether a definite and satisfactory correlation exists between sentences and referents is a type of investigative enterprise that goes beyond any kind of language activity. On the other hand, if semantics constitutes only an occupation with sentences the differences between the syntactic and semantic phases vanish.

The insistence that the semantic form of symbolinguistic logic is a new type of logic signifies primarily that logicians sooner or later discover the weakness inherent in systems made up of language elements. Because of the symbolinguistic tradition, however, they do not at once see that they cannot go much further with the rules for properly organizing sentence systems. Certainly the organizing of sentence systems cannot serve the purposes of scientific investigation. The lack of novelty in semantic logic becomes plain when we consider that even the traditional Aristotelian system was employed by its proponents in such a way as to take account of the correlation between things and sentences. The important criticism, therefore, is not that Semantic logic is not new, nor that it fails to establish any sort of close relationship between logic and truth, but rather that the exclusive preoccupation with sentences does not constitute an adequate logical procedure.

Symbolinguism and Scientific Linguistics. Despite the fact that symbolinguistic ideas can be precisely dated historically, symbolinguists display a notorious reluctance to consult the records of history. So great is their desire to avoid the pseudoproblems of traditional philosophy that they disregard the cultural matrix of their thinking and consequently work with abstracted linguistic materials which they regard as self-existent things, though subject to given rules.

More unfortunate, perhaps, than their inattention to historical origins is their disregard of current scholarship. Because of the great emphasis symbolinguists place upon words and symbols we might expect them to show considerable interest in modern linguistic science. As a matter of fact, however, symbolinguism fails to comport with the findings of scientific linguistics. Precisely because symbolinguists neglect fields in which symbols are set up and used, including the circumstances which give rise to these operations, they cannot properly associate their systems with thinking or science. When we observe the actual operations of logicians or mathematicians, we do not find it feasible to consider symbols as merely conventionally established words of a given text. Nor can syntax be taken to consist of two kinds of organizational processes—namely, simple rules of (1) formation and (2) transformation of sentences.¹¹ No more can semantics be viewed as the basis of simple relations between sentences and referents.

Symbolinguistic logic is based upon several mistaken notions. One is that the fixed and objective character of symbols is made plausible by mathematics. Surely we have here an undesirable confusion. It is true that when we start with a mathematical system the elements take on a definiteness and a set of interrelations which are constant for the system. But we cannot overlook that the postulates and operations have been deliberately selected.

Again, mathematics and symbology are implicitly regarded as language. Clearly, the processes and activities of mathematical enterprises are not discriminated from the products and descriptions of such enterprises. Not to observe that mathematicians embrace problems of particular sorts, and perform all kinds of processes such as counting, measuring, discovering, and organizing relations, is like reducing architecture to the hauling and piling of stones.

To confuse the fact that we may abstract ourselves from mathematical conditions, and attend merely to involved processes, with the notion that mathematics or logic is the manipulation of contentless symbols is an excellent instance of disembodied thinking. True enough, we may operate with contentless symbols as imaginary objects, but there is nothing more definite than the operations of speaking of such symbols or manipulating them by substitute

¹¹ Carnap, *Logical Syntax*.

processes. Even independently existing Platonic ideas are definite constructions in specific operational situations. When we are concerned with words, signs, or number symbols, it is impossible to regard them as anything else than things with which persons interact.

Actually, of course, no logician is content to regard his work as the production of tautologies or systems of trivial relations. At some point the most abstract and formal logicians are ambitious to discover the essential processes of reasoning, to understand the basic principles of mathematics or perhaps to develop formal processes useful in science. Some symbolinguistic logicians go only so far as to promise "a sharpening of the concepts of science."²² Others regard logic "as the name of a discipline which analyzes the meaning of the concepts common to all the sciences, and establishes the general laws governing the concepts."²³ Tarski, whom we have just quoted, immediately admits that in the empirical sciences to date semantic logic has not found any specific or fertile application. Quite properly he observes that the empirical sciences include human behavior as well as asserted statements.

At this point it is not necessary to enter into a comprehensive criticism of the untoward results of the symbolinguistic conception of logic and science. We merely suggest that even in the science of mathematics the symbols and equations used are actually results of experimentation or reflection, though these activities are fundamental interbehaviors with relations and not with things. Both Leibniz and Newton could develop the calculus, though they used different symbols. Although their calculative aims and products were common, the different symbolisms they used were the results of detailed differences in activity. Whereas Newton was influenced by his interest in fluxions or variable quantities generated by the continuous motions of points, lines, and planes, Leibniz was more particularly interested in such geometric problems as the determination of tangents to curves.²⁴ Familiar illustrations of the part played by workers and their problems can be discerned in the various alternative forms produced for common logical relations such as $a(1-b) = 0$, $a \subset b$, $a = b$, all a is b .²⁵

²² Quine, *Mathematical Logic*, p. 8.

²³ Tarski, *Introduction*, p. xiii f.

²⁴ Cajori, *History*, p. 196 f.; and Boyer, *The Concepts*, chap. 5.

²⁵ Cf. Lewis and Langford, *Symbolic Logic*, p. 15.

We hasten to add, however, that we thoroughly disagree with those who condemn symbolinguistic treatises as "compendia of nonsense."¹⁸ Abiding, rather, by Spencer's dictum that every serious theory has some merit, we may agree that symbolinguism has a basis in adequate and competent criticism. When this criticism, however, leads to positive construction concerning the nature of language, logic, and mathematics, the whole movement misses its mark completely, as we shall see when we later contrast the inter-behavioral hypothesis of language with the formalistic view concerning symbols and sentences.

B. *Mathematical Logic*

Centered in the complex called mathematical logic is an intricate combination of traditional cultural trends, current theories and motives, and particular techniques. Above all, mathematical logic, as the name implies, is cultivated by persons interested in mathematics, although some of its proponents simply regard mathematical logic as a development of Aristotelian formal logic toward greater generality. In mathematical logic deduction has broken the bonds of the syllogism, so that instead of being confined to the typical system all M is P, all S is M . . . all S is P, logic moves on to: If a is b, and b is c, then a is c. Moreover, rigorous inference under the aegis of mathematics progresses from the subject-predicate proposition and simple classes to the propositional function and complex relations.

Two Phases of Mathematical Logic. Because of the varying, specific, cultural influences operating upon the proponents of mathematical logic they differ in their ideas concerning its origin, nature, and significance. We may then differentiate between two variant aspects of mathematical logic which we designate as the logistic and the algorithmic.

(1) *Logistic Logic.* Under this rubric we place those ambitious formulations designed to reduce mathematics to logic. Logistic logic is not intended to be merely a special symbolization of mathematics as Frege¹⁹ and Peano²⁰ attempted to make plain. Especially

¹⁸ Cf. Black, Introduction.

¹⁹ Grundgesetze, and Grundlagen.

²⁰ Formulaire.

Peano and his collaborators wished to add to the work of translating mathematics into a rigorous logical symbolism a demonstration that each branch of mathematics must be precisely deduced from a small number of assumptions. Russell, as a proponent of logistic logic, goes even further and expresses his belief that logic is "the elementary part of mathematics and the logical prerequisite of all the rest."²⁹ Indeed, he continues until he reaches the position that not only does mathematics have a logical basis, but that mathematics is nothing but logic. In the preface of his *Principles of Mathematics*, Russell undertakes "to prove that all pure mathematics deals exclusively with concepts definable in terms of a very small number of fundamental logical concepts and that all its propositions are deducible from a very small number of fundamental logical principles." Thus the program Russell sets himself appears to be an annexation of the postulational technique known as hypothetico-deductive systematization. But this is only part of the program. The other "is the explanation of the fundamental concepts which mathematics accepts as indefinable."

Logistic logic therefore is not merely mathematics as the work and products of technical mathematicians, but a particular type of philosophy as well. In a unique sense it is foundation material presupposing the existence of about eight or nine logical constants in a timeless realm of Being. Among these logical constants are terms like any or every class, denoting, implication, relation, relation of a member to its class, the notion *such that*. Systems or orders are definitions developed as a process of reporting the existence of these logical entities. Definition, then, is enumeration. Pure mathematics consists of series or classes of propositions, p implies q defined in terms of logical constants.

If we take such a system as typical of logistic logic we find its basic characteristic to be an *a priori* self-contained system of inter-related propositions. Its validity does not lie in its comprehensiveness or cohesiveness, but in the immutability and revelatory character of numbers. Insofar as such a system is at all hypothetico-deductive the latter process begins only after the adoption of the absolute numbers and their progression or relations.

Exhibited here is the profound conviction that logic is ultimate and unitary. Thus, when Russell becomes sceptical of the Platonic

²⁹ Theory.

character of his logical constants he does no more than attempt to correct the system. The result is doubt and inconclusiveness. Russell himself dramatically describes his own changes of view. As he proceeds from the *Principles of Mathematics* to *Principia Mathematica* he is obliged to drop out not only specific constants but to modify his conception of their nature. Of primary importance is the abandonment of classes.

The symbols for classes, like those for descriptions, are, in our system, incomplete symbols; their *uses* are defined, but they themselves are not assumed to mean anything at all. . . . Thus classes, so far as we introduce them, are merely symbolic or linguistic conveniences, not genuine objects.²⁰

Because cardinal numbers had been defined as classes of classes they became "merely symbolic or linguistic conveniences." As Russell declares, under the influence of Whitehead he abandoned points of space, instants of time, and particles of matter, substituting for them "logical constructions composed of events." Despite the apparent growing appreciation of the importance of construction in logical work the self-subsistence of logic is maintained, and while there is an expanding recognition that entities are linguistic constructions Russell is unable to accept this view wholeheartedly.²¹

Probably a valid conclusion is that logistic logic consists fundamentally of a process of system building of which the materials are mathematical products and the manipulative procedures that of description. In other words, certain views of mathematics are adopted and descriptions are created in a symbolic medium. Then mathematics is regarded as identical with the descriptions. It is difficult to escape the conclusion that we have here the fallacy of confusing descriptions with the objects described, in this case mathematics. Notice an instructive analogy with the syllogistic theory of logic. Just as syllogistic logic selects certain inferential processes and exhibits them in syllogistic form, so logistic logic takes certain mathematical materials and shows how they can be set up in the form of symbolic deduction.

(2) *Algebraic or Algorithmic Logic*. Whereas logistic logic is regarded as the foundation of number and space concepts or as

²⁰ Whitehead-Russell, *Principia*, vol. I, p. 71 f.

²¹ *Principles*, Introduction, p. xi.

identical with mathematics, algorithmic logic consists essentially of the application of mathematical procedures to the development of logical systems. Algebraic logic constitutes the generalization of traditional formal logic through the adoption of the processes and general laws of algebra. The development of this phase of mathematical logic can be traced directly to the work of De Morgan, Boole, Schroeder, Jevons, Peano, and Peirce, among others. These beginnings are not merely of historical interest, since they are deeply incorporated in present day logic. For this reason and because the kernel is simpler it is not inappropriate to indicate the nature of Boolean logical algebra.

Fundamentally Boole was concerned with the improvement of Aristotelian or classical logic. As he explains,²² although from time to time he had been interested in logical speculation he was especially stimulated to formulate and publish his ideas by the 1847 De Morgan-Hamilton controversy concerning the quantification of the predicate. It occurred to him that if logic could be looked at from *without*, that is, as connecting itself through number with the intuitions of space and time, it could also be regarded from *within* as based upon facts connected with the constitution of the human mind. In adapting algebra to logic Boole resorts to a process of generalization. First, the two kinds of logical relations—that among things and that among propositions—must be generalized and reduced to relations among propositions. Next, the things and propositions must be reduced to elements in such a way that “the *premises* of any logical argument express *given* relations among certain elements, and that the conclusion must express an *implied* relation among those *elements*, or among a part of them.”²³ All this can be accomplished by the process mentioned above, namely, by adopting particular symbols as part of the language which manifests the operations of reason.

Recalling Poincaré’s remark about the fecundity of mathematical logic in engendering paradoxes we find at the very inception of algebraic logic that the generalization actually constitutes a specialization. A fundamental feature of Boole’s logic is that it should deal with no other numbers beside 0 and 1, 0 being interpreted as nothing and 1 as the universe. Furthermore, by contrast

²² *Mathematical Analysis*, Preface.

²³ *Laws*, p. 8.

with general algebra in which xx equals x^2 , in Boole's system xx equals simply x . The basis for this selective construction lies no doubt in the declaration that "it is not of the essence of mathematics to be conversant with the ideas of number and quantity."³⁴ The algebra set up is concerned with classes instead.

The operation of language as an instrument of reasoning can be carried out by three classes of symbols.

(a) Literal symbols, x , y , z , etc., representing things as subjects of conceptions. These are symbols of election, for by means of them classes of nouns, qualities or circumstances are elected for logical arrangement and calculation. The negative or 0 of these symbols is $(1 - x)$, $(1 - y)$, and $(1 - z)$, for the not- x 's, not- y 's, etc., are everything except the x 's, y 's, etc.

(b) The second or operational class of symbols is already presented in the negative symbols, representing the notion of non-selection or nonaggregation. For example, if x represents man and y Asiatic men, $x - y$ stands for "all men except Asiatics." The symbol $+$ standing for "and," "or," is used to bring parts together to form a whole. $x + y$ aggregates "trees and minerals" or "barren mountains or fertile valleys."

(c) The symbol $=$ expresses relation and forms propositions. "The stars are the suns and the planets" may be symbolized $x = y + z$. Simple equality is represented by $x = y$, or $x = x$.

Since this algebra is like general numerical algebra except for the proposition that $x^n = x$, it is governed by the general laws of distribution, commutation, and association, and, of course, the general law of transposition. Moreover, since $x^2 = x$ can be written $x - x^2 = 0$, and this is the same as $x(1 - x) = 0$, the laws of contradiction and excluded middle are fundamental to the system.

With the modification in symbology such as the substitution of $-x$, $-y$, etc., for $1 - x$, $1 - y$, and the improvements introduced by Jevons, Peirce, and Schroeder, the Boolean type of logic may be taken as a fair sample of algebraic logic.

Further light is cast upon the nature of algebraic logic by considering the large number of particular systems which may be evolved. In one situation two of Peirce's students (Ladd-Franklin

³⁴ Ibid., p. 12.

and Mitchell) developed two new modifications of such algebras following five which had already been formulated.³⁵

Despite the large number of such algebraic logics possible the field is still pervaded by the notions of ultimacy and generality. Algebraic logic is presumed to constitute a single exclusive system, which through its generality is not only a model but the very essence of reasoning and also embodies in itself the fundamental character of reality. As is clear in Boole's writings, this algebra or calculus mirrors the fundamental nature of thought, but at the same time it is likewise a reflection of the fundamental nature of things. In this sense mathematical logic is in part based on the Pythagorean notion of numbers as the essence of all things. Furthermore, deduction, as a basis for geometry as well as algebra, constitutes the essence of reasoning.

The Relation of Logic and Mathematics. Although both symbolinguistic and mathematical logicians pay their respects to classical formal logic³⁶ it is undeniable that today they are more concerned with mathematical forms than with logical ideas. Why is this the case? Because logic must be totalitarian, rigorous, and objectively independent, and apparently it is assumed that only through mathematics can these qualities be attained. At once the question arises whether in view of the recent arguments concerning the foundations of mathematics logic can become absolute through mathematics. Indeed, the difficulties inherent in the mathematical foundation suggest that the circumstances are quite reversed and that mathematicians are intent upon achieving the rigor and certainty of mathematics through various cementing qualities of logic. Obviously, both mathematics and logic require reformulation as well as the relations between them.

For the most part both mathematicians and logicians display an unsalutary deviation from the operational principle. They build up their theories not so much from observations of mathematical and logical work as from various historical principles. Logic and mathematics certainly reflect a type of thinking which is not merely cosmic, metaphysical, and absolutistic, but even theological. How overlook the large place that infinity studies have contributed to

³⁵ Peirce, *Studies*.

³⁶ Tarski, Quine, Eaton, Cohen and Nagel, etc.

their general development! Consider but the contribution of Bolzano, Royce, and Cantor with their mystic interest in the absolute.*

Basic to the interrelationship of mathematics and logic is the hope of each discipline to derive from the other some powerful and useful support as a guarantee of its potency and pervasiveness. The logician who seeks to root his system in mathematics plants himself in the soil of well-established techniques, confident in traditional values and in the record of solid, practical achievement. But since the logician is a totalitarian he takes mathematics to be a generalized system beyond the particular work and accomplishments of mathematicians. Too often the logician overlooks the possibility of misinterpreting mathematics when its character and foundations are assumed without regard to its history and development. Accordingly, to build upon mathematics may result in inadequacies, uncertainties, and highly undesirable contradictions.

Likewise, the mathematician who builds upon logic similarly seeks some fundamental prop for his subject, some generalized certificate of validity and perennity. Sometimes he hopes to secure the laws of reasoning or the ultimate character of objective relations. The mathematician thus accepts certain basic, absolutistic, and reliable principles. What he frequently gets is a mystical philosophical theory, for example, a Platonic realism which is thought capable of explaining undefinables, irrationals, and other puzzles of historical mathematics.

One consequence as certain as it is simple follows from the longing glances cast at each other by logic and mathematics in the hope of obtaining general and absolute principles. It is the failure of both to achieve this end. In the meantime the faults of logic are attributed to the difficulties of its mathematical foundations, whereas in turn the instability of mathematics has been ascribed to the inadequacies of its presumed logical groundwork.

Underlying all attempts to bring mathematics and logic together are various presuppositions concerning the similarities of aim and technique in the two disciplines. But immediately we ask: Is mathematics a universal form of reasoning or inference? Is it a single kind of process of any particular sort? Inherent in all identifying or relating enterprises is the process of simplifying and abstracting

* Klein, *Vorlesungen*, part I, pp. 52, 56.

the alleged relata. Today it is universally recognized that there are three distinctive interpretations of mathematics, the formalistic, the intuitive, and the logistic.

Certainly we must challenge the notion that mathematics is an exclusively formalistic and ultradeductive enterprise. Is it not improper to ignore its synthetic and constructive character! In this connection one of the authors of the *Principia Mathematica* complains that the formalistic notion of mathematics as represented by Hilbert overlooks the content.³⁸ The formalist, Russell asserts, forgets that arithmetic has practical uses and that, while the application of numbers to things lies outside arithmetic and logic, a theory which makes this application *a priori* impossible can not be right.

Russell as a logician in mathematics, however, does not consider its instrumental and practical character. By sheer definition and selection he excludes counting, calculation, and so forth. Through similar arbitrary procedures he makes mathematics ultimately and invariably deductive, and consequently emphasizes certain descriptive features and regards them as covering the total.

Essentially the logistic mathematician sets up his own theory of mathematics and considers that formulation the essence of the subject. Whatever merits may be ascribed to his theory accrue to it because it does cover certain important aspects of mathematics, such as its system-making feature and systematic products. This, however, is not the whole of mathematics. To be added are the problematic and solutional factors. An incomplete and inconclusive theory automatically results from considering system products without regard to the processes necessary to build up those systems. A possible defense here is that deduction or implication covers the whole set of problematic and solutional processes. The issue is plain. Either the logistic mathematician simply uses the word deduction to cover the other processes, though he does not employ them or show that they are covered, or else he leaves out all the important constructional features.

Does deduction-implication comprise the principles which such mathematicians as Kroneker, Brouwer, and Weyl regard as the essential feature, whether or not it is called construction?

Theories of mathematics like other types of theory may be divided into two general sorts. The first consists of construction

³⁸ *Principles*, Introduction, p. vi. Cf. also Ramsey, *Foundations*, p. 2.

of principles derived from the essential processes of mathematics and which in their essence simply describe fundamentally what those processes are. Such theory may well contribute to the further development of mathematics and not merely refer to certain features of the finished products.

By contrast the other sort of theory does simply the latter. It consists of analogical description; the construction comprises a set of terms at worst exhibiting the finished products and at best hinting at the principles leading to those products. As such it is considerably more arbitrary in the selection of essential features. Even though such a theory is of some service in indicating certain mathematical processes, the selective criteria are directly based upon certain presuppositions concerning the place and nature of inference in mathematics.

It is hardly a valid criticism of the logical theory of mathematics to ask how it came about that so much important mathematics was developed before that theory ever became formulated. At the same time we must consider whether this theory has been or might be conducive to mathematical development. Certainly, logic as system building and as the theory of system building sustains a certain correlation with mathematics which also involves system building. But, as we have already indicated, there are other features to be considered, such as the actual processes of making these systems as well as numerous other mathematical processes and results. Again, we ask: Have the constructive and formalistic consistency theories no merit as descriptions of system construction?

Grant, then, that some features of mathematics are logical, but this is far from tantamount to an identity of the two disciplines. The objection is twofold. As suggested, we are equally opposed to making all mathematics simply logic, even if not formal and tautological logic. On the other hand, logic as a complex and diversified enterprise can in no way be identified with mathematics. Only one type of logic builds systems out of relations which are the fundamental material of mathematics. An issue like the present one throws into relief the interbehavioral principle which stresses the work of a system maker either in following certain traditional rules or in building up new rules depending upon the kinds of elements related. On this basis logic must be freed from the rigid

limits of abstract implication or deduction. Nor can logic be limited to systems of propositions or propositional functions, the organization and relation of classes or empty relations. Whether or not one regards logic as the activity of reasoning, its ample and varied scope in dealing with all sorts of things and problems must be allowed fair play.

An important basis for attempting to bring logic and mathematics together is that the latter has been progressively approaching logic, which has always been thought to be the most generalized and formal of disciplines. But consider how mathematics has been constantly increasing its repertoire, moving from positive and negative integers and fractions—the natural numbers—to irrationals, imaginary, complex, hypercomplex (quaternions, vectors) and transfinite numbers, and even absorbing non-Euclidean and abstract geometries. Such generalization is frequently referred to as an emancipation from numbers and from intuition of time.

But is generalization the same thing as logic? Even interpreting this generalization as system building does not connect logic and mathematics. Every human activity, not excluding art and the practices of daily life, involves generalization and system building. The most we can say, then, is that some logical procedures are common to all activities. Similarly, all complex and some simple human activities involve mathematical exercises like counting, measuring, etc.

As to the formality of mathematics, here again we describe a discipline on the basis of some partial and contributing characteristic. Actually neither logic nor mathematics is formal or empty. To make mathematics formal one must select out pure as distinguished from applied mathematics. But even pure mathematics is not formal; it is a series of operations upon certain kinds of relations. There is no satisfactory warrant for characterizing relations as *no things*; surely from the standpoint of operational fields it is a disastrous procedure.

Furthermore, most complex affairs of ordinary life involve formalization and abstraction from an actual plenum. Our very conversational language, our writing, and reporting demand abbreviation, short cuts, symbolization. Any identification of mathematics and logic, therefore, seems highly inexpedient.

C. Logic as the Science of Order

When the science of order is well formulated it is legitimate to regard it as a new and distinct development in contemporary logic. In doing so we do not, of course, overlook the fact that the science of order is fundamentally interrelated not only with symbolinguistic and mathematical logic, but also with other logical theories as phases of the general cultural complex called logic.

The distinctive feature of order logic is its emphasis upon the ordering of relations as an essential system-building process, with its hierarchical development of principles and products. The kernel of such logic is the stress of the postulational or hypothetico-deductive principle. In essence this implies that system building is definitely regarded as the work of logic, although order logic is often considered as too closely allied with the distinctive motives found in various mathematical and symbolinguistic logics.

Order logic as acknowledged system making may well be taken to illustrate the fundamental logical characteristics. For instance, notice how large a place is given to such problems as the consistency and completability of systems. Another of its merits is the stress placed on the multiplicity of possible systems when particular hypotheses are adopted.

Unfortunately the facility with which systems can be constructed on the order-logic plan is usually interpreted as the discovery of a basic technique when actually it reveals the principle of persons at work, setting up various organizations or structures. No doubt this technique is regarded as rooted in fundamental psychic processes or as yielding final systems. Some of the proponents, for example, Royce, assume that ordering technique, whether in the form of propositions or classes, must always yield a hierarchical product with some sort of absolute or infinite at the apex.²⁹ Such writers miss the point that, after all, the systems are the products of certain premises or presuppositions and hence do not achieve any greater validity than those presuppositions.

Order logicians frequently ignore the fact that the primitive elements or axioms obtain their status from a projected system, such that the theorems are in some sense implied in the primitive propositions or axioms. The closed circle here exhibited is not

²⁹ Principles, sec. 3.

necessarily vicious. When proper regard is had to the presuppositions and the need for making such systems one may avoid trivial tautology or a tangent toward the infinite. It is only necessary to guard against interpreting the symbols and propositions and other features of the system as having their own status independent of the system maker.

A transparent error is to use the term *true* or *false* and then become oblivious to the fact that these are simply arbitrary classifications for a particular system, thus unwittingly giving the system a greater scope and validity than it actually warrants. On the contrary, any system can be properly evaluated only as a particular enterprise, and as such it may be very useful and altogether valid.

D. *Humanistic or Nonformal Logic*

Though humanistic logicians do not question the efficacy and ultimacy of logic they do not agree with formal logicians concerning the nature of reasoning and knowledge. Logical humanists being more impressed by the humanistic aspect of our culture than by the need for precision and exactness are sensitive to the perennial resurgence of human life and are thus unwilling to make logic abstract and formal.

Of the many aspects of humanistic or nonformalistic logic let us first consider the negative one which consists of a vigorous attack upon traditional Aristotelian logic. Basically this attack concerns Scholastic verbal logic, though the Scholastics' own name for it—Aristotelian logic—is retained. But the objections are generalized and made to include symbolic logic as well.⁴⁰ To the defense that symbolic logic may be close to and valuable for mathematics the rejoinder is offered that mathematics itself is formalistic and limited and in general a minor feature of human life.⁴¹ Even Aristotelian formal logicians object to symbolic logic on the ground that it is concerned exclusively with classes and relations. Logic which deals with thought of things, they hold, cannot limit itself to secondary subjects of thought.⁴²

Because humanistic or nonformal logic is primarily critical and rejective the available expositions are rather vague and sparing

⁴⁰ Schiller, *Formal Logic*, chap. 24.

⁴¹ Schiller, *Logic*, chap. 19.

⁴² Joseph, *Introduction*, p. 12.

of positive details. As we have indicated, the criticisms on the whole are made in the interest of concreteness, practicality, and use. The humanistic logicians would not, of course, admit that they lack positive principles. On the contrary, they insist that the rejection of formal tightly knit arguments such as syllogistic systems is itself a definite and positive principle. Schiller defines logic as a systematic evolution of actual knowing and thinking.⁴³ The fundamental significance the definition is intended to stress is the exclusion of errors.

In his own defense the humanistic logician argues that a new interpretation must be put upon the term *positive*. Allowing for the differences in purpose and need humanistic logic is systematic, coherent, normative, and hence positive. The corresponding change necessary in the conception of logic is to substitute for an extreme intellectualism some sort of voluntarism—the activity of the person in his general experience or life conditions, rather than the limited rationality of the formalists. In line with this view the humanistic logician is satisfied to compensate the paucity of detail in his exposition by referring to all the developments of science.

By far the easiest type of intellectual exercise is to uncover criticisms in established theories. Still, one may regard as valuable much of the criticism which the humanistic logician directs against formal logicians. The acceptance of his alternative, however, is another matter. Since voluntaristic logic itself is based upon the assumption that logic is one, the issue is not whether it can substitute for and replace formal logic, but rather whether or not one of the worst faults of formal logic is that it is exclusive. Actually, neither of these types of logic is exclusive. Voluntaristic or humanistic logic, if it is logic—that is, involves system-building procedures—at most stands beside formal logic as another kind of system-building enterprise. Obviously, then, it is a serious weakness of voluntaristic logic that its proponents regard it as the only logic and that it supersedes all others. Another criticism is that the technique of establishing the value of humanistic logic consists of an attempt to assimilate many virtues which belong to no logic, but to science and other disciplines as well. No other than the universalizing and rationalizing tendencies of logicians lead the voluntaristic logician to assume an identification between his type

⁴³ Logic, chap. 2.

of logic and all successful and valid intellectual disciplines.

Despite the claims of humanistic or nonformal logic to represent all experience and in general the concreteness of human life, it, too, consists to a great extent of propositions and their organization. Furthermore, the proponents of nonformal logic assume that logic is the record of how people actually think. Yet what could be more formalistic and improperly abstract than the assumption that there is a single, general pattern of thinking!

It may well be denied that logic is exclusively intellectualistic. But voluntarism does not escape either the stigma of exclusiveness or intellectualism. Voluntarism like intellectualism formalizes abstract logical procedures and replaces the specific operations of individuals who build systems. Whenever a system is erected on the basis of definitely voluntaristic criteria a voluntaristic logic is established. If there is such a type of logic it takes its place among other products of system-building procedures, no matter how different from the others it may be.

E. Instrumental Logic

Whatever may be the fundamental characteristic of a particular logical view it is bound to share features common to others. In the case of instrumental logic, however, it is extremely difficult to isolate what is common to it and other logics and what is its unique contribution. True, the proponents of this logic make definite descriptive distinctions between it and other logics, but in their actual expositions these distinctions lose their point.

If Dewey's treatise be taken as an exemplar of instrumental logic we find that he plainly sets forth its goal as the achievement of warranted assertions.⁴⁴ Basically logic is described as an instrument for carrying out the purposes of practical life and science, with a possible exception made of mathematics. As in humanistic logic the principle is stressed that logical processes are fundamentally biological and social in character, and therefore the basic purpose of logic is that of correlating means and ends. The claims of common-sense and human experience are generously granted in instrumental logic by defining it as the theory of inquiry.

At the bottom of this inextricable mixture of old principles and new assertions is the fact that instrumental logic is a phase of the

⁴⁴ Logic.

perennial logico-philosophical tradition.⁴⁴ Accordingly its asserted novelty and uniqueness lie in the criticism of its correlated members in the tradition, whereas the essential characteristics of instrumental logic are very much like the preceding logics.

In characterizing instrumental logic, therefore, it is essential to indicate that its critical feature is far more fundamental and unique than its positive and systematic factors. Indeed, probably the most valuable aspect of instrumental exposition is the detection and description of the faults and difficulties in older logical systems. The positive and constructive details which align instrumental logic with traditional logic can be summed up in the statement that the logical domain covers the relations of propositions to one another and that logic is concerned with form rather than with matter.⁴⁵ Although Dewey states that his treatise applies instrumental logic to the forms and formal relations which are the standard materials of logical tradition, we may characterize his logic as propositional operationism. Discourse and not concrete things are his subject matter. The basis upon which instrumental logicians criticize other logics is that, in the latter, form is separated too far from matter, whereas instrumentalism deals with forms of matter.⁴⁶

Crucial questions arise. Do the proponents of this type of logic make good their claim to set aside formalistic and deductive systematizations in favor of actual inquiry? Or, do they transform events into abstract relations quite in the manner of formal and mathematical logicians? And above all, does inquiry concern propositions or linguistic methods rather than actual things, radiation, energy transformation, growth, etc.?

So far as its critical phase is concerned instrumental logic no doubt reflects a unique characteristic of current culture. More than any other type of logical theory it celebrates the tremendous technological and general constructive character of our civilization. An important aspect of our age is the fecund development of techniques and processes necessary for an elaborate utilization of natural things and events and the mass production of goods of all sorts. Undoubtedly the reaction to these events impresses us with the increased ability of men to manipulate things and achieve

⁴⁴ Ibid., p. 2.

⁴⁵ Ibid., Preface, and p. 105.

⁴⁶ Ibid., p. 372.

enormous creative powers. The upsurge of ideas concerning operations and operational processes excellently reflects in the mirror of theoretical formulation what is happening in our civilization. This operational principle is stressed not only in the practical and theoretical sciences, but likewise in the examination of intellectual theory and construction. The resulting emphasis upon the constructional processes of individuals points to the worker's increased place in all sorts of situations, in contrast with other periods when emphasis upon man meant a minimization of working materials. In contemporary thinking to the contrary, it is impossible to disregard the materials worked with, even when these are products of earlier operations. In logic, then, an emphasis must be placed upon the linguistic and symbolic construction both as materials to operate upon and as products of such operation.

Envisaged from the standpoint of its critical phase instrumental logic makes a decided appeal. A pity that the discoordination between its two phases irreparably injures the system. The reiterated plea to abandon the *a priori*, if thoroughly heeded, would alone mark the instrumental view as a landmark in logical thought. Unfortunately, although this attack upon the *a priori* is never relinquished it certainly is weakened in the development of the system. Equally meritorious, too, is the assertion that logic is naturalistic and progressive and not a finally formulated enterprise.⁴⁸ Although this view articulates with the notion that logic is exclusively the theory of scientific investigation it does recognize the fallible and operational character of logic as a concrete human undertaking. On the other hand, it would not be surprising if this fallibility were interpreted as an expository concession, since there are positive signs that instrumental logic is taken to be "the only view that provides a constant, logical interpretation of facts."⁴⁹

If instrumental logic really were constructed on the basis of its criticism of other systems it doubtless would possess a very different character from what it now has. For instance, it would not stress form as compared with matter even on the plea that it deals with form *of* matter. It would be more concerned with concrete problems without adopting the grand manner of traditional logic. The instrumentalist, in other words, would not, like other logi-

⁴⁸ Ibid., p. 14.

⁴⁹ Ibid., pp. 453-4.

cians, be dominated by the ultimacy principle and therefore resort to a generalized formal pattern of inquiry. Not only would he not regard logic as dealing with forms, propositions, and organization of propositions, but he would scarcely consider logic as taking over all the activities of reasoning, problem solving, and scientific practice.

Again, if instrumentalism actually were concerned with concrete inquiry or problem solving it would not be exclusively occupied with the organization of propositions but would include the organization of other things also. And this not merely by assertion but by practice. Otherwise, products are stressed instead of processes, and the actual contacts with things give way to paradigms, to descriptions of what logic does or ought to do. Furthermore, instead of making logic into a general theory stressing the relations of ends and means it would be more closely integrated with specific and detailed system building. As it is, the specificity of systems and system production is overlooked, and reasoning and inference are made into abstract patterns of action.

To regard logic as the theory of "inquiry as such," or as the "generalized account of the means by which sound beliefs on any subject are attained and tested"²⁰ is to incline more to a form of dialectics than to concrete activities in specific situations. Despite the pretense of dealing with things the subject of logic becomes a set of rules—necessarily indefinite and vague, of course—for thinking or thinking properly. Actual human situations, however, do not lend themselves to general and comprehensive rules. Probably the worst consequence is not the great discrepancy between promise and performance, though this is great enough, but the transformation of things into objects of knowledge, a transformation which encourages subjectivity, verbalization, and even the spiritualization of things. To look upon logical forms as the special language or set of symbols necessary for competent science,²¹ and to regard these forms as accruing to subject matter after inquiry is instituted, is to remove logic a considerable distance from real things.²² The assertion that logic deals with forms in no way describes the fact that logicians of every persuasion fundamentally

²⁰ Ibid., p. 335.

²¹ Ibid., p. 334.

²² See McGilvary, *Professor Dewey*.

are system builders. The value of a logic can be rated on a scale indicating convergence to or divergence from a universal or cosmic system.

Linguistic Factors in Instrumental Logic. Interrelated with the totalitarianism and universalism as well as the formalism of instrumental logic is the implied view on language. Both because we are dealing with the theory of logic and because we are concerned with an expository treatment it is necessary to have linguistic materials prominently before us. This linguistic material, however, proves a stumbling block to the instrumentalist.

Quite properly Dewey asserts that things have to be formulated.⁵³ In this sense language may be regarded as an instrument or set of symbols for the effective manipulation of ideas. But it turns out that, after all, these linguistic factors lose their instrumental character and are actually products, elements of the system. In other words, there is a retreat from things to linguistic abstractions. Dewey is constantly shifting from language as communicative, as referential acts or their transcription, to language things constituting the fixed materials of logic. Logic, then, is made to deal with descriptions rather than with investigative events. Descriptions, to be sure, are necessary in expository enterprises, but the terms of description assume a rigidity and definiteness which are intensely reminiscent of the old logics. In this connection it seems anomalous for the instrumentalist to eschew symbolism and to inveigh so forcibly against formalism in logic.⁵⁴

The descriptions with their verbal and other linguistic materials may well be regarded as constructions. That is to say, the linguistic materials constitute ways in which the expositor of logic either refers to or symbolizes the items which he is describing. To respect this distinction between construction of theory and the thing theorized about would, in short, help to maintain a proper perspective toward logical forms, propositions, and other features of logical theory.

The Psychological Implications of Instrumentalism. The instrumentalist insists that his psychology is naturalistic. In fact, Dewey asserts that he scorns anything but biological psychology. Nevertheless, there appears to be considerable confusion in dealing

⁵³ Ibid., pp. 283 ff.

⁵⁴ Ibid., pp. 374 ff.

with knowledge and the contact of the inquirer with objects. Logic according to Dewey has to do with knowledge which is established by making an indeterminate situation determinate by means of judgment. Accordingly, the logical characteristics of objects are asserted to be established through inquiry or investigation. Here the proper position is taken that knowledge can only be of objects known. Soon, however, objects known are confused with objects. This skirts dangerously upon Berkeleyian mentalism. And so unless the instrumentalist is asserting that in inquiry one moves from known objects to other known or better known objects, which amounts to a tautology, he clearly jumps to the view that objects are created in the act of knowing or as a result of inquiry.

Is Logic the Theory of Inquiry? Despite the instrumentalist's own view to the contrary, and despite long and honorable tradition, it would not be objectionable to use the name logic for a discipline whose work it is to theorize about inquiry or investigation. Those who incline toward this view would, of course, find it feasible to use another name for logic as described and practiced by still other workers. As it happens, however, instrumental logic does not deal with actual inquiry or investigation. It deals with propositions and the relations of propositions.

Perhaps, after all, there is no great merit in shifting the term *logic*. Since that term is well established to mark off a soundly established discipline, we are possibly merely obliged to indicate specifically with what materials any particular discipline is concerned. The theory of inquiry, in the sense of building up a system of views or propositions concerning inquiry, is by all means a type of logic. It cannot, of course, eliminate the other systems.

CONTEMPORARY GENERAL AND SPECIFIC LOGIC

The fact that we find so many different trends and trend variants in contemporary logic simply illustrates the law of complication in culture. The inevitable changes taking place in continuing cultural items merely emphasize the presence of those items. The large and small variations resulting from individual interactions with these institutional items help to maintain and increase the number of institutions. Just as a particular language differs in detail in the spoken behavior of individuals, so items of worship, technological practice, and logical construction serve to enlarge the

general cultural complex. At the same time, the accumulative principle of culture is demonstrated in the coexistence of various major and minor modifications within the parent trends.

A trenchant implication for logic derived from observing the operation of these principles of culture is that the influence of a logical trend upon a logician is neither absolute nor inevitable. Though every logical system or viewpoint is an outgrowth from a cultural matrix it is still possible for logicians to vary their particular systems in detail. An outstanding technique is to combine several existing systems or parts of systems and by that very process to bring about the emergence of a new system. The multiplicity of systems not only makes possible the criticism of particular logical constructions but also the projection of new ones.

On the whole, every competent critic is justified in his attacks upon existing logical constructions. The new developments which he himself projects, however, may not be entirely immune to such destructive criticism. As we have frequently pointed out, the solid criterion for judging destructive criticism and the new construction proposed is to be found both in the task the logician sets himself and the way in which he carries it out. The strongest attacks against logical theories can be directed against those logical constructions which purport to build up systems that are either extrapolated too far from or are completely unrelated to natural events. Under the heading of natural events we include, of course, also the constructive activities of man in culture.

A conclusion that forces itself upon us in surveying either the doctrinal succession of logical construction or any particular temporal or situational cross section is that the attitude of comprehensiveness and unitarianism appears to be the most strongly entrenched. This is true no doubt because those who engage in system-building enterprises tend to look upon their particular systems as the only systems. It matters not whether the systems are metaphysical, epistemological, humanistic, mathematical or methodological, whether they are completed by verbal technique or by mathematical tautology. Some regard only metalogical or metamathematical systems as exclusively logical. Actually, there are many systems which are adequate and valid within their particular cultural frameworks and on the basis of the different materials which they systematize.

Summas and systems are not only inevitable under current cultural circumstances but are quite proper. What is wanting is the view that they are specific and special. Furthermore, an attitude of toleration is needed with respect to the materials, purposes, and methods of system builders. There are many kinds of logic. No single one is *the* logic.

Logical systems, then, are to be judged according to the situation in which they are developed. Perhaps it is fundamental to logical study to stress the rationality factor. Certainly in our own day rationality is strongly emphasized. There exists a powerful deductive trend, as though complete and thoroughly interrelated systems were the ideal of logic. Probably it is a better view that any system which answers to the needs of the system maker is equally as good as any other system yielding the same satisfaction. The point is that no type of system whether mathematical or empirical is to be preferred. All good or competent systems are equally good or competent.

CHAPTER V
THE INTERRELATION OF LOGIC
AND PSYCHOLOGY

HOW IS LOGIC RELATED TO PSYCHOLOGY?

WITH respect to the interrelation of logic and psychology logicians are generally in disaccord and in specific instances assume opposite positions. This situation is owing to a serious dilemma which they face. On the one hand, logical processes are acts of persons. In other words, conceiving, judging, inferring, implying, proving, and inquiring are certainly in some sense psychological activities. As Royce¹ says: "Everyone will agree that throughout its history logic has been concerned with the conduct and with the results of the thinking process." On the other hand, because logicians are dominated by the belief in the ultimacy and objective rationality of logic they cannot become reconciled to the idea that their discipline can be influenced by the principles of such an empirical science as psychology. A typical assertion is that psychology like all other sciences can only establish its results by making them conform to the rules of logical inference.²

Logical literature, therefore, reveals all three possible variations of accepting, neglecting, and rejecting psychology. Particular writers, of course, vary in the details of their adherence to one or another of these attitudes. For example, among the logicians who reject psychology some simply draw a sharp line between the two disciplines on the ground that logic is too exalted and significant to be connected with the comparatively simple and fleeting events with which psychology is concerned, whereas others like the Realistic logician insist that the existence of mind is totally irrelevant and the subject matter of logic would be equally true if there were no mind.³

Similarly, of the writers who are sympathetic toward psychology, some like Mill make psychology basic to logic, while others completely assimilate logic to psychology. Typical of the

¹ Principles, p. 68.

² Cohen and Nagel, Introduction, pp. 118 ff.

³ Russell, Discussion.

latter view are the pronouncements of Lipps and Angell. In the words of Lipps:

Die Logik ist eine psychologische Disziplin, so gewiss das Erkennen nur in der Psyche vorkommt und das Denken, das in ihn sich vollendet, ein psychisches Geschehen ist.⁴

Angell declares:

If the knowledge processes are of value to the organism, it obviously must be because of what they do. No one questions that they serve to reflect and mediate the external world, and this they can only do effectively provided they distinguish the true from the false. It would seem fairly clear, therefore, that a functional psychology in any event, however the case may stand with a structural psychology, cannot possibly avoid a consideration of this aspect of the cognitive activities. But the problem to which this view leads is essentially identical with the accepted problem of logic.⁵

Again, those who disregard the connection of logic and psychology do so on different grounds. Some incline entirely toward mathematical processes and relations, and regard reasoning and thinking about them as sheer manipulative procedures. Others confine themselves to linguistic and symbolic things without definitely considering the psychanthropic origin and nature of symbols and sentences. They assume the existence of a logical syntax which provides rules for the formation and transformation of sensible sentences, just as grammar gives rules for the formation of conversational and literary sentences. Such writers assert: "In any case thinking is not an object of logic, but of psychology."⁶

The writer submits that a considerable portion of the dissension among logicians concerning the nature and aim of logical work may be traced back to the dilemma concerning the relation of psychology and logic. It is not an unreasonable suggestion then that a clarification of the interrelationship of logic and psychology may help to remove some of the obstacles to the development of logic as a science.

The unsatisfactory situation concerning this relationship may be

⁴ Lipps, *Grundzüge*, p. 1 f.

⁵ *Relations*, p. 13.

⁶ Carnap, *Philosophy*, p. 34.

accounted for by the espousal of inadequate ideas concerning the nature of both disciplines. As to logic, as we have seen in the preceding chapters, it is persistently regarded as absolutistic and universalistic. Whether it is considered as exhibiting the rational structure of the universe, the discovery of the invariant relations constituting the order of nature, or a unique general process of achieving a view of reality, logic is articulated with some traditional form of totalitarian philosophy. No metaphysical system of this sort comports with the fact that all logical work constitutes specific activities within a particular cultural frame of reference.

As to psychology, with hardly an exception logicians harbor traditional philosophical conceptions concerning psychological events. Current logical writers display an intolerable conservatism in cleaving to outworn psychological theories. For the most part, logicians accept, reject or neglect psychology on the basis of sympathy for or aversion from the correlation of logical processes with the operation of an impersonal *Nous*, as exemplified by Husserl's "pure (transcendental) consciousness," or with associations of fleeting psychic states. Almost completely lacking among logical writers is an appreciation of recent scientific developments in psychology. Certainly logicians who concern themselves with the nature of reasoning and thinking processes and their relation to language and symbols do not treat them as actual events in which persons interbehave with things within specific situations.

Since the presence of psychological factors is inevitable whenever reasoning or any other logical operation occurs, an acquaintance with scientific psychology is essential to logic. Not that there is any question whether or not to make logic psychological. We wish merely to obviate the notion that logic is nonpsychological or that psychology is indifferent to logic. The desideratum, then, for attaining a satisfactory interrelationship of logical and psychological happenings is to regard them both as psychanthropic events. On this basis logical as well as psychological activities constitute definite interbehavior with things, and for that reason differ only in aim and procedure.

Because of the importance of the present problem we devote the present chapter to a study of some representative views concerning the interrelationship of psychology and logic. We shall not, of course, be so much concerned with men as with doctrines, though

for convenience of reference we find it expedient to specify what author is primarily responsible for a particular doctrine.

A. *Logicians Accept Psychology*

I. *Kant*. The proposition may be defended that Kant brought logic into close connection with if not under the domination of psychology. This he accomplished by way of his Copernican revolution. Recall that fundamentally this intellectual revolution consists in assuming that objects must conform to our mode of cognition. We have already observed that Kant's primary role in logic is that of synthesizer, in that he attempted to connect *a priori* and infallible principles with the concrete facts of experience.

Kant was, of course, a Rationalist and Absolutist. As a philosopher and logician his doctrines may be traced back to the general Continental rationalistic tradition, especially as formulated by Leibniz. Essentially, this tradition is based upon the acceptance of a reasoning faculty which orders the universe. This viewpoint articulates with the doctrine that reason is a mirror of the order in the universe. The logic developed on this ground is simply a single, rigid, cosmic method which produces absolute results because it implies an absolute mind.

But Kant was also a scientist. Certainly the Kantian philosophy articulates with Newtonian natural science. The synthesis, then, which Kant effected was to regard laws of nature as established by the organization of the scientist's objective materials through aesthetic and logical forms. Accordingly under the influence of the tremendous success of the physical science of his day Kant became sensitive to the claims of experiment and concrete observation. It was in this connection that he was awakened from his dogmatic slumber sufficiently to consider the merits of British empiricism. Furthermore, this awakening led him to stress the doctrine that experience tells us what is, though it cannot inform us why things must be so.

The fundamental point here is that the empirical factors in Kantian thought are what made Kant sensitive to the interrelationship of psychology and logic. An unadulterated rationalistic view traditionally deals with objects on a cosmic plan without too much regard to the particular circumstances of observers and formulators of scientific laws and principles. With the formulation that the

mind contributes to the existence and nature of things Kant established a connection between logic and psychology which has maintained itself to our own day though not without numerous modifications. The formulation and persistence of this interrelation, it must be said, does justice to the constructional character of logical processes, but because of Kant's interest in the *a priori* and the omnipotence of logical processes he gets no further psychologically than an absolute and transcendent Reason. The question is in order, then, whether this rationalistic conception is not really anti- or at least nonpsychological, if by psychological one understands the concrete activity of individuals in contact with actual things.

II. *Boole*. Though Boole is a mathematician he definitely considers the question of the interrelationship between psychology and logic. It is patent, however, that his ideas of psychology are philosophical and not technically scientific in a present-day sense. When Boole designed his famous treatise "to investigate the fundamental laws of those operations of the mind by which reasoning is performed" he made plain not only that he believed that a "science of the mind" is possible, but also that he knew the results its observation had yielded.⁷ Boole believed that the ultimate laws of logic were mathematical (L. T. p. 11) and that its basic forms and processes were likewise so (L. T. p. 12). He was convinced that he understood well the ultimate constitution of the intellect and that the ultimate laws of thought were mathematical in form (L.T. p. 407).

Moreover, Boole certainly connected psychology and logic when he discussed the details of the relationship of language, in the form of words, signs, and symbols, to reasoning. As a mathematician, naturally he stressed symbols and propositions, but he leaves no doubt that he was concerned with the actual operations of the mind. That he concluded there was a close analogy between the operation of the mind in general reasoning and its operation in the particular science of algebra may be ascribed to the double influence upon him of the work of Hamilton, Whately, and Thompson, and of his own mathematical interests. But the fact that he believed that the laws of these independent types of operations were in exact agreement (L. T. p. 6) reflected his ideas of the nature of mind. When mathematicians become interested in the reasoning and logi-

⁷ *Laws of Thought*, p. 3, Hereafter cited as L.T.

cal techniques which they employ they incline, of course, toward the formalistic processes concerned with abstract relations and not those stressing things. It was in this sense that Boole believed that though logic is conversant with two types of relations, that between things and facts, the latter could for logical purposes be resolved into relations of propositions, since facts are expressed by propositions (L. T. p. 7). In this manner there is a partial withdrawal from the cosmos with emphasis upon the work of the mind itself. It is clear that mathematicians begin with tightly knit systems of relations and then account for these by the inevitable way in which reasoning has to operate. Unfortunately the particular systems constructed by mathematicians are subject to correction, and as mathematics advances the mathematician becomes aware that no particular type of system answers to the fundamental ways in which the mind operates. For example, Boole's own logical algebra received important corrections by De Morgan, Jevons, Venn, Peirce, and Schroeder. For one thing, Jevons modified the expression $a + b$ from a symbol of simple exclusion to one of inclusion, so that it stands not merely for a or b , but also for both a and b . As Lewis^o indicates, there are important advantages. Again, the operations of subtraction and division were eliminated from the system which carried the systematization toward the logical and away from the mathematical domain. Then, too, Peirce added the relation of including one class in another, with the result that numerical coefficients were eliminated and the bringing together of sums and products. To maintain that the corrected system mirrors the fundamental operations of the mind is simply to insist that operations resulting in systems satisfactory at a given time are fundamental. The same thing is true when one selects the remaining unimproved part of the system with the addition of new improvements as indicating the fundamental processes. Taking competitive logical systems into account one is bound to admit that there are different kinds of fundamental operations or else give up the notion of limited and unique universal operations altogether.

Whether or not a direct connection may be traced between Boole and Kant through the logicians mentioned, there is no question that Boole's type of psychology is not only formal but ultimate. Boole declares that his conclusions concerning the laws of thought

^o Survey, p. 73; Lewis and Langford, *Symbolic Logic*, p. 14 f.

are not merely probable or analogical but truths of science (L. T. p. 402). Even if he modestly refrains from assimilating the cosmos he is certain that he has the secret of the human mind through a knowledge of its inevitable and proper operations.

III. *Mill*. Kant and Boole we have taken as varying instances of formalists who accept psychology as a discipline basic to logic. Kant we may regard as an epistemological formalist, while Boole may be designated as primarily a mathematical formalist. In the case of Mill we likewise have an instance of a logician espousing psychology. But with two differences. In the first place, the appeal to psychology is for the purpose of setting aside the intuitive and formal character of logic. Mill does not merely accept psychology because it points to certain rigid and inevitable operations of mind such as conceiving, judging, and reasoning which discover the order and validity of things. On the contrary, he believes such order and validity originate in the actual operation of mind. Mill's notion of psychology, like that of other British Associationists, is diametrically opposed to an organizing and unifying mind. Reasoning has to do only with the association of ideas and, insofar as mind and the cosmic order are correlated, the cosmos is ultimately reduced to sensations.

In the second place, and following from the above, psychology is not simply related to logic by Mill, but logic is considered essentially psychology. Recall Mill's declaration:⁹

Logic is not a science distinct from, and co-ordinate with, Psychology. So far as it is a science at all, it is a part, or branch, of Psychology; differing from it on the one hand, as a part differs from the whole, and on the other, as an art differs from a Science. Its theoretic grounds are wholly borrowed from Psychology, and include as much of that science as is required to justify the rules of the art. Logic has no need to know more of the Science of Thinking, than the difference between good thinking and bad.

Mill's idea of the relationship between logic and psychology, no less than that of the formalists, implies general and universal principles. But in the present instance such principles are reduced to successions and resemblances of psychic states. The power of logic arises, then, from the fact that the objects and relations with

⁹ Examination, vol. 2, pp. 145-6.

which logic is concerned are ultimately mental states. This is also true of the reasoning processes which relate and order them.

The psychological theories in the case of both the formalists and materialists are not in conformity with current information concerning actual psychological processes. In each instance the writer appropriates a traditional conception of psychological events which satisfies his particular logical theory. From our present point of vantage we can see how each logical theory with its corresponding and implementing psychological doctrine constitutes a factor in a changing cultural matrix. The same is true of the humanistic and instrumental logicians with their psychological correlates which we now consider.

IV. *Humanistic logicians.* In the humanistic logic of Schiller, psychology stresses fundamentally the place of man, his needs, and conditions as factors in a philosophical system. Now it is Schiller's idea that psychology simply represents the mentality of individuals which, of course, plays a large part in every human enterprise including the logical. The objection against Schiller's psychology is that it is in no sense a scientific construction. Such a vague notion of psychology allows for no significant discrimination between the aims and processes of psychology and logic respectively.¹⁰ Essentially, Schiller's view is simply that logic is human and therefore ought to take account of mental processes.

V. *Instrumental logicians.* So favorably inclined has instrumental logic been to psychology that its opponents have accused it of being simply a form of psychology. As a matter of fact, this acceptance of psychology has been made with definite reservations.

Pragmatic or instrumental logicians have favored psychology primarily as a strategic aid in their attack upon absolutistic logic. For the most part they have been interested in stressing the point that philosophy in general, as well as logic in particular, must take account of the fact that there is a natural history of thought, and that therefore logicians should not remain complacent with a formula of thinking in general or a system of logical ideas without regard to human circumstances.

Historically the instrumental viewpoint is based upon the evolutionary or biological psychology developed in the Darwin-Spencer-Bain tradition. The basic notion is that mind must be regarded as a function of the organism serviceable in its adaptations

¹⁰ Castro, *The respective standpoints*, p. 72 ff.

to its environment. Psychology, accordingly, is closely integrated with biology. Especially James stressed reasoning as a form of adaptation.

In general, we may characterize this type of logic and psychology as the off-shoots of a particular sort of philosophical viewpoint. The philosophy deals with a general world view, which is not founded simply upon purely intellectualistic construction but is presumed to have its roots in the biological nature of man. The corresponding psychology does not obviate comprehensive system though it stresses a human basis for values and norms of all sorts.

VI. *Other Sympathizers.* Although our present aim is to illustrate the trend which inclines logic toward psychology and not to present a history of the trend, it may be enlightening to indicate a few further items. First, to be mentioned is the tradition originating in Fries. As historians of logic point out,¹¹ this Kantian carried out the ideas of his master to the point of defining logic as the science of laws of thought. Essentially, Friesian logic is concerned with thought activities which are always centered in human circumstances. This Friesian psychological trend has recently been continued by Nelson and his students.¹²

Of especial interest here is Wundt¹³ who is influenced by British biology and associationistic psychological trends. He starts with the assumption that logic reflects the evolution of psychological processes which have attained a certain state of human knowledge, the stage of developing principles of norms and validity which can be applied to the work of the sciences both exact and social.

As a final illustration of the trend to relate logic to psychology we may cite Peirce. Note especially that he is in the predicament of both accepting and rejecting psychology. Though essentially a formal logician himself, Peirce objects to a lack of fundamentalness in a set of eight simple propositions set up by De Morgan. Peirce writes: "But formal logic must not be too purely formal; it must represent a fact of psychology, or else it is in danger of degenerating into a mathematical recreation" (C. P. 2.710).

B. *Logicians Reject Psychology*

Although the propositions (a) logic and psychology are different disciplines and (b) logic is nonpsychological or completely un-

¹¹ Cf. Ueberweg, *System*, p. 52.

¹² Linke, *Logic*, pp. 391 ff.

¹³ *Logik*.

related to psychology, are completely independent, some logicians still assume that *a* implies *b*. Naturally the criteria and the arguments for separating the two disciplines take on many forms, whereas the logicians who reject psychology clearly display the particular intellectual and cultural influences which sway them.

To begin with, in the development of logic on the Continent we find a counter-revolution against Kant's Copernican revolution. For various reasons a tremendous opposition sprang up toward the Kantian logic. Primarily this may be localized in the too hospitable reception which Kant accorded the sensationism of the British thinkers, especially Hume. Since the trend on the Continent had always been totalitarian, it was no accident that a forceful opposition arose against Kant's assimilation of the British individualistic trend of thinking. Intellectually speaking, the totalitarian emphasis amounted to a greater stress of rationalism, and Kant turned out to be not rationalistic enough.

Doubtless a basis for upsetting the Kantian tradition was found in the developments of non-Euclidean geometry. Certainly it became more and more evident that the apparently subjectivistic or personal intuition of space and time was not satisfactory. The emphasis that the non-Euclidean geometries placed upon postulation and deduction suggested that transcendental reason could dispense with the given psychic sensations.

Non-Euclidean developments, to be sure, are not confined to geometry. Witness the upsurge of mathematical analysis which can be looked at from such different angles as absolute formalism or a revival of medieval Platonism and as an attempt completely to separate mathematical work from psychological entanglements.

Another change of culture and intellectual conditions on the Continent was the Renaissance of medieval Scholasticism through the efforts of Brentano and his disciples. From the psychological standpoint this development constituted a preference for the totalitarian mind against the individualistic states of consciousness propagated by Helmholtz and Wundt. From a metaphysical standpoint it represented a stress of the ontological or realistic view against the phenomenalist or positivistic theory of the nature of things.

On the side of the British logicians the rejection of psychology followed two general lines. First, it produced an upsurge of the

aristocratic and theological emphasis upon reals, which certainly since the Cambridge Platonists has constituted a prominent feature of British thought. Secondly, this absolutistic and antisensationistic trend of British thinking was implemented by the formalistic development in mathematics. This movement is excellently illustrated by the Russell-Whitehead adoption of the ultrarationalistic development of mathematics, well symbolized by the notion that mathematics itself is logic.

I. *The Essence Criterion.* A long list of logicians has differentiated between logic and psychology on the basis that logic has to do with essences in some form similar to Platonic reals or ideas. From this common attitude various divergent views have ensued. For example, some opponents of psychologism have simply assumed that logic is concerned with ultimate essences which they discover or otherwise handle by rational processes. Others start with finite inferences or other rational procedures and believe they can attain to ultimate essences, whether substances or relations.

It is clear that the realistic or rationalistic logicians simply adopt a metapsychological mentality of some sort. Such a psychology, of course, can only be created or fostered by the use of language symbols, and furthermore can only exist in the rarefied atmosphere of ideal essences. Thus Bolzano¹⁴ could build system after system of things and concepts in themselves—that is, independent and subsistent entities. Similarly, a logician like Meinong could luxuriate in a real world of golden mountains and round squares because forsooth he was able to talk about them.

One of the most vigorous attacks against psychologism was launched by Husserl on the basis of essences. Husserl, building upon various supporters of a revived Platonism and the Intentional psychology of Brentano, constructed a colossal cosmos of essences. By means of a set of submissively compliant words he erected for himself a world of essences which could not be touched by natural man and his logic, but required a transcendental consciousness which was capable of intuiting anything. Thus, by an exaggeration of his own verbal and symbolic products he could forget that he himself made them and could proceed to adore them.

So evident is the fact that logical items are activities of persons that it is bewildering to find writers of great competence persisting

¹⁴ *Wissenschaftslehre.*

in the belief in processes and principles which determine the existence of things and the value of propositions about them. A sort of dissatisfaction with concrete facts, whether such facts consist of the interbehavior of persons (scientists) with objects or events (natural or cultural) or of the objects and events themselves, seems to generate a belief in absolute relations and eternal existences, with the consequent search for unchanging essences. Now strange to say, nothing is more intolerable to essence logicians than the notion that they are concerned only with personal beliefs and statements. But the situation is hardly improved by asserting that all existence is established, all science guaranteed, by prior logical principles—that there are necessary conclusions from premises independent of logicians and scientists. Such a situation leads to the paradox that discontent with the particularities of specific things and personal thinking in turn leads to the triviality of tautological propositions and empty syllogisms. What is more unfortunate is Husserl's intuition (*Anschauung*), which when vigorously exercised by all transcendental egos enables them to shuffle off the irrelevancies of scientific objects and investigation and forthwith to attain in *phenomenological community a corpus of eternal truths at the basis of which is the knowledge that the world is the validly constituted ideal-objective sense of transcendental consciousness or ego.*"

Besides this transparent verbal creationism, the logic of those who simply deal with empty syllogisms appears completely innocuous. Moreover, such logicians may comfort themselves by the fact that the form in which they describe their syllogisms allows for considerable content. Thus, all y's are z's, x is y, then x is a z starts from a system of interrelated factors, so that y's are identified or characterized as z's and x is restricted as a y. The apparent emptiness of a syllogistic system is simply a deviation from the conventional description which indicates more particular qualities. The emptiness, then, is merely a matter of degree of quality or property specification. Light is thrown upon this situation by the belief of those grammarians who declare that every noun in any ordinary language is a general symbol. "The apple is red" according to them symbolizes an indefinite number of apples if only they are red.

The problem of fullness and emptiness of logical elements or

■ *Logische Untersuchungen, and Formale und Transzendente Logik.*

symbols is further illuminated by the notion that mathematics deals with nothing definite as though precisely defined relations are nothing. Such a notion exists simply because of the prejudice based upon the above common or perceptual properties. To assert that every curve contains an infinite number of points is to deal with precise relations. The fact that such points are merely indicators of positional relations and are fundamentally constructions does not detract from the argument.

II. *The Objective or Impersonal Criterion.* Many logicians agree that logic like psychology deals with thinking, activities of persons, but they insist that the two kinds of thinking are different. Psychological thinking is personal and private going on in the head¹⁶ or a process going on in someone's mind.¹⁷ Logic, on the other hand, is presumed to be concerned with objective thinking—that is, thinking or reasoning controlled by various external criteria that exist independently of particular individuals. Naturally the objective and impersonal type of thinking is more important and more significant. By comparison psychological thinking is vigorously depreciated. If there can be a psychological logic that logic is not valued.

Among those who insist that logic is independent of psychology some stress the existence and importance of thinking and reasoning, which processes are concerned with objective relations of things. For them logic to a great extent has to do with the discovery or establishment of certain rational connections among things. Of course, such a view is greatly influenced by various mathematical considerations. Ultimately, then, logic deals with order among entities, as foreshadowed by mathematical equations.

Quite a different phase of the objective criterion was that fostered by Frege who took the position that a psychological logic simply must miss the one unalterable truth. He argued that a psychological logic implies relativity—namely, there might be beings who think in accordance with logical laws different from ours. This would put into jeopardy the truth, as indicated in the following.

The person who, on the other hand, means by logical laws such laws as prescribe how one ought to think, that is, laws of what is true (*wahrseins*) and not the natural laws underlying ideas as to what is

¹⁶ Eaton, *General Logic*, p. 6.

¹⁷ Cohen and Nagel, *Introduction*, p. 18.

true (*Fürwahlhaltens*) will ask "Who is in the right? Whose laws with reference to what is held to be true are in accord with the laws of what is true?" The psychological logician cannot ask this question; otherwise he would acknowledge that there are laws with regard to what is true which are not psychological in character.¹⁸

Frege was, of course, a mathematicologist who believed in the Platonic reality of numbers. In consequence he could speak of *der verderbliche Einbruch der Psychologie in der Logik*.¹⁹

Peirce adopts both phases of the objective criterion in his rejection of psychology.

Logic is not the science of how we *do* think; but, in such sense as it can be said to deal with thinking at all, it only determines how we *ought* to think; nor how we ought to think in conformity with usage, but how we ought to think in order to think what is *true*. That a premiss should be pertinent to such a conclusion, it is requisite that it should relate, not to how we think, but to the necessary connections of different sorts of *fact* (C.P. 2.252).

And again, "How we think, therefore, is utterly irrelevant to logical inquiry" (C.P. 2.55).

Logic for Peirce was a very different thing from psychology and from the natural history of thought, as he pointed out in a review of Dewey's *Studies in Logical Theory*.²⁰ Clearly Peirce's strictures upon psychology are based upon the conventional notion of mind prevailing at his time and indeed in our own—the notion, briefly, that mind is an evanescent thing or process (C.P. 2.179 et passim). Reasoning, of course, he thought he knew considerable about and primarily because he accepted mathematical operations as reasoning processes in the best established form. "Mathematics," he declared, "is not subject to logic." On the contrary, "logic depends upon mathematics" (C.P. 2.191).

Whether or not one accepts Peirce's interpretations of mathematics or his evaluation of its fundamental significance it cannot be denied that it constitutes a potent and effective type of logical system. That mathematics is basic to logic may be questioned. Incidentally we suggest that mathematics as a logic is a specific type of

¹⁸ Frege, *Grundgesetze*, I, p. XVI.

¹⁹ *Ibid.*, p. XIV.

²⁰ *The Nation*, New York, 1904, 79, 200.

system or set of logical systems, whereas there are many logics which may or may not be related to the mathematical logics. Despite Peirce's pragmatic and fallibilistic views he was unable to free himself from the notion of totalitarian and monopolistic logic.

To be sure, if psychology were actually concerned with certain "processes in the head or mind" we could hardly avoid such unfortunate distinctions as logicians make. But such an assumption is a travesty on psychology. Fortunately psychology need no longer be considered as dealing with psychic or even subjective or internal processes. Psychological data, rather, are the interbehavioral events taking place between organisms and the objects and processes with which they are in contact in definite behavioral fields.

III. *The Normative and Regulative Criterion.* Logicians also reject psychology on the basis that psychological thinking and reasoning are simply subject to uniformity laws—namely, ways in which men normally think, whereas the laws of logic are regulative and authoritative. In other words, logical laws afford "criteria by the aid of which valid and invalid reasonings may be discriminated."²¹ Keynes asserts that psychology deals with actual thinking, while logic deals with ideal and cogent reasoning. A variant statement is that psychology is concerned with the origin of beliefs, logic with their validity.

All psychological activities of thinking and reasoning inevitably involve norms and validity. Hence it is an obvious mistake to think that logic alone is concerned with validity. As a matter of fact, all complex human activities in all phases of life require the consideration of norm and validity problems. Furthermore, the logical domain no more than any other is concerned with absolute norms, since all norms and principles are derived from the concrete system-building activities of persons.

In brief, absolutism in logic issues in two fallacies. In the first place, in order to get away from psychological happenings the quest for absolute logical validity leads to abstractions and formalities. Since logicians wish to avoid this undesirable consequence their abstracting and formalizing procedures end in mistaking formal descriptions of psychological events for the events themselves. The result is a serious misconstruction of events by the formalization process.

²¹ Keynes, *Studies*, pp. 5-6.

In the second place, logic turns away from the study of logical processes as specific events, despite the fact that only when we deal with specific events and situations has logic any real significance. Even though a logician may be interested only in the most abstruse relations, he is after all not fundamentally dealing with symbols and the interrelationship of sentences but the subject matter or referents of such sentences. Clearly this leads us back to what individuals actually do in the form of interbehavior with concrete stimulus objects. Perhaps the inevitable emergence of probability problems is sufficient indication of the point we are making. When logicians realize that psychology deals with the contacts of individuals with actual objects and conditions and not with discrete psychic contents there will be no urge to draw a sharp distinction between psychology and logic.

IV. *The Product Criterion.* Logic is also divorced from psychology on the ground that the latter is concerned with thought processes, while the former deals with thought products¹¹ or with objects of thought.¹² Two observations may be made here. First, although it is true that logical products may be separated from logical action as a process of thinking, inquiry, or investigation, such products must be intimately related with the processes which produced them. Secondly, we must guard against the traditional differentiation of conceiving, judging, and inferring as psychic processes from terms, propositions, or syllogisms as objective linguistic products. Both of the above distinctions overlook the essential fact that logic is a form of activity performed by persons with respect to specific problems and conditions in particular fields. Norms, criteria, as well as products of logical situations, are concrete things interrelated with persons in their contacts with natural or cultural events.

V. *The Comprehensiveness Criterion.* Many logicians fully aware of the place of the individual in logical situations still reject psychology as not comprehensive enough to have much effect upon logical problems. This metaphysical doctrine is essentially based upon the belief that logic must deal with some form of absolute reality. Historically this comprehensive view of logic is associated with the epistemological doctrine that logic is concerned with items

¹¹ Keynes, *Studies*, p. 4.

¹² Eaton, *General Logic*, p. 7.

of knowledge. There are two forms of comprehensive logic; the earlier dialectic phase makes logic a mental process which functions absolutistically and comprehensively, thus ultimately embracing the whole universe. Whether this type of logic is exemplified by the Hegelian dialectic or the more personalized Bradley-Bosanquet variation, in either case the logical process is presumed to begin with particular judgments that ultimately comprehend the universe. Comprehensive logic thus becomes a system which soars beyond specific and localized psychological activities.

The proponents of the instrumental and pragmatic variations of this dialectic claim that, whereas psychology is of some importance to logic, the latter is so much more comprehensive that on the whole it is independent of psychology. The implication is that psychology deals with processes localizable within the organism, that while psychological activity constitutes adjustments to external things which are mirrored in mental processes, it lacks such interrelationships between the subjective and the objective as are required to reconstitute reality. Even though instrumental logicians reject the absolutistic feature of the Hegelian dialectic, they nevertheless regard logic as so comprehensive that judgments and inferences operate to reconstruct the world. A typical, relevant statement follows:

Psychology may demonstrate the effectiveness of the judgment as an instrument of adaptive behavior, nevertheless the adaptation secured by means of the report which the psychological idea gives of reality is *to* a reality which is unaffected by it. Thought, if efficient in attaining knowledge, is so by way of learning new facts about a reality which is entirely independent of it. The judgment, to repeat, is a psychical act whose reconstructions occur entirely in the mind. Logic, on the other hand, makes of the judgment, not an inner act, but the dissolution-resolution process by which reality itself is active, changes, and develops.²⁴

Instrumental logic as Professor Dewey has developed it²⁵ stands in sharp contrast to the absolutistic logic which originally gave it its impetus. Nevertheless, instrumental logic is strongly opposed to psychological influences. Dewey has constantly stressed that logic is a form of action, a process of resolving problems, an inquiry into

²⁴ Castro, *The respective standpoints*, p. 52.

²⁵ Cf. chap. 4.

inquiry. Still, he declares, "as regards ultimate subject matter logic is a branch of philosophic theory."⁸⁶ In other words, its universal character is stressed. Though Dewey's logic is always put into the form of a thinker's concrete action, logic is regarded "as the generalized account of the means by which sound beliefs on any subject are attained and tested."⁸⁷

Logic, on such a basis, is not concerned with *particular* inquiries, with *specific* situations. Though it is given a noncosmic description the stress on generality can mean but one thing: one draws away from the concrete activities of particular individuals. Let us grant Dewey's contention that logical characteristics or forms result from transformations occasioned by inquiry. The question still remains whether logical work is exclusively concerned with situations in which "logical forms accrue to subject matter when the latter is subjected to controlled inquiry."⁸⁸ According to Dewey:

Inquiry effects existential transformation and reconstruction of the material with which it deals; the result of the transformation, when it is grounded, being conversion of an indeterminate problematic situation into a determinate resolved one.⁸⁹

Is the insistence upon problem solving and inquiry as described in the above quotation anything else but a modernization of historical philosophical dialectics? Even if we do not urge the principle of single system building we must contrast this comprehensive, philosophical logic with a nonphilosophic logic concerned with particular situations in which, through inquiry, one discovers the characteristics of objects and events that achieve their properties by various interactions of particular factors in a field. Only such inquiries and not cosmic or general philosophic ones can be specific and in line with the operations of the scientist.

In working out this totalitarian logic Dewey assumes that logic's independence of psychology arises from the fact that the latter after all has to do with personal if not subjectivistic or mentalistic events. For himself, he⁹⁰ claims a biological or behavioristic type of psychology—a psychology, moreover, which cannot sustain too

⁸⁶ Logic, p. 2.

⁸⁷ Ibid., p. 535.

⁸⁸ Ibid., p. 101.

⁸⁹ Ibid., p. 159.

⁹⁰ *Essays*, p. 31; Logic, p. 36 et passim.

close a relation to logic and certainly cannot be influential in shaping its character. One may conclude that inquiry is not the tentative and specific activity of the laboratory, but a decidedly more comprehensive form of action, even though Dewey³¹ declares that logic is a progressive discipline and not capable of final formulation.

A careful analysis of instrumental logic reveals at least two attitudes with respect to psychology. The first is to reject or at least to neglect psychology on the basis that the requirements of inquiry demand only that an inadequate psychology be avoided. On the other hand, the objection to absolutism makes inquiry human and problem solving. The result with respect to psychology is really the opposite of that intended. Instead of rejecting it instrumental logicians really reduce logic to psychology, at least to a certain type of psychology. Nor is this procedure concealed by making logic a matter of language. To reduce inquiry to the handling of terms and propositions is not to avoid psychology, but merely to overlook the fact that both logic and psychology constitute concrete interbehavior with stimulus objects, whether things, propositions, or relations.

VI. *The Formality Criterion.* Logicians also reject psychology or at least make logic more or less completely independent of it on the ground that logic is formal and therefore remote from psychology which is concerned with content. Formalists in logic assume that they are operating primarily with thinking and reasoning and on that very ground are constrained to mark sharply the line separating logic from psychology.

As in most other cases the formal distinction implies various alternatives. Consider first the notion that logic deals with the formal in the sense of the laws and principles of thought rather than thought itself. This is not to be confused with the distinction of laws as normative but simply with the formal character of thought standing over thought as content. The emphasis here is upon principle rather than upon instances, and in a sense amounts to an abstraction on the basis of sentential formulation. Certainly some proponents of this formalistic view do not mean sharply to sunder thought from what is thought about.³² They do not therefore insist upon a single form for thought; it may have different

³¹ *Logic*, p. 14.

³² *Joseph*, Introduction, p. 5.

forms depending upon the things thought about. Consequently there are levels of forms. At any rate, in all cases logical thought is a process to be distinguished from more or less accidental operations regarded as the province of psychology.

Another variant of the formalistic principle is that logic deals with pure thought without regard to application. The assumption is made that there are rules or principles no matter what the subject matter might be and that when regard is had to this subject matter logic becomes applied. In either case the principles of thinking are formal in contradistinction to the haphazard activities of thinking when it occurs as the casual responses of individuals.

To a great extent this second view of the formal character of logic articulates with the notion which separates logic as science from logic as art. The study of the pure and unapplied principles of thinking or reasoning is scientific, whereas the application is art. The art of reasoning involves bringing together the formal processes of thought and the particular instances in which the things thought of are stressed.

C. Logicians Neglect Psychology

Since those logicians who accept and reject psychology do so only with some disadvantage to their theory of logic, one might conclude that it is just as well for the logician to keep clear of psychological issues altogether. Those who take this last view may well find support in the relative independence of the two disciplines. Unfortunately, however, logic and psychology can only be independent when neither one violates principles developed in the other. Perhaps we must even go further and say that logic and psychology are independent when logic takes account of the psychological participation in system-building processes. This in no wise means, however, the reduction of logic to psychology nor even making the latter the foundation of the former. On the whole, those logicians who neglect psychology do so at the risk of misinterpreting the nature of logical data and the place of reasoning, language, and symbols in logical situations.

1. *Symbolinguists.* One essential technique of evading psychological entanglements is to decline commerce with thinking and

reasoning and confine oneself to language. On this basis, logic becomes the organization and transformation of sentences.³³ The criteria for the manipulation of sentences consist of deliberately chosen systematization principles, for example, the conventions of grammar or rules of some particular type of symbol system. In the latter case the customary logical criteria of truth and verification are substituted for by consistency and order.

Now this separation of logic and psychology is certainly a legitimate procedure, but only on a proper basis. It is proper to systematize words, symbols, and sentences or order them on some acceptable plan if one can do so without prejudice to the materials or process. One is not ordinarily obliged to stress the psychological implications of such system-building processes.

As it happens, however, those who form and transform sentences usually acknowledge the fact that sentences are not, as noncultural things may be, entirely free from characteristics given to them by those who make and employ them. No one engaged in the serious work of logic would care to be concerned with mere sentence manipulation were it not that howsoever sentences are formalized they still are in some way referential or designative. All the more is this true since the syntactical logician is frequently interested in the logic of science. To handle sentences as though they lacked such functions he would regard as indulging in the most trivial employment.

Upon what basis, then, may the logician properly neglect psychology? Since logic and psychology constitute two different disciplines an individual may certainly work in one and not in the other. The situation here is no different from working with one rather than another problem within any single one of these disciplines. Logicians may neglect psychology on two levels. On the crude-datum level, for example, he can describe a symbolin-guistic enterprise simply as the work of a class of men, i.e., professional logicians. Also on the level of scientific construction he may, of course, superficially treat sentence construction and reconstruction without regard to the intimate stimulus and response functions involved in such procedure. When, however, the scientific description penetrates deeper into the system building one cannot

³³ Cf. Carnap, *Logical Syntax, and Philosophy*.

without serious misinterpretations neglect the logician's activities in choosing or producing materials (words, sentences) for system building.

Among such misinterpretations of the symbologist, at least in his syntactic phase, is the assumption that it is possible to separate the psychological meanings of sentences (thought, images, etc., connected with them) from their logical meaning, the latter regarded as other sentences which are consequences of employing rules for transforming the original sentences.²⁴ This assumption is tantamount to the view that sentences, that is, combinations of words or symbols or the rules of their combination, possess properties belonging intrinsically to the words or symbols and are not products of interbehavior. This belief has been referred to as a chemistry of symbols.²⁵ Whence do the transformation rules come? The fact that there are restrictions to the transformations permissible should suggest the interbehavioral and even the psychological processes involved. The neglect of psychology, therefore, simply amounts to a self-appointed license to deal with psychological materials without the necessity of consulting the scientific findings concerning such items.

While it is permissible to suppress the acts of the workers with symbols in order to deal with the symbols and systems themselves, it is not proper to do so by making psychological events consist of parallel psychic thoughts and images. Another unsatisfactory consequence of neglecting the psychological processes is to misinterpret the actual interbehaviors involved which obtrude themselves on the symbolinguist as nonpsychological events. Furthermore, to identify certain acts, productive of sentential products and their rearrangement, with the whole of logic is another undesirable outcome of neglecting the psychological components of logical work.

The treatment of symbolinguistic things as independent objects is a consequence of reducing the products of linguistic acts to symbols. Originally, of course, words as units of enunciated patterns constitute psychological activities which when transcriptively recorded result in a textual product. Similarly, writing acts directly produce textual results. As products, of course, the words or

²⁴ Cf. Carnap, *Logical Syntax*, p. 42.

²⁵ DUCASSE, *Concerning the status*, p. 323 f.

symbols stand alone, but to begin with they are interrelated with referential or designatory activities. Unless one keeps this fact alive one is prone to reach erroneous conclusions about (1) logic, (2) language, and (3) the relations between them.

The assumption that logic is syntax³⁶ not only results in misinterpreting logic, by reducing all system making and ordering to the manipulation of meaningless sentences, with behavior factors surreptitiously slipped in, or by connecting sentences with pseudo-psychological ideas or psychic meanings, but at the same time falsifying psychology and language as well. How the latter is accomplished it is interesting to observe.

Ducasse asserts that in the sentence "Napoleon was short" the verb "was" formulates (symbolizes) the attitude of belief, while the words "Napoleon," "short" formulate the propositions believed.³⁷ This is certainly an arbitrary analysis and interpretation of a sentence. Superficially the sentence may be criticized, since it is hardly a proper transcription of any attitude or proposition. Nor is it likely that it is even an important assertion. Ducasse has merely chosen a piece of text suitable for his purpose. Certainly the sentence as it stands is not susceptible to division in the manner indicated. Furthermore, it is illegitimate to identify a proposition with the event—namely, Napoleon's shortness. The style of verb certainly suggests time of event, since one refers to or designates an historical fact.

We may conclude, then, that symbolinguism does not effectively separate psychology from logic in the sense of obviating all connection, though a proper interpretation of all the factors does not demand any illegitimate identification of logical analysis with psychological description.

II. *Rationalists*. Another technique of avoiding psychological problems in logic is to regard logic as a process of generalized and objective reasoning. On this ground reasoning becomes a more or less mechanical process of organizing systems on the basis of relations existing independently of psychological activities. For the most part the proponents of this rationalistic view, who happen to be mathematicians, follow a suggestion of Peirce.

It is true, that logic does not go into the business of doing people's

³⁶ Ducasse, *ibid.*, p. 324.

³⁷ *Propositions*, p. 704.

reasoning for them; so far as it can be relegated to anybody, mathematics undertakes that business (C.P. 2.532).

After all it is only a minor fault not to distinguish between mathematics and logic. A more serious difficulty is that mathematical processes are made to assimilate psychology by the scheme of deducing the nature of psychological events from logical or mathematical processes. An instance in point is the view of Carmichael who seeks in mathematical processes the invariant character of human nature, especially the reflective and rational processes, as indicated in the following.

It is an eternal truth that every integer is the sum of squares of four integers, and there is unanimity as to this fact and as to its demonstration. The persistence of mathematical theorems and the continued agreement as to their proof indicates a profound unity in the characteristic thought processes of those who contemplate them, exhibiting one fundamental phase of human nature.²⁸

Such a statement overlooks the constructional character of any logical system. Howsoever true it may be that certain relations of numbers inhere in the numbers and are discovered by the mathematician, we cannot overlook the latter's activity of exhibiting the relations as constituents of a system. The view indicated above gives not the slightest hint of all the variations among mathematicians concerning (1) the nature and relationship of numbers or (2) the character of mathematics as a whole, and certainly does not suggest the similarities and differences in mathematical theory as influenced by cultural conditions. And finally, Carmichael overlooks his own assertion that logic is solely concerned with reasoning, with the laws of inference, and that mathematics is the primary or best exemplar of reasoning. As a result he deduces the ultimate character of human nature from the operations and products involved in some abstract manipulations.

The view of Carmichael and others that reasoning is a sort of nonpsychological or cosmic process is not unrelated to a theory concerning mind. For Carmichael mind is essentially a mystic entity, perhaps correlated but certainly not identical with physicochemical processes, and which carries on various enterprises of invention and discovery. In this connection Carmichael correlates scientific proc-

²⁸ *Logic*, p. 253.

esses, which can only deal with "the more mechanical or determined or measured aspects of nature," with another process, a "direct sympathy," only through which we can attain the deepest understanding of the "inner aspects of reality."⁸⁹ Here we run into a familiar exaltation of mathematics which not only makes it into a form of art occupied with the aesthetic character of formal structures, but into a transcendental philosophy and religion as well.

COORDINATION AND COOPERATION BETWEEN LOGIC AND PSYCHOLOGY

As human behavior logical activities involve psychological factors. On the other side, psychology as a science must involve logic as system-building procedures. What is called for, then, is a workable notion of the interrelationship between the two disciplines, since perhaps all intricate human situations involve both logical and psychological activities.

Be it granted forthwith that no science is fundamental to any other, though any one science may cooperate or work in conjunction with almost any other. Furthermore, no science or discipline is prior to or more important than another in subject matter or principles. We may say, then, that in actual situations both logical and psychological processes constitute isolated phases of a larger complex. In fact each phase must be isolated from this complex in order to receive proper study. Naturally, too, the student who is especially occupied with one of these phases tends to overemphasize its importance in the intellectual household. Notice, however, that specialization implies neither inclusiveness—that is, the absorption of the other domain—nor any absoluteness. It is traditional to regard logic as the more important or prior discipline because relations, abstractions, and generalizations are considered the exclusive concern of logic and more fundamental than other happenings. Whenever we work with independent and objective relations, however, we derive them from complex situations and elaborate them by a process analogous to mathematical extrapolation.

We propose, then, that logic and psychology be taken as coordinate enterprises. Nor is this a unique relationship. Both logic and psychology are coordinate and may be cooperative enterprises with physics, chemistry, biology, and any other science. We may go

⁸⁹ Loc. cit. p. 200.

further and suggest that logic should always be a cooperative discipline with every science. As an illustration of the advantage to be derived from such cooperation we can at once point to the regulative influence of logic upon some phases of scientific work. Since system building involves (1) attention to frame of reference, (2) investigation of postulates and the principles upon which they are set up, and (3) control of investigation by the postulates, the place of logic in science is definitely located.

INADEQUATE PSYCHOLOGY OBSTRUCTS LOGIC

The unsatisfactory relations between logic and psychology undoubtedly are based upon inadequate conceptions of the latter. Unacceptable notions of psychology certainly lie at the basis of faulty constructions concerning language and symbols as well as thinking and reasoning.

1. *Dualistic Psychology.* Underlying most logical theories we find some form of dualistic psychology. Fundamentally this means that psychological processes are regarded as existing in a domain different from the natural or spatiotemporal field of events. Dualistic psychology allows for constructions of all sorts of sub-sistential realms, since mind is something either potent enough to create worlds or limited to its own phantasms. These constructions as imaginal creations, of course, must be extrapolated from concrete conditions, but in the end attain to realms utterly divergent from the starting point.

For example, logicians build up systems of intimately interrelated propositions, so that one proposition can be used to anticipate another or to refer back to another. These forward or backward moving operations are then elevated to the dignity of implicative or deductive principles. Later, under the influence of specific kinds of problems and general cultural conditions logicians tend to endow both the deductive or implicative processes and the things upon which they operate with a primacy and ultimacy altogether out of proportion to the actual situations involved. Thus begins the tradition that principles and universals are prior to or more important than the contents and particulars of the propositions or the relations between them.

This inquiry into the behavior of logicians immediately brings to the front the problem of the logician's place in the logical situa-

tion. Because behavior is productive—that is, results in a product—some logicians hypostatize sets of propositions or principles and regard them as independent universes of discourse, forgetting the activities of generating and establishing those universes. Other logicians, impressed with the fact that they have had a large role in the development of these systems have overemphasized their work and, as a consequence, built up subjectivistic conceptions, finally arriving at the notion that there are no worlds independent of the logician's action. Thus they have denied universals and made particulars into figments of their own imagination. This polarization of logical behavior has given us the historical quarrel between the deductionists and inductionists.

Further illustrations of the consequences of dualistic psychology are the faiths established in the universal and absolute character of constructions, for example, the belief that certain items such as protons, electrons, and neutrons are more basic and fundamental than biological, psychological or other types, or that geometric and general mathematical operations constitute the most effective forms of reasoning. In general, such commerce with absolute constructions bespeaks a lack of faith in man and the natural things with which he interacts. Logicians working on such a basis cannot tolerate the tremendous detail of work necessary to the investigation and moderate control of actual events.

As we have indicated, however, the dualistic notion of psychology is a cultural heritage from political and social conditions originating in the Hellenistic-Roman world, and no longer represents the status of psychology. Like all other sciences, psychology, has gradually found it feasible radically to depart from such formulations. Taking these objective developments into consideration we no longer find logicians forced to admit, on the one hand, that logical operations are the specific activities of human beings, and, on the other, that they necessarily comprise a different and more valid domain of science or thought.

2. *Biological Psychology.* It is only natural that a reaction to dualistic psychology should appear. But the form it took was not altogether a happy one. In effect, those who were concerned with the objection to dualistic psychology simply rejected the psychic aspect and set up what they called a biological psychology. While the improvement does obviate errors attaching to psychic states or

transpatial mind, it can hardly do justice to the psychological data involved in logical work.

Certainly there is no way in which biological principles or facts can be employed to describe the actual processes of thinking, reasoning, problem solving or system building. At most, biological psychology is a negative doctrine; it excludes the psychic. Accordingly, all of the actual performances involved are substituted for by linguistic materials which can at best be regarded as products of actual psychological activities. Those who espouse a biological psychology naturally gravitate toward the view that the proximate subject matter of logic consists of the relations of propositions (sentences) to one another, for example, affirmation-negation, inclusion-exclusion, particular-general, etc. In this connection Dewey writes:

No one doubts that the relations expressed by such words as *is, is-not, if-then, only, (nono-but), and, or, some-all*, belong to the subject-matter of logic in a way so distinctive as to mark off a special field.*

No matter, then, how superior biological psychology may be to a psychic doctrine it favors the construction of a unique formal system which actually is out of touch with concrete inquiry or specific activities of system building. The manipulation of words and sentences is substituted for all the concrete interbehavior of particular individuals with particular problem situations. At best, sentences and sentence organization constitute some products of logical work. To reduce logical activities and products, therefore, to organization of sentences is to miss most of the essentials in logical situations. The following type of psychological view attempts to solve some of these problems.

3. *Interbehavioral Psychology*. Perhaps the most fundamental contribution of interbehavioral psychology to logical theory is that in addition to the elimination of all powers of mind, all intuitions, it takes into account the concrete interactions of individuals with things. Mind from an interbehavioral standpoint itself consists of activities definitely developed through contacts of organisms with particular objects. In this sense psychology places a distinct check upon any flights into mythical, psychic domains. In fact, it

* Logic, p. 11 of, also, p. 371.

limits us to the actual activities of persons, including the accumulative effects of such activities in cultural institutions.

More specifically, interbehavioral psychology indicates how improper it is for logicians to construct a particular doctrine of reasoning suitable for their own purposes. Constructions concerning reasoning events must be derived from observations of actual interbehavior. Though these interbehavioral suggestions concerning reasoning do not seem difficult to accept, a different situation prevails with respect to the facts of language and symbols.

According to interbehavioral psychology language events are definite activities of individuals and constitute references to actually existing or nonexisting referents. When we keep to concrete events it should not be difficult to differentiate between references to existing and nonexisting things. Even the ancients understood the means language provides for the construction of mythological entities. Thus Xenophanes declared that if horses could talk they would delineate their deities in their own dimensions. In point here, too, is Spencer's pronouncement that perhaps mythology, and certainly metaphysics, is a disease of speech.⁴¹ The question can be definitely settled in specific situations whether what is thought, believed or said concerns something that exists aside from the belief or assertion. In many cases constructions based on belief or assertion not only refer to and represent but are also identical with the objects allegedly dealt with. And so we must distinguish between purely analogical construction and definite references to things. Linguistically to construct something out of nothing leaves nothing but words as a product.

Although interbehavioral psychology can fairly well control the linguistic constructional process, it runs into greater difficulty in the symbolic situation, especially in the mathematical domain. Recall here the confusions arising from the ability to symbolize infinitesimals and infinities or constant recurrence and continuity. Much of mathematical foundations consists of the substitution of symbols for the symbolized. But even here a definite criterion can be discovered if we recall that in the final analysis all symbolization as the manipulation of objects is the activity of individuals who are in contact with cultural cumulations.

For example, in infinity discussions it is obvious that the starting

⁴¹ Principles, vol. II, p. 502.

point is simply recurring operations, say, of adding *one* to a *one* started with, a process which can be continued for a very long time. Now, these *ones*, whether regarded as originally described or referred to or freely constructed, are products which may be looked upon as existences illimitable in number and therefore reaching out into an infinite or illimitable domain. The rule not to forget the derivation of these products or units may be a salutary check upon absolutistic and mystic creations.

CHAPTER VI
INTERBEHAVIORAL PSYCHOLOGY AND
SPECIFICITY LOGIC
THE ROOTS OF SCIENCE

ALL science presupposes a natural history in which human organisms becoming interested in transpiring events study these events with varying degrees of precision. The perfect illustration is the evolution of astronomy from the casual interest in the stars. Such studies may be scaled as casual observation, field observation or minutely controlled experimentation. The results of these investigations in the form of casually described or critically defined formulations constitute constructions offering records of the origin, rhythm, relations, and dimensions of the crude data. Under the most favorable conditions such constructions are primarily derived from the contacts of the investigator with the events, and with a minimum projection into those events of characteristics stemming from traditional beliefs or conventions.

As we have seen, logic either in the form of processes or products constitutes crude data continuous with other humanistic or natural events. Unfortunately, however, the differences between logical and other happenings (interbehavior of astronomical or physical bodies or the interbehavior of organisms with their environments) have been magnified into absolute differences of kind. Thus logic has been regarded as concerned with ultimate powers and absolute relations, and as a result has been totally lifted out of the continuum of psychanthropic occurrences.

In the preceding chapter we indicated that underlying this fallacious construction of the nature of logical data is an equally fallacious notion concerning mind and mental processes. Psychology, however, as a definite natural science consists of constructions derived only from the observation of organisms in interbehavior with stimulus objects in specific fields or situations. It is hoped, then, that the treatment of psychological data as concrete, natural happenings may be serviceable in the study of logical theory.

PSYCHOLOGY THE SCIENCE OF INTERBEHAVIORAL FIELDS

The crude data for psychology are exclusively interactions of organisms and particular stimulus objects which may be other organisms, inorganic things, relations or other cultural and non-cultural events. These interactions constitute fields or situations in which the actions of both individuals and objects are important factors along with others such as the settings or circumstances under which the interaction takes place.

This interbehavioral-field construction with its emphasis upon actual events radically departs from the traditional construction which implies that objects are *creations* of a mind or psychic power. Interbehavioral psychology assumes that organisms and objects exist before they become the subject matter of the various natural sciences. All the sciences are late comers upon the earth and the transformations they have helped to achieve were only made possible by observing things and events previously evolved. During the cultural evolution of science, workers gradually became specialists concerned with particular kinds of interaction. Physicists preempted the domain of inorganic things, biologists that of living organisms, and psychologists took over the study of the intimate interbehavior of organisms and stimulus objects. The psychological event or interbehavior became described as the discrimination, sensing, or perceiving of objects and their qualities by organisms, their performance of feeling or thinking reactions to such stimulus objects, etc.

Interbehavioral psychological fields, implying a genuine behavioral dynamics, must be described in terms of coordinate response and stimulus functions. These functions are built up by the organism and object under specified conditions. For instance, an organism develops a salivating function when a buzzer takes on a corresponding function in the conditioned reflex situation. Psychological events, therefore, are historical and involve a definite durative factor.

The unit of psychological interbehavior called addition may be taken as another example of response and stimulus functions. An adding situation or field is set up by bringing a child into contact with an adding stimulus object, $2 + 3 =$. If the child now interbehaves properly with the stimulus object we may confidently describe the event on the basis of the child's prior contacts with

such a situation. During his arithmetical training he has built up adding behavior with respect to such an object, whereas the object has acquired the mutual and reciprocal adding stimulus function. In the immediate situation this function previously evolved now operates because of its presence in the field.

Especially to be emphasized is the concrete character of every psychological event. This means taking into account the specific situation in which it arises and operates. Accordingly, we consider not only the organism's biological and physicochemical character and the object factors in the situation, but also the specific cultural influences upon both organism and object.

In a psychological frame of reference the individual is simply an enormously large sum of response functions which are coordinate with an equal number of stimulus functions. Just as the stimulus functions are localized in specific objects, so the response functions are localized in the activities of persons. It is perhaps an inevitable convention to regard the action of the individual as simply the operation of muscles, bones, nerves, etc. Such an arbitrary isolated response has no place in psychological description. On the contrary, the psychological event consists essentially of particular responses inseparably related with the actions and functions of objects in event fields. The muscle-bone-nerve construction merely continues the age-old dualistic belief that mental happenings have correlates or substrata in organic events.

REACTIONAL BIOGRAPHY AND STIMULUS EVOLUTION

Two fundamental constructions of interbehavioral psychology consist of reactional biography and stimulus evolution. Through such field events as we have described, called an individual's reactional biography, organisms develop a vast number of responses. Correspondingly, objects undergo an evolution in assuming their stimulus functions in connection with the acts of particular individuals. Accordingly, there is excluded from the psychological field any generalized type of mentality as well as self-existing psychic states.

An individual's interbehavioral fields are, of course, not confined to natural situations, since there are many response and stimulus functions which are conventional and have to do with humanistic reactions and artificial properties of objects.

PSYCHOLOGY EXCLUDES BOTH MINDS AND SENSATIONS

Psychological data as objective events in no sense encourage such classical constructions as mind, intelligence, sensation, consciousness, or private and inaccessible processes such as psychic states or secondary qualities. All these constructions are definitely dated psychanthropic formulations. How, then, can a scientist committed to the investigation of natural happenings harbor notions concerning materials which by hypothesis are transpatial? The fact that he does so, however, has been amply demonstrated by the difficulties encountered in both psychological and logical studies. Even though severe measures have been resorted to from time to time in order to overcome this difficulty, so powerful are cultural traditions that the very measures adopted for eliminating psychics have resulted in their perpetuation, such as the sensations which figure so prominently in mentalistic psychology.

SCIENTIFIC OBSTACLES TO NATURALISTIC PSYCHOLOGY

Science Implements Faulty Psychology. It is an old tradition to attribute the faults of psychology to its close connection with some form of philosophy or metaphysics. Despite the truth it reflects, this tradition does not tell the whole story. For one thing, psychology itself in its mentalistic phases, with its doctrine of mind, reason, psychic qualities, etc., is the foundation and support of metaphysics. Then, too, there is the striking paradox that the closest association of psychology with the well-established sciences has failed to make it less metaphysical, but, quite the contrary, strengthened its transcendental fabric.

Those who are puzzled by such a situation might reflect that our general cultural atmosphere exerts its influence upon science as well as upon other phases of our civilization. That the votaries of the allegedly best-established sciences do not become immune to transcendent thinking is attested by the spiritualistic literature produced by physicists and astronomers in the last few decades. Naturally, then, psychology has suffered as much from the influence of a metaphysically conditioned science as from the more direct metaphysical influence itself. For this very reason the efforts of psychologists to make their subject scientific by drawing on other sciences and adopting laboratory or experimental methods have not

helped psychology because laboratory workers have implemented dualism in the process. Despite their good intentions, then, psychology has remained more philosophic than scientific.

A clearcut illustration comprises the events leading to the rise of modern laboratory psychology out of the work of the 19th century philosophical physiologists. Recall the efforts of Müller, Helmholtz, Lotze, and Wundt to discover a basis for the psyche or psychic state in the physiological processes of organisms. The story begins with Helmholtz and other physicists and physiologists attacking psychological problems. In general, psychic qualities were regarded as necessarily mediated by end-organs and neural structures. More especially the attempt was made to set up covariations between (1) stimuli, whether radiating energies, chemical changes or mechanical forces, (2) physiological changes in end-organs, specific neural conduction and brain termination and (3) sensation qualities.

What has happened, then, is that excellent work and valid organic data have been employed to substantiate and confirm the dualistic heritage of our culture. Our task, therefore, is to put the specificity principle into operation and differentiate between the presence and value of particular, authentic, scientific discoveries on the one hand, and the harm arising from improperly connecting them with psychological events, on the other. The process of perpetuating psychic states by relating them to physiological happenings has persisted from those days when spiritualistic philosophy was first implemented by physiological findings up to the present moment. A powerful factor in addition is the fact that psychologists have ceased even to reflect upon the havoc this situation has wrought to their science.

Incidentally it is interesting to note that this failure of psychologists to consider the unsolved problem of the connection between physiological processes and psychic entities is duplicated in other sciences. Struik¹ remarks that astronomers after three hundred years have not solved the problem of the ratio of the mean radii of the planetary orbits, though they have stopped talking about it. Psychologists, however, have in addition a different mode of procedure. They simply declare that the "psychic" problem is not theirs, that it belongs to metaphysicians.

¹ Struik, Kepler, p. 45n.

Dualism Implemented by Experimentation. To make psychology a study of natural events has been an ideal among psychologists since the development of experimental techniques in the biological sciences in the early decades of the 19th century. Unfortunately, as the history of psychology demonstrates, this ideal has been far from fulfilled. It is most significant that Fechner, one of the founders of experimental psychology, attempted by psychological experimentation to establish the mystic and religious correlation between the spiritual and material aspects of the cosmos. Though his work did not square with his beliefs, for experimentation can never be anything but the controlled study of natural events, his procedure became a model for the tradition that psychic states can be experimented upon and that what can be experimented upon is natural. All the more was this tradition supported by the fact that Fechner's work did lead not only to the development of valid methods of statistical and experimental procedures, but also to a formula or law of interbehavior. The secret of Fechner's success, of course, is that in practice he worked with the crude data of reactions of persons to stimulus objects, while in his constructions (his hypotheses and interpretations) he fitted into the age-old dualistic tradition. As the writer has pointed out,³ every record of an experimental investigation must be made in terms of the interbehavior of organisms with objects, events or conditions.

Even today experimental psychologists allow various psychic constructions to intrude themselves into their work, despite their careful and wholly exemplary experimental operations. An illustration is provided by the work of Stevens and Davis on hearing.⁴ With the employment of the latest and most effective electrical (amplifiers, oscillographs) apparatus they have secured records in a most refined and valid manner concerning what occurs in the nervous system when one hears certain sounds. Then they proceed to assert that "the centers seem to code their messages in a new language and we are still seeking the clue to its translation" (p. 307). This language is, of course, the psychic. The precise technique for injecting these psychics into objective techniques is to follow the tradition that the nervous system is somehow the physical or natural parallel of the psychic. Other techniques consist

³ Can the psychophysical experiment.

⁴ Hearing.

of asserting that mentalistic constructions like percept, image, and idea can be operationally defined. In other words, just as in the case of Fechner, what is being done is unwittingly or habitually translated as something else, because of the perpetuation of the dualistic dogma.

Whether experimentation reveals or suggests a dualism in psychological events awaits our answer to the question whether such events consist of the interbehavior of organisms and objects in actually existing fields or of unobservable functions of the organism or of the nervous system presumed to be identical or correlated with the operation of observable biological processes. We propose that carefully to interpret our results only upon the basis of our operations with organisms and stimulus objects will obviate the necessity of transmuting what we observe into traditional dualistic constructions.

Dualism Implemented by Constructionism. So thoroughly is dualism entrenched in our culture, however, that its proponents manage to turn every scientific device to serve its advantage. An interesting example concerns the principle that all science is constructional. Psychologists especially argue that since all science consists of concepts (constructions), why not hypothesize sensations. Then to support such psychic qualities they proceed to construct all sorts of presumably necessary physiological processes. Unfortunately this argument overlooks the fact that constructions may be false and useless as well as true and valid.

Dualism Implemented by Operationism. The operational principle which has proved so illuminating and fruitful in modern scientific methodology has likewise been employed to support dualistic interpretations in psychology. In brief, this principle was proposed to guard against the inroads of mystical tendencies inherent in mathematical formulation in physics.⁴ Its basic tenet is that scientific conceptions must be constructed upon the ground of actual operations upon events.

In three distinct ways the operational principle is utilized to support psychological dualism. The first is to point out that there are cases in which operations, especially technical laboratory procedures, are not possible, for instance, in building up scientific constructions concerning astronomical facts or in situations in which

⁴ Cf. Bridgman, Logic.

instruments are not available. The assumption is made, then, that free constructions are allowable, that is, not founded on observation. Immediately, elements having no analogue in spatiotemporal fields are admitted into science.

Secondly, the assumption is made that any manipulation of crude data presupposes a traditional spiritistic construction. For example, in experimenting upon the perceptual reactions of organisms to visible or acoustic objects it is presumed that one has been operating upon psychic processes somehow connected with neural or other biological events.

In the third place, dualism is implemented on the basis of the operational principle by supposing that "Parallelism would automatically reduce to a double-aspect formula, because where two sets of defining operations coincide perfectly they become identical operationally."⁶ The justification for this view is revealed in the following quotation.

We now know that operational definitions can always transform a psychological description, expressed in terms of consciousness, into a description in behavioral or physiological terms.⁶

In view of the fact that no psychologist has ever investigated anything but the interbehavior of an organism with stimulus objects and conditions it is plain that the operational principle is violently transmuted into a technique for establishing traditional theory. Those attempting this transmutation overlook the fact that when we study sensory discrimination we have two definitely objective sorts of investigation. For instance, when we say we study sensation we are really investigating the organism in interbehavior with sounds, colors, etc. On the other hand, when we study neural-action currents we are observing particular features of the organism's action in such interbehavior. In neither instance is there any scientific justification for assuming any dualism.⁷

Dualism Implemented by Verbal Conceptualism. As a final technique for maintaining psychics in psychology we may cite the verbal-concept argument employed even by authentic experimentalists. This technique involves the use of language in the form of

⁶ B. B. B. Changing Views, p. 379.

⁷ Boring, Titchener on meaning, p. 94.

⁸ Cf. Kantor, Operational principle.

propositions and descriptions to substitute for and perpetuate Berkeleyan metaphysics in psychology. The detailed procedure is to deny that one is concerned with psychic states but rather with language which embodies knowledge. It is thus assumed that nature is the same as the knowledge of nature. One can then say that the existence of psychic states can be established by reducing (translating) assertions about or definitions of them to language referring to natural events, as in the above quotation from Boring.

Certainly there is a vicious circle here. The powerful momentum of dualistic metaphysics is assumed as a basis for perpetuating the same dualism in psychology. Curiously enough those who strive to eliminate all this metaphysics are themselves branded as metaphysicians. For example, Stevens⁸ has asserted that the interbehaviorist who stands for the construction of descriptions from the interbehavior of organisms with actual objects is a realist, and therefore espouses a realistic metaphysics. In view of the fact that every psychologist begins and ends his observations with such interbehavior or crude data he need not start with (1) dualism as a general cultural presupposition and (2) traditional physical or physiological constructions as correspondents or bases for psychic states. In that case it is in no sense necessary to bring in any question of metaphysics.

Furthermore, since everyone must agree that scientific language consists of constructions summarizing our knowledge concerning natural events, neither these constructions nor the nature of knowledge itself are the natural events known or described. Knowledge is, of course, the interactions of persons (scientists in the present instance) with natural or humanistic crude data. To interact with anything beyond crude data is to interact with a construction. Hence the necessity of keeping our constructions in line with crude data.

Curiously enough, Stevens thinks it is significant to ask whether any operations can be formulated which will either prove or disprove metaphysical propositions. If we were interested in a metaphysics or the metaphysical character of nature, obviously there would be no such operations, but as we have indicated, no scientist need occupy himself with such questions. In the meantime, Stevens fails to consider whether any operation could be formulated to deal

⁸ Psychology, p. 231.

with psychic states of any sort or with the language the clue to which he himself admits he is seeking. Because this language is the language of spiritualistic metaphysics we venture to assert he will never discover the clue. The lesson is plain—namely, by the frail process of language manipulation ideas are established with which no scientific enterprise is in any manner concerned.⁹

THE ELIMINATION OF THE MIND-BODY PROBLEM

Scientists in general should have no difficulty in getting rid of psychic constructions. In no phase of their technical work, whether contacting crude data or describing the results of their observations do they have the remotest connection with anything beyond spatio-temporal boundaries.

It is true, of course, that historically natural scientists under medieval influences have assumed that their data were coupled with psychic epiphenomena. Even such scientific titans as Galileo, Newton, and Boyle divided the cosmos into spatiotemporal or primary qualities, on the one hand, and nonspatiotemporal or secondary qualities, on the other. Historically it was the moral as over against the natural philosopher or psychologist who was presumed to wrestle with the nonspatiotemporal factors, and which he did by talking about mental powers or a source of knowledge. This historical tradition is kept alive by such natural scientists as Jeans and Eddington who ultimately reduce natural events to spiritualistic materials.

Although it is now possible for psychology to become a natural science it has not yet completely given up commerce with non-spatiotemporal factors. This means to say we are still living under the shadow of the Renaissance doctrine of the bifurcation of nature. As of but yesterday a biochemist writes:

Empirically it seems obvious that the spatial relations of material processes are important, but the spatial relations of mental processes, if any, unimportant. In other words, the spatial extent of mental life is very minute or very difficult of observation and need not as a rule be taken into consideration.¹⁰

Technical psychologists, of course, disclaim all interest or pre-

⁹ Excepting, of course, the psychological and cultural investigation of the origin and maintenance of such theories.

¹⁰ Ritchie, Whitehead's defence.

occupation with metaphysical problems. Nevertheless, they keep this dualistic tradition and the psychic alive by various modifications of the mind-body doctrine. If they do not favor outright parallelism of the psychic and the organic they resort to such subterfuges as psychocerebral or psychosomatic constructions, which actually have no other effect than the misinterpretation of both psychological and biological events. Today, of course, we should not expect psychologists to talk about psychoses paralleling neuroses, albeit, only their language and not their essential thought has changed. Psychic states are called experience, sensations, or intelligence, but they are nevertheless epiphenomena dependent upon and functions of biological, especially neural, structures and their physiological functions.

Throughout the period of scientific dualism psychologists have leaned heavily on biological principles. This is as it should be, for certainly no psychological interbehavior is performed by nonbiological objects. But where are the biological structures or functions connected with psychic states? As the writer¹¹ has frequently pointed out the sole basis for the interpretation that sensations or intelligence are functions of biological structures lies in the fact that the destruction of parts of the organism results in the lowering of its interbehavioral efficacy. Obviously the mutilation of an organism affects one of the primary factors (the organism) in an interbehavioral field, but it grants no support to any psychic doctrine.

We have already indicated of what psychological interbehavior consists. Biological facts constitute different kinds of interbehavior. As we have implied, biological events, of course, always participate in psychological events because the latter involve the activity of organisms, but there is no basis for making psychological facts the epiphenomenal functions of biological facts. More specifically, there has never been any evidence that psychological interbehavior consists of psychic states mediated by the brain, as the historical dogma has presupposed.

A brief glance at the history of the brain dogma is important for its evaluation. To go no further back than Descartes, we observe why some part or the whole of the brain has been regarded as the seat of consciousness. But what is more important is to see that it

¹¹ Operational principle; Nature of psychology.

is the same kind of cultural tradition which endows the brain with the unobservable psychological (psychic) functions of thinking, reasoning, etc., besides the observable physiological activities of conduction, coordination, and integration. Now it is only when psychological data are considered as vague, transpatial "functions" that such a doctrine seems plausible. When psychological occurrences are taken to be complex field events, the brain and the other parts of the nervous system have their proper place in psychological happenings, where they act in the coordination of the organism's activities. They play no special psychological role; they are and remain biological. Though one cannot overlook the essential function of the nervous system in psychological interbehavior, it is no more essential than the circulatory or any other biological system. It is a fundamental fallacy of the brain dogma, therefore, to transform biological facts into explanatory principles.

For a naturalistic psychology it is no paradox to say that the "body" of the mind-body construction exists no more than the "mind" or the combination of the two. This assertion, far from denying anything about a biological organism, securely places it in its proper biological domain. The point is that in traditional psychology the body or the organism is really not dealt with as a biological object participating in psychological interbehavior, but as a set of constructions not derived from the study of either biological or psychological events, but definitely taken over from a cultural tradition, established, as we have seen, before the days of modern science.

THE CONTINUUM OF NATURE

Though psychologists are undeniably guilty of imposing dualistic constructions upon their crude data they are not alone in their guilt. The dualistic dogma has infected every type of scientist. The psychologist along with the physicist was adversely influenced by the scientist's early adoption of the notion that nature was divided into two parts, the extensional and the psychic. Fortunately for the physicist who concerned himself with extension, he could build up constructions concerning mechanical and vibrational happenings which gave workable if not altogether satisfactory systems concerning the nature of physical facts. Because of an intellectual division of labor he could, even with a clear conscience, declare

that colors and sounds and other so-called secondary qualities were not natural but belonged to the unreal psychic realm. This circumstance, on the other hand was fatal to psychology, since the psychologist blandly undertook to build himself a science out of unrealities, on the assumption already discussed, that he could correlate or parallel sensations with physical vibrations and physiological, especially neural, functions.

The study of actual events, on the contrary, allows for no such bifurcation, but decidedly suggests a continuity of happenings. All sciences deal with specific forms of interrelations between objects or events. In each case the particular scientist, say a physicist, biologist or psychologist, selects a distinct form of interaction for his investigation. The physicist works with interactions which he can formulate into equations, which on the whole yield a principle of energy equivalences. The biologist, on the other hand, is concerned more with the complicated processes in which energies are stored up and expended without definite equalities of operation in the two interacting objects. The psychologist studies a still different type of event, namely, one in which the two interbehaving objects participate in a field situation on the basis of their prior interactions in similar situations.

To appreciate the fact that nature is continuous one need only divest oneself of metascientific notions, keep strictly to the procedure of studying observed objects, relations, and conditions, and build hypotheses solely on the basis of such events. Probably one of the best criteria of intellectual efficiency is to be able to sunder sharply the authentic character of events from the traditional cultural accretions which have become attached to them.

VARIETIES OF PSYCHOLOGICAL EVENTS

Psychological events cover a large range of interbehavioral fields displaying specific organizations of factors including types of organisms and objects with their various settings. Also, there are many forms of interrelationship or contacts between these factors. In order to illustrate the interbehavioral principle we select the following representative set of such interbehaviors.¹²

1. *Sheer Contactive Interbehavior.* The simpler psychological interbehaviors may be described as sheer contacts or adjustments

¹² For an analysis of these interbehaviors see Kantor, Principles.

between an organism and an object or between two organisms. Excellent illustrations are reflex action and elementary perceptual interbehavior. The former is a field event in which the organism merely performs a differential response because of its own natural properties and those of the stimulating object. In the more elaborate differential response, conventionally called perception, the contact may be described as a discrimination of the character of an object and its differentiation from other objects and its surroundings, not only on the basis of its natural properties but its characteristics achieved through the organism's prior contacts with it under other conditions.

2. *Manipulative Interbehavior.* When an organism manipulates things the contacts are more intimate than simple discrimination, approaching or withdrawing; they also display discernible consequences. Manipulations may be mere handling or rearrangement of objects, or complex throwing, tearing, and destroying actions.

3. *Implicit Interbehavior.* An exceedingly important type of psychological interbehavior is that in which the field organization involves substitute stimulus objects. All complex situations require adjustments in which the adjustmental stimulus object is absent. The technique of interbehaving with this absent object is by means of contact with a substitute stimulus. In such fields the individual nevertheless operates effectively though he is not in direct contact with the adjustment stimulus object.

Typical adjustments of this implicit type are the imaginal and conceptual form of interbehavior. In some cases the adjustment depends upon one's prior developments of reactions to things not now present. In other instances it involves reactions to acts of things not occurring at the moment but which are known through records or other substitute stimulation.

4. *Organizational Interbehavior.* Putting parts of things together into wholes or organizing sets of objects into patterns or systems on the basis of some kind of criterion, desire or need obviously comprises complicated fields including simpler interactions. In organizational activity, therefore, the emphasis is upon actual procedure, upon what is done and with what; thus a satisfactory description of such fields necessitates the specification of particular component discriminations and manipulations.

5. *Creational Interbehavior.* Among the direct contact fields the most complex are the constructive or creational handling of things.

Two distinct types may be differentiated. In the first, the construction involves relatively more direct contact with manipulated objects, whereas the other type emphasizes the imaginative engendering of a variety of objects. To suggest the differences in creational interbehavior brings to the front the concrete and specific character of the field.

6. *Referential Interbehavior.* A unique form of psychological field is that in which the organism refers to some stimulus object. A referential field is unique, we say, in consisting of at least three primary factors instead of the usual two. In addition to the acting organism and the object referred to there is another factor localizable in the thing or person to whom the reference is being made. This third factor may be the original referor; in other words, the person speaks to himself. In ordinary conversational speech references may be made by pointing, gesturing, or by elaborate, conventional linguistic behavior.

7. *Memorial Interbehavior.* Fundamentally this type of interbehavior involves contact with absent objects or situations, and thus entails the implicit type of response earlier described. The precise operational technique is thus to interbehave with something serving as a substitute stimulus. Typical examples are the ordinary remembering, recollecting, and reminiscing actions in which we carry out projects previously arranged, or look back to events in which we have participated at former times. As in all interbehavioral fields, the individual's exact activity varies enormously. Memorial action may be consummated by crude, overt movements, verbal and gestural actions, and even subtle responses of realizing the absence of some thing or the nature of an unfulfilled performance.

8. *Speculative Interbehavior.* Again we have a type of constructive or creative interbehavior. Given some object or event the field involves creating a model or analogy concerning its nature, origin or significance. To be emphasized here is the individual's great freedom of action, though it is inevitable that he should be connected if only by a tenuous bond with the original stimulus object.

PSYCHOLOGICAL AND SOCIOLOGICAL INTERBEHAVIOR

The continuity of natural events is well reflected in the conviction of all scientists that they are students of interbehavior. The chemist thinks of himself as an investigator of chemical substances

in reaction, the physicist of the behavior of nuclear or electronic waves or particles. Thus scientists adopt similar descriptive techniques for the results of their investigations—an advantage not to be misprized. Nevertheless, the variations of the interbehavior studied and the different conditions under which it takes place need to be kept distinct. Certainly in the investigation of psychological events and their relation to logical work we must distinguish sharply between genuine psychological activities, which are always specific interbehaviors of individuals, and sociological behavior described as group action.

This distinction is especially important in view of the fact that logicians convert conventional beliefs or knowledge into things, and in addition look upon such beliefs as supporting the existence of the objects believed in. Hence we have objectivity guaranteed by public knowledge or what is sometimes called intersubjective belief. Such views have obviously arisen because of the prevailing notion that psychological actions are intangible processes or at least private and not objective interbehavior. As a consequence verbal or other formulations are mistaken for established and valid existences.

To illustrate the difference between genuine psychological and sociological behavior consider a person multiplying a pair of two place numbers. This particular process action may be regarded as a custom, a sociological item abstracted from the details and summed up in description. In a sense, sociological behavior is class behavior, actions performed by a sociological unit; it is also class action in the sense of a formulation or universe embodied in a set of descriptive terms. As the action of groups and not of individuals sociological action is essentially statistical.

The fundamental distinction between sociological and psychological action may be clarified by comparing the hesitant and random actions of individuals in inventing or discovering the multiplication process with the completed and finished performances according to rule or practice. All original inventive and discovery action is individual performance, though it may be duplicated by many individuals if they are in contact with similar situations under comparable backgrounds of interbehavior.

Sociological and conventional activities in the logical domain become established and perpetuated through behavioral products

embodied in texts and treatises. These products operate for longer or shorter periods depending upon the extent to which new members of the logical community use them in their thinking. As we have seen, these institutions become modified and varied by cultural changes.

Between the essentially individual and unique psychological actions are the social psychological performances which are common and similar because of the interrelation of persons affording precepts and examples.¹³ Social psychology covers the cases in which the stimulus functions of objects take on institutional character—that is, they operate in fields in which they are correlated with response functions common to various individuals. Social psychological groups consist of similar actions performed by members of schools, clubs, families, religions or other aggregations of persons with respect to given objects and situations.

INTERBEHAVIORAL PSYCHOLOGY AND THE BOUNDS OF LOGIC

Scientific psychology carries important implications for marking out the bounds of logic. To begin with, if a logician is sensitive to the nature of interbehavioral fields we may expect his logical work and products to become a scientific or natural discipline occupied with actual events, that is, the organization of systems of existing or created things and the products of such organizing actions. If, on the other hand, he is involved in some form of logos, absolute reason or principles, we know that he is concerned more with arbitrary constructions and metaphorical assumptions than with materials finding a place in a scientific order of things.

Among such arbitrary constructions are numbered all absolute and inevitable laws of thought or laws of absolutistic mathematics, also principles of being which are universally applicable because they are concerned with the general characteristics of a subject matter, that which it has in common with everything else.¹⁴ To be added also are principles of inference that must be accepted by all, as well as those which cannot be proved because what is required in every proof cannot itself be demonstrated. There are likewise the mathematical infinite and the logical gateways to infinity, un-

¹³ Cf. Kantor, *Outline*.

¹⁴ Cohen and Nagel, *Introduction*, p. 186f.

less they are looked upon as heuristic constructions for the achievement of some definite purpose.

Behind such views is the absolutistic tradition which carries us back to intuitive and obscurantistic trends. Of course, one does not go beyond certain limits in proof, but that is because there are no absolute principles, not because there are. In actual logical work we need only go so far as to organize a system within the limits of a given framework. Having done this, our work is completed.

The alternatives here really boil down to some sort of arbitrary creation, on the one hand, with an asserted absoluteness and certitude, and, on the other, concrete and particular action which can issue in necessary and satisfactory adjustments. To keep within the bounds of scientific psychology means giving up the power to operate with metaphysical systems such as Nature, the World Order, etc., presumed to exist in the sense of being discovered by transcendent powers of Mind through such processes as giving Laws to Nature, etc. Such comprehensive and absolutistic realms are capricious constructions, in the most reasonable cases rooted in human circumstances and, in the least reasonable purely autistic developments, based upon sentimental reactions of fear, hope, and desire. Though some logicians may regard the naturalistic rational alternative as imposing serious limitations, actually it leads to an increased efficiency in dealing with natural and cultural facts.

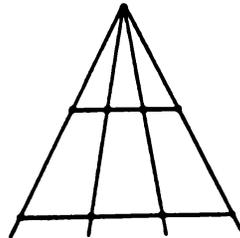
Because of its relational subject matter, mathematics affords us excellent illustrations of the two alternative procedures. Even the most abstractive constructions start from concrete events, either from a progressive series, for example, the natural numbers, or from the activities of individuals building up systems or arrangements. It follows that infinities are cuts or boundary points between what can and can not be done, though the process is indicated. For instance, we count or arrange series of numbers, but only so far and so long; we then set up a symbol or declare the existence of the construction of the series or process. We achieve here a description of a three-phase series of conditions, a potency and an impotency with a cut which may be called a partipotency. In the case of convergent series the process is bounded, while in divergent series the boundary is lacking.

In analogical construction mathematics offers another excel-

lent example of how to operate in infinite domains. When the logician or mathematician makes use of the following devices:

$$\begin{array}{cccccccc}
 1, & 2, & 3, & 4 & \dots & \dots & n & \dots & \dots & \dots & n+1 \\
 2, & 4, & 6, & 8 & \dots & \dots & 2n & \dots & \dots & \dots & 2(n+1)
 \end{array}$$

Lines related
as 1:2, with



biunique point
correspondence

for establishing a one-one correspondence of an infinite part and a whole, the product is enormously useful, but in all its heuristic perfection it is simply a unique construction, evolved in interbehavior with things and for a purpose. These constructions are not to be discounted merely because they bespeak powers limited by human circumstances or a bounded graphiopotence or linguipotence, since the fundamental criterion is the accomplishment of some type of system.

INTERBEHAVIORAL PSYCHOLOGY AND THE DEFINITION OF LOGIC

No better way to define logic, of course, can be found than to observe what logicians actually do. Following interbehavioral principles we look for the kind of stimulus objects with which the logician is in contact and the type of interbehavioral field in which he operates. In this manner we may discover the basis for defining logic as (a) scientific method, (b) mathematics, (c) the study of the laws of thought or things, or (d) the process of building up a cosmic dialectics.

Four Methods of Defining Logic. Since logic in the final analysis is an interbehavioral enterprise we may relate the various views concerning its nature to the emphasis of one or the other of the three primary factors in interbehavioral fields. Thus logic is usually defined on the basis of either subject matter, process, or product. This gives, then, three bases for definition, all implying some appreciation of the system-building enterprise. A fourth method, which is independent of that enterprise, results in simply defining logic on the basis of some general characteristics.

(1) *Definition by Subject Matter.* To define logic as the study of the inductive process, theory of inquiry, scientific methodology and the logic of mathematics is to emphasize subject matter. Whether or not the intrinsic work of organizing systems is acknowledged there is a strong inclination toward particular kinds of materials out of which systems are constructed.

(2) *Definition by Process.* Those who stress the laws of thought or inferential or implicational techniques exemplify the emphasis upon process.

(3) *Definition by Product.* Logicians who define their field by product produced vary greatly in what they understand logic to be. The two extremes probably are marked by mathematical and general cosmic systems. Keeping before us the fact that all logicians are particular workers environed by complex materials and cultural situations we may expect them to adopt various definitions the while they insist upon a single exclusive formula.

(4) *Nonsystem-building Definitions.* The definitions of logic set up in greater or lesser isolation from actual system-building enterprises cover a large range. This is not surprising since the actual work done and the materials worked upon are somewhat remote from the definer's immediate point of vantage. As a consequence the many definitional criteria available can only be derived from general philosophical interests.

The Logical as the Reasonable or the Rational. Writers who identify the logical with the reasonable doubtless hark back to some form of action, though they regard themselves as concerned with some sort of ultimate thing or power called reason. This principle is presumed to possess a fundamental law-giving or discovering capacity with respect to any kind of material. Since reason is envisaged as independent of the actual interbehavior of persons with things, it operates on a plan of universality and *a priori*. Naturally there can be no rigid determination of what is reasonable in this sense, though actually we can trace the attributed characteristics to conventional attitudes and the magnification of processes observed in everyday life.

On the whole, the rational refers more to the character of objects dealt with. The very fact that certain objects or conditions exist, especially if they are important, wins for them this characteristic. Classic examples are suggested by the Leibnizian Principle of Sufficient Reason and the Hegelian Real as the Rational. These

definitions clearly reveal the acts of creating and projecting cosmic entities on the basis of culturally influenced personal attitudes.

The Logical as the Pervasive, General or Universal. Whatever other definitions logical writers adopt most of them also harbor notions of pervasiveness, generality, or universality. When objects are stressed the logical is assumed to be the most pervasive and general character of existing things and also of what might exist. When actions are uppermost in the definition universality is presumed to apply to the potency of processes and techniques to deal with every imaginable subject matter in a rigorous manner.

The Logical as the Conventional or Acceptable. In many instances what is called logical is simply what is conventionally adopted as a matter of social habit or custom. Mathematicians, for instance, regard certain well-established notations or operational processes as inherently more logical than others. Similarly, the historical difficulties encountered by mathematicians with negative, irrational, and imaginary numbers and with noncommutative relations are further samples of the coincidence of the logical and the conventional.

The acceptable, or what is agreed upon, as a basis in defining the logical field is excellently illustrated when logical systems are rejected because they do not conform to approved models. A classic instance is Saccheri's belief that his new geometric system was not logical and concealed some contradiction because it did not conform to the established Euclidean system. A more technical logical parallel is the inability to drop the principle of excluded middle from the accepted laws of thought.

The Logical as the Theoretical. A rather common attitude is that the theoretical coincides with the logical. It is frequently asserted that the logic of mathematics is concerned with the theory of the subject as over against (1) its practice or operation or (2) the relational subject matter. Likewise, the theory of physics or any other science, the theory of law, etc., are taken as the logic of the subject. The implication is that theory involves the rationale and the systematic organization of the materials concerned.

The Logical as the Formal. One of the most widespread definitions of the logical identifies it with the formal. There are, however, many notions concerning the latter which are not always distinguished. In some instances the formal as ideal is contrasted with the actually existent. Again, the formal, whether or not

envisaged as the ideal, is distinguished from the factual in that it is formulated into propositions, whereas the factual is considered independent of such formulation. In this connection Dewey¹³ regards *if then* propositions as concerned with relations that are non-temporal and nonexistential. The basic distinction apparently involves the separation of linguistic from nonlinguistic existential things.

Another view of the formal is that things and events possess forms distinct from their contents or materials. For example, the integral event of perceiving is constructively segregated into (1) sense data, "the things that are immediately known in sensation," (2) sensations, "the experience of being immediately aware of those things," and (3) the "physical object" inferred from (2). These formal aspects are set over against the perceiving event as material or content. In many instances, at least some of the factors are regarded as independent existences as in Platonic realism.

The Logical as the Necessary. A prominent factor ascribed to the logical is the character of necessity. This necessity may be localized either in rational processes, as, for example, the inevitable procedure from premises to conclusions in deduction. One is reminded here of Benjamin Peirce's definition of mathematics as "the science which draws necessary conclusions."¹⁴

On the other hand, the necessary and hence the logical may inhere in abstract relations or relations between things. The former is illustrated by Aristotle's sea-fight alternative.

A sea-fight must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place to-morrow.¹⁵

Logical or necessary relations between things find their locus in the fittingness and appropriateness of units in total schemes or situations. Classifications of things it may be insisted are or are not logical on the basis of whether or not the things really belong or fit together or are appropriately connected.

The Logical as the Systematic. Implied and sometimes expressed in several of the above definitions is the system motif.

¹³ Logic, p. 271.

¹⁴ Peirce, *Linear*, p. 1.

¹⁵ *De Interpretatione*, 19a, 30.

Certainly this is the case in the rational and necessary definitions, even though the systematic is envisaged only as an autonomously existing condition which is discussed or recognized. Among the most acceptable systems or structures are the interrelationships of natural objects or mathematical relations. What is lacking, however, in such appreciations of system is the actual process of organizing the structures as activities based upon chosen criteria. Moreover, though such definitions come very close to some factor of the logical enterprise they stress too much either materials or products. If the construction of the system-building procedure could be added, the approach to logic as a definite field event would be readily available.

SYSTEM BUILDING AND ONTOLOGY

The impact of interbehavioral psychology upon logic results in bringing logic as well as other human enterprises into the sphere not only of natural but particular events. In consequence logic becomes a system-building enterprise focused upon a particular system construction, just as nautical engineering constitutes a specific ship-building enterprise whenever a particular ship is designed. How important this interbehavioral consequence is may be convincingly demonstrated in handling the ontological problem that has proved so troublesome to logicians.

Briefly stated this problem concerns the relationship between logical work and products and actual things and events. In Dewey's words: "The basic issue concerns the relation of the logical and the ontological, the formal and the material or existential . . ." ¹⁸ On the whole, Dewey denies that "things have the formal properties that characterize ideas that are employed in reasoning about them and that are necessary for consistent reasoning. . . ." ¹⁹ More definitely the question is raised whether the laws of thought and particularly that of excluded middle are directly applicable to things.

Historically, of course, this problem arises from a type of psychology which allows for the conventional polarization of things and thoughts. Follows then the dilemma of applying logical thoughts, sentences, or forms to existing things.

This problem naturally assumes different forms. We do not, however, propose to consider the ultrametaphysical question how

¹⁸ Sphere, p. 705.

¹⁹ Ibid., p. 702.

a logical system can be related to the universe. It is sufficient to remark that the *reductio-ad-absurdum* of this form is to be found in the Hegel-Bradley-Bosanquet conclusion that, after all, thoughts and things are identical.

Instead we consider only those logicians who are concerned with concrete things and events and their relation to logical propositions and systems. Here we find a definite opposition. Among those who deny the relationship there is Russell, for example, representing the formalistic view that ". . . logic aims at independence of empirical facts. . . ." ²⁰ And, again, the instrumental logician takes the stand that the laws of thought are laws of logic and not of things. ²¹

On the other side there is the contention that:

. . . the laws of logic are unique in that they turn up in every conceivable inquiry, and in that no alternative postulates have ever been successfully applied; one suspects, therefore, that they represent factors invariant in *every* subject-matter. ²²

We propose that interbehavioral psychology with its basis in specific interactions, such that every thought or system-building operation constitutes definite contact with things, enables us to resolve the dichotomy problem. Essentially the resolution consists of taking strict account of the field in which the logician is operating and especially the particular kinds of stimulus objects he is handling. On this basis we can determine whether the system is built of (1) things which can or cannot be rigidly formalized, (2) observed or constructed relations, or (3) symbols or sentences which may be thoroughly consistent but unrelated to actual events.

Those who believe that logic is ontological argue that the propositions organized are operationally derived from and hence continuous with things. It is probably not straining a point if we interpret this view as a general recognition of the specific psychanthropic character of all logical work. But to agree that all enterprises are enclosed within and limited to direct or extrapolated contacts with things should also include the recognition that there are all sorts of systems. One need not assume that only things or propositions concerning existing things can be systematized. One

- Peirce, *Logic*, p. viii.
- Dewey, *Sphere*, also *Applicability*.
- Nagel, *Can logic*, p. 709.

can systematize sentences themselves as in formal syntactics and also sentences designating imaginary objects. What the logical ontologists fail to consider is the range of system-building materials.

In what sense can logic deal with nonexistents? Logicians often assume that there is a domain of logical existence different from natural existence, as when they assert that a thing, a quality, or relation is only logical. The ontologist is right in believing that round squares, dimensionless points, subsistent essences, sense data, etc., are constructions derived from contacts with actual things. But he is wrong in rejecting systems constructed from sentences designating such creations. Doubtless the motive for overlooking everything but scientific and methodological systems is that logicians overvalue logic and think of it as something more than sheer system building.

Logicians who separate the logical from the ontological, the formal from the existential, certainly stand on solid ground, but only on a specific interbehavioral basis. For example, when Dewey declares that “. . . an abstracted relation is purely conceptual—or subsistent and universal—in character, it does not exist by itself in *rerum natura*”²⁸ he gives the impression of confusing nonexistence with abstraction products as materials. Every abstractional process, however, definitely displays the continuity of interbehavior with the original things and with the products. When we abstract from some thing or event is the original stimulus object totally dissipated? In what sense is any universal sentence, for example, “Reduced pressure lowers boiling point” concerned with something outside *rerum natura*? Assume that at any moment there is no reduced boiling-point event. Nevertheless, there still remains an abstracted product originally derived from that happening; the difference is that one now remembers the event, or believes or reads the statement about it; in short, an interbehavioral process is what we have rather than an abstracted relation lacking spatiotemporal character.

It might well clarify this issue to differentiate between an event occurring and the linguistic reference to that event. From an interbehavioral standpoint this is a question to be determined entirely by interbehavioral operations. Even in an experiment in which we have to report that the results we have obtained are true only

²⁸ Sphere, p. 704.

on an average, how can we insist that the law or rule describing the events is purely logical? To do so is to build up a realm of sheer metaphysical essences.

From an interbehavioral standpoint the nonontological logicians are entirely correct when they assume that systems can be built without regard to particular objects. Certainly the materials for system construction may be ideas or concepts. Anyone who collects or organizes attitudes or opinions builds a statistical system of such materials. Incidentally, such materials in themselves, as contingent facts, are not subject to the deterministic rules of contradiction or excluded middle. Furthermore, there is no objection to these materials for system-building purposes even if they are isolated reactions, that is to say, abstracted from the total interbehavioral field in which they are located.

On an interbehavioral basis, however, we can never assume an autonomous mentality ideating or conceiving in a vacuum. Rather, ideas or conceptions constitute reactions to things. Though we can deal with the reactions more or less independently of the stimulus-object things, at some point we have to take account of the origin of the reactions. Practically speaking, we separate the reaction by means of records or sentences transcribing the remarks of those who inform us of their ideas and concepts. Incidentally such materials as we are now considering comprise constructions less remote from concrete interbehavior than are those concerned with round squares and similar subsistential entities.

Let us not forget either that even though logical systems can be built of detached responses handled by records or sentences, the system builder is always located in an interbehavioral field where he is in immediate contact with the reactions as stimulus objects. In a double sense, then, logic is ontological—namely, the reactions as materials are derived from interbehavior and the logician himself is definitely operating in an interbehavioral field.

For the most part ontological questions arise from the fact that logicians inevitably begin with sentences or linguistic elements, and, moreover, do not consider that such linguistic interbehavior is on a par with the other sorts of interbehavior performed by individuals. Accordingly, in many instances we may ask: What is the relationship between the linguistic interbehavior and the operational contact with things as the source of that linguistic interbe-

havior? Three general possibilities suggest themselves. In the first place, the referential interbehavior may be very exact—that is, it has a very definite correspondence to an operation upon some object or event. We expect a scientist's description of an event to correlate rigorously with that event. In the second place, the individual may be free in his inferential interbehavior and verbally pile up characteristics for the objects interbehaved with. A typical example is a false report of what one has witnessed or an innocent adding of interpretation to the facts observed. And, finally, the individual's contact with an event may consist of using it merely as a jumping off place for an elaborate construction, perhaps completely unjustified by the event. For example, a person admiring the construction of an animal or plant immediately assures us that it has been created by divine intention.

Are logical characters—universals or abstractions—inherent in things? The answer, of course, depends upon the specific character of this question. When Dewey, for example, argues that logical functions cannot be transformed into antecedent existences he means to point out that when propositions are made concerning events as descriptive of certain contacts with things we must not regard the results of such contacts as having been localized in the things. Undoubtedly this is a correct view; in other words, neither a reaction to things nor a description of that interaction exists antecedently to the interaction. It would be an error, however, to deny that the result of contact with things is a necessary consequence of such a contact.

Opponents of this view, on the other hand, point out that propositions concerning things may be asserted which afterwards are verified by the things. Thus the verification is regarded as establishing the discovery of certain properties or conditions of objects. Further, such thinkers are intent upon showing that even relations which are abstracted and set up into propositional form may exist objectively and antecedently to such abstraction and description. For example, McGilvary²⁴ points out that Einstein's idea, that the path of any light ray that passes through the gravitational field of the sun is deflected during passage by a certain amount, proved to be a statement about events existing antecedently to the description.

²⁴ Professor Dewey, p. 564.

INTERBEHAVIORAL PSYCHOLOGY AND SPECIFICITY LOGIC

Granting the fundamental principle that all logic in the final analysis consists of (1) the activities of individuals who build systems and (2) those systems as products, there is immediately removed every basis for the presupposition that there are universal and exclusive principles, independent of concrete system building, which determine the nature of logical work. No *a priori* limits can be placed on the number and type of systems built. System builders are conditioned only by various technical and cultural circumstances. Even those logicians who are intellectually circumscribed by certain traditional or conventional bounds produce different systems on the basis of the objects worked with and the variations in pattern which the systems assume. All this variability and specificity may be illustrated by the following arrangements of system types.

1. *Cosmic and Noncosmic Systems.* Cosmic systems are illustrated by Hegelian dialectics or by the logics which presume to exhibit the uniformity and interrelationship of nature. Noncosmic systems, on the other hand, include those which do not presume to systematize the universe.

So far as dialectical logic goes the system builders start with the assumption that psychological or logical processes consist of psychic activities which in some form are a part of, and taken altogether constitute, a total spiritual whole. The fundamental technique is to use linguistic materials as symbols or representatives of entities which though they may have no actual existence are justified on the basis of carefully chosen, commonsense analogies. An important analogical model is the fact that every proposition we make or every act we perform has numerous implications and interrelations.

The instrumental logician simply throws overboard the enormous complex of hypothetical connections in the interest of specific jobs. This antibloc or antimetaphysical presupposition in many ways is much more in line with the concrete activities of scientists and other workers with particular events. Because of the more limited scope of their problems such workers may well be critical of the totalitarian views of the dialectical logicians, since the former are not obliged in order to carry on their work to build upon such premises. The instrumental logician, however, fails to free himself entirely from the implication of totalitarian or dialectical

tical logic, since his criticisms cover only type of material and product, but not the exclusive or totalitarian principle. In consequence, he, too, proposes a generalized type of system and regards logic as a unique organon for various purposes. Although we might well expect him to regard his logic as concerned with specific system-building events, since he is opposed to metaphysical ontology, he still overemphasizes the formality of his logical procedure and logical propositions.

2. *Formalistic and Contentistic Systems.* In this dichotomy the principle of division is whether the material employed is formal, that is, propositions, syllogisms, symbol systems, or whether it is presumed to be actual things. In the latter case it is assumed that one is organizing facts of a concrete sort into a scientific or other type of system.

Symbolic logic on the whole appears to avoid the cosmological difficulty, though at the cost of reducing logic to systems of tautological sentences which may be pure but infertile.²⁵ Here it is obvious that the withdrawal from ontological or existential problems is a fatal deficiency. Fundamentally the faulty analysis involves the assumption that mathematical relations, which are after all the fundamental pattern and justification for this type of logic, do not constitute existing things. This is simply ignoring the fact that relations constitute stimulus objects in precisely the same way as electric currents or masses. The only difference is that the symbolic logician is interbehaving with abstract stimulus objects, which, though lacking thing dimensions, are nevertheless existential.

A fundamental implication of the interbehavioral principle is that symbols or linguistic facts do not constitute the exclusive materials interbehaved with. Symbols are substitute stimuli for things and relations in the interbehavioral field. Obviously the symbolic logician is not merely interested in the sheer arrangement and order of symbols; he is further concerned with problems of implication and interrelationship of various abstract elements which in the final analysis are themselves relations.

3. *Concrete and Abstract Methodistic Systems.* Concrete methodistic systems aim at basic techniques for organizing objects and events. System-building principles articulate with content in the

²⁵ Except in the sense Poincaré intended when he asserted: "La logistique n'est plus stérile; Elle engendre la contradiction." Cf. *Foundations*, p. 483.

sense that the methods are presumed to apply to scientific data. The procedures, however, are universalistic and not specific, although methodistic logicians claim to validate their systems by taking their departure from concrete organizations of events.

Abstract methodistic systems are constructed on the basis of conventional and established mathematical procedures. The aim is to produce abstract systems by proceeding from chosen axioms or postulates and developing a set of consistent theorems which permit a coherent structure or system. The theorems of the system are required to be related by a method of deductive implication.

AUTHENTIC SPECIFICITY LOGIC

By contrast with all universalistic logics specificity logic never departs from the principle of particular operations. No other generalization is presupposed than that system building goes on. Thus an indefinite variety of systems is possible depending upon stimulus objects, criteria, purposes, and needs of the logician, as well as the availability of system-building facilities.

That all system building actually is specific is sufficiently attested by observation freed from theory based upon philosophical influences. Such emancipated observation results in distinguishing the essential differences between intuitional and nonintuitional geometries, whether Euclidean or non-Euclidean, or between analytic mathematical systems which do or do not allow transfinite numbers. Similarly, the domain of physics is replete with fundamental variations depending upon the adoption of postulates of classical mechanics, either rational or empirical, or relativity principles as a starting point. The evaluation of systems likewise demands the selection of a type of physics as a starting point. The observer can readily determine whether a system builder has adopted some type of physics as an absolute basis for judging physical systems. Needless to say, the same situation prevails in every field in which systems are built or in which logical processes are employed.

Obviously it is not intended to imply that one cannot group and classify systems to form a totality. Nor do we deny that such organizations of systems are allowable. What we do insist upon is the recognition that such generalizing processes are constructional

enterprises developed in specific situations for definite purposes.

Once more we repeat that because the interbehavioral principle is concerned with system-building procedures and not with any particular system product it is neutral and tolerant toward all systems. The student of system-building operations is indifferent to whether a system is cosmic in scope or scientifically established. In each case the question concerning its nature and value must be determined upon the basis of the specific conditions under which the construction takes place. We have pointed out sufficiently that interbehavioral principles allow for the construction and implicit manipulation of nonexistent things and have likewise suggested the actual operations employed in such systematizing procedures.

CHAPTER VII
THINKING AND REASONING AS LOGICAL
OPERATIONS

THINKING, REASONING, AND SYSTEM BUILDING

IF logic constitutes a domain of actual system building there is naturally no place in it for data-transcending powers called thinking and reasoning. Moreover, the field or natural-science view of psychology disallows the assumption that there are any such powers altogether. In order then to indicate the place of thinking and reasoning in system-building enterprises we face two preliminary tasks. In the first place, we propose to show that although thinking and reasoning under the traditionally dualistic domination of logic and psychology have been treated as psychic processes, actually they comprise classes of interbehavioral operations upon particular stimulus objects. Secondly, we must indicate how unsatisfactory are the two processes—namely, (1) the separation of thinking from logic altogether, as is the case with the logical positivists,¹ and (2) the interpretation of all logical interbehavior as simply problem solving or inquiry.²

Crucial for the dissociation of logic from transcendent powers is the consideration that logical operations are not invariably connected with thinking and reasoning even as concrete actions. Beyond a doubt some logical enterprises may be independent of thinking or reasoning interbehavior altogether, since obviously among the large number of system-building operations there are many which do not extend beyond the manipulation and rearrangement of things to form a system, pattern, or order. Certain simple mathematical operations constitute nothing more than the organization of elements on the basis of an already accepted set of rules.

It may be argued, however, that in such instances the rules or the general evolution of mathematical systematization presuppose thinking or reasoning. There are two answers. First, the great

¹ Cf. Carnap, *Philosophy*, p. 34.

² Dewey, *Logic*.

stream of thinking and reasoning events which have occurred during the accumulative evolution of certain processes and products are not actual performances in the present field. Secondly, this argument reduces thinking and reasoning, which are immediately occurring events, to items of description or pure construction. We must not confuse the authentic discoveries of originating mathematicians with the manipulations of school children who go through the steps of geometrical demonstrations. The differences here must be respected even when comparing slight but genuine inferences, as in original school proofs, with the more profound development of fundamental techniques.

When we follow through each logical enterprise on its own account we find as a matter of fact that all *complex* system building necessarily involves thinking and reasoning. Indeed such enterprises also include many other processes, for example, various intellectual operations such as believing, doubting, hypothesizing, etc. Going still further, we find in the logical field what are conventionally regarded as the more humble perceiving and manipulating interbehaviors.

Our objection to the interpretation of logic as problem solving is two-fold. First, problem solving is either a term used indifferently for various kinds of events, or worse, these events are misinterpreted as all of one sort. From a naturalistic and specific standpoint the solving of one problem may not be at all like the solving of another. Howsoever far our description necessitates generalization and fixation, the actual procedures comprise important and unique details. Certainly genuine interbehaviors are not general procedures. Secondly, and even more seriously, the reduction of thinking and reasoning to generalized problem solving reveals a tendency to cleave to the undifferentiated crude data and to avoid the intricate analysis necessary to appreciate the character of the complex events involved. Despite the naturalistic suggestion that such activities are interbehaviors or actual tasks performed, such a reductive process is unacceptable.

In the present chapter, then, we attempt to analyze the salient facts concerning intellectual operations, thinking, problem solving, and reasoning, as a necessary step in indicating their relationship to system building.

THINKING AND REASONING AS SPECIFIC INTERBEHAVIORAL FIELDS

The fine balance that must be observed between construction and data is excellently illustrated in the investigation of thinking and reasoning. The obvious necessity to formalize and generalize the events we study carries the risk of confusing descriptions with original data.

More than likely this confusion is accompanied by another—namely, identifying products with processes. For example, propositions are regarded as forms of thought, whereas syllogisms are confused with reasoning. When propositions and syllogistic systems involve thought or reasoning they constitute products of thinking and reasoning activities respectively. To call propositions and syllogisms thought and reasoning is simply to formalize and conceal actual events and to reduce them to static structures.

A classical example of the translation of acts and operations into static things is the ubiquitous parallelism of concepts or ideas with terms, judgments with propositions, and reasoning with syllogisms. This procedure is not only owing to the dread of psychological processes, as we have seen in an earlier chapter, but is also the result of staticizing logical events themselves.

It may not be uninformative to trace back the staticizing tendency in logical writing to the historical connections between conventional logic and Greek geometry. To do so is to find a close parallel between concepts, ideas, terms, and geometric points. The judgment or proposition is derived from the figure, problem or theorem, while the syllogism, proof, or reasoning certainly spring from geometric demonstration. Whether or not the parallels are convincing they do suggest a basis for the identification of reason with logic even when one does not describe the latter as system building.

Another factor in this confusion is taking the position of the observer or describer and overlooking what the actual participant in the thinking and reasoning field does in interaction with the objects in the field. Inevitably ambiguities and transformations of the original events are slipped in.

Instead, then, of accepting traditional construction as the data of thinking and reasoning, we propose to consider actual operations, the numbers of which are legion and the specific charac-

teristics of which vary in significant detail. To further our enterprise, therefore, we shall mark off a series of these operational types. Thinking and reasoning will be separately considered and contrasted with problem solving and various intellectual operations which, we have noticed, are sometimes substituted for the former.

Above all, we cleave to the principle that we are investigating persons in action. Certainly there are many advantages in deriving our descriptions of the psychological and logical factors from the observation of persons performing logical work and producing logical products. For one thing, while we keep before us the fundamental psychological processes involved, at the same time we do not neglect coincident factors such as the presence of certain natural objects as well as the anthropological, sociological, economic, and other factors which play their part in every system-building enterprise.

Though thinking and reasoning events constitute specific interbehaviors our discussion need not be limited to particular acts. We may also take into account the statistical grouping of such interbehaviors in the sense of conventional or social thinking or reasoning. Judging and inferring acts occur as the expected manner of attack by school members upon certain stimulus objects. For instance, *a prioristic* logicians react deductively to inferential situations. Of all inductive logicians it can be predicted that they will proceed from particulars to generals or universals.

To describe thinking and reasoning as actual interbehavior not only dissociates such activities from the traditional psychic processes, but also from biological processes of restoration³ and new integration⁴ states.

INTELLECTUAL FACTORS IN LOGICAL SITUATIONS

Intellectual interbehavior consists of a large series of operations involving remote or substitutive contacts with things, and carries such conventional names as attitudes, beliefs, hypotheses, assumptions, etc. Such activities are performed in different ways and at different points in logical situations. In system building they occur primarily with respect to postulates, premises or criteria. In more

³ Rignano, *Psychology of Reasoning*, pp. 6, 11, 31.

⁴ Dewey, *Logic*, Chap. 2.

conventional logics the intellectual factors concern the nature and operations of logic. The four types of intellectual interbehavior mentioned above will serve to illustrate the relevant principles of all such psychological activities.

1. *Intellectual Attitudes.* It will not be amiss to describe attitudes as the activity of assuming a stance. When we proceed to lift a heavy stone we take a favorable position with respect to it, stand in a particular way, and do what may be described in everyday language as using oneself as an instrument or machine. Intellectual attitudes are similar forms of interbehavior with relevant differences derived from the nature of the field. Instead of immediate manipulation the action rather is one of noting the possibilities of manipulation or its advisability. In general, the individual develops a position with respect to the object in question. Essentially attitudes arise as results of prior contacts with objects which leave a residue of appreciation or evaluation concerning the importance, relevance, or significance of the things.

In logical situations intellectual attitudes may be illustrated by the general positions held. For example, is logic the study of the laws of thought or the laws of things? Or is it a general science of relations or order? Again, one may hold the attitude that logic is a general epistemology or science of knowledge, in the sense perhaps of warranted assertibility or the ultimate methods for scientific investigation.

Because of their generality, intellectual attitudes may also be illustrated by the question whether one views logic as constructive work or as an enterprise of becoming acquainted with order among Platonic entities. Such attitudes are interrelated with the question we considered in Chapter 4, whether logic is to be identified with mathematics or whether mathematics is identical with logic. Within the mathematical domain we note the formal, logistic, and intuitive or constructive attitudes.

2. *Believing Interbehavior.* Basically believing is characteristically more specific than attitudes, since the former are somewhat more tightly integrated with stimulus objects. Believing implies a readiness to do some particular thing with respect to some object or situation, or the readiness may merely attach to the acceptance of the belief itself. The primary feature of the actual interbehavior with the stimulus object is a condition of unsettled-

ness because of the paucity of evidence concerning its nature. Frequently the believing situation occurs in the partial presence and partial absence of objects and processes interbehaved with.

Logical literature is replete with beliefs concerning particular processes and symbols demonstrating the place and influence of believing activities in logical development. A pertinent illustration is the reactions of logicians to the introduction of the symbols $+$, $-$, \div , \times into logic. The discussion manifests the varying beliefs that these symbols are mathematical and not logical, that they are signs of operation which are indifferently mathematical or logical, or that they are both, because of the identity of mathematics and logic. Another example of such believing interactions is Jevons's view that Boole is concerned not with a system of logic at all, but a system of mathematics.

Boole's views concerning the relationship between logic and mathematics afford us further instances of logical beliefs. Venn countered Jevons' belief by his assertion that Boole's processes were logical.⁶ Harley, on the other hand, believes that Boole kept his logical and mathematical systems altogether distinct despite the employment of the same symbols in both.⁷

3. *Assuming Interbehavior.* Assumptions though closely related to attitudes differ from them primarily by being more closely interrelated with a particular task in hand. In other words, they have more to do with the manipulation of things during the course of system building, whether the materials are objects, sentences, or other items. In conventional logical work assumptions are concerned with axioms and postulates, especially in the sense that they are the most deliberately set-up statements or assumptions. Besides axioms, logical work comprises in addition assumptions which may or may not be deliberately espoused or definitely appreciated by the workers.

Examples of nondeliberate assumptions are that probability is one type of process or event, that logic must be intensional or extensional, etc. Another excellent illustration is the assumption by Russell in his *Principles of Mathematics* (p. vii) that implication is the fundamental process, which led to his further assertion that

⁶ *Principles*, p. 71.

⁷ *Symbolic Logic*, p. xvii.

⁸ Jevons, *Principles*, p. 155.

"pure mathematics is the class of all propositions of the form 'p implies q'." As he explains, the basis of his assumption, which he later gave up, was the observation that since in geometry both Euclidean and non-Euclidean systems must be included it could not be asserted that propositions are true because axioms are true, but only that axioms imply the propositions.

4. *Hypothesizing Interbehavior.* Logicians thoroughly committed to deductive principles hardly yield a prominent place to hypothesis making and using. Inductive logicians are more sympathetic while instrumental logicians grant hypothesizing a special and important function in their systems. In detail we must differentiate between the hypothetical statement of conventional logic and the hypothesis formation of the scientist. In point here is Dewey's contention that the assertion that logic is so-and-so can in the present state of logical theory be offered only as a hypothesis.*

In all system building, hypotheses play a tremendous role. At every step in a logician's work he is called upon to frame tentative plans and procedures for dealing with his materials both in direct and indirect connection with his system-building enterprise.

THINKING AS A LOGICAL OPERATION

Despite the widespread notion of the close connection between thinking and logic there is no uniform view concerning the nature of thinking. It is no rare incident to discover that eminent logicians still regard thinking as some sort of ethereal process going on in a private consciousness or mind unlocalized in space and time. More cautious writers avoid such palpably unscientific views by dealing with terms and propositions. Propositions, however, are regarded as expressed in sentences, so that it is clear that the word *proposition* simply does duty for the mental process formerly called judgment.

Studied as an objective event the most intimate personal thinking as well as the most elaborate problem solving in science constitutes specific interbehavior. No matter whether the objects with which the individual interbehaves are as concrete as the knocking of an automobile motor or as abstract as the nature of the infinite

* Logic, p. 3.

or the continuum, in each case we have in principle the same sort of interbehavior. In the so-called abstract situations the individual uses substitute stimuli, either things, events, words, diagrams or symbols, as his proximate manipulatory or other interbehavioral materials. The term thinking, therefore, refers to a series of operations upon objects on the basis of modes of action developed in one's own lifetime and within a framework of a particular culture. To stress this unique interbehavioral process with a definite origin in an individual's lifetime excludes any sort of vague power antedating contacts with objects.

Although it is not necessary for our present purpose to describe thinking activity in detail^o we should note that there are many particular thinking fields. For example, we can differentiate between planning, judging, analogizing, synthesizing, etc. In each instance we have definite contacts with things, whether natural or cultural objects, or constructions. The conventional contrast in logical writings between thoughts and things is absolutely extruded.

Whether thinking interbehavior occurs in logical or nonlogical situations it is subject to the same general principles. This applies to the problem of validity as to all others. Certainly we cannot say, as we have already pointed out, that nonlogical thinking is not, while logical thinking is, normative. That trait will and will not apply to specific instances of either type.

In specific instances nonlogical thinking may be much more valid than thinking accepted in logical situations. Undoubtedly there are inconsequential situations possessing little interrelation with other situations in which rigor and validity are not essential. Of course, when important issues are at stake, nonlogical criteria of solution and rigor of operation may be much more evident than in system-building procedures.

It is implied here that all thinking occurs under specific circumstances and that the character of those circumstances has a tremendous influence upon the operations. Though it is true that the individual is considerably swayed by his thinking habits and by various authorities these disturbing factors operate in all situations whether logical or nonlogical. Even in the most consequential of situations thinking individuals exhibit effects induced by the his-

^o Cf. Kantor, *Principles*, vol. II. chap. 21.

torical culture in which they are reared and the schools in which they have developed their modes of behavior.

Thinking and System Building. Adopting the hypotheses of specificity and system building, it follows that in any system-building enterprise the type and amount of judging, analogizing, planning, predicting, etc., depend upon the particular logical field. In simple fields, as we have indicated, there may be a minimum of thinking of any kind. The field factors more or less form themselves into a system, with a paucity of planning or analysis on the part of the system builder. The construction of the classic square of sentence opposition required no more than the simple manipulation of sentences on the basis of direct inspection of their forms.

In more complex situations many specific thinking activities must be performed in order to achieve the required system. The building of the system may itself be designated as a planning enterprise. In certain fields then the system building is part of the planning process, while in other cases it is larger than the thinking operations and may involve many of them. Not having any fixed and *a priori* specifications for system building we can discover the character of situations and the system-building constructional processes involved only after working with those situations.

PROBLEM SOLVING AND SYSTEM BUILDING

The identification of logic with problem solving calls for approval and tribute. The sponsors of this view have the commendable aim of eliminating the *a priori* from logic. More positively, they are interested in making logic a definite activity in actual situations. To identify reflective thought with objective inquiry certainly makes logic a naturalistic enterprise. Of special merit is the conception that the logicizing individual is in contact with a problematic situation, so that inquiry can be defined as the determination of an indeterminate situation.

This conception stems from the Darwin-James doctrine that mental and intellectual happenings are features of biological adaptations. Thus there is a suggestion here of the naturalistic view that reasoning constitutes a form of adjustment. Another source of this doctrine is the British empirical tradition which stresses scientific method as a basis for logic. In general, the in-

ductive theories of Herschel and Mill are emphasized as over against the rationalistic doctrines of classical logic.

Unfortunately the problem-solving conception becomes vitiated when its proponents adopt the classical notions of universality and generality. As a result, this type of logic like the classical systems allows descriptive construction to swallow up the events under investigation. Although this theory does stress the activities of individuals in logical enterprises it still places too great an emphasis upon the general description and not enough upon the actual processes. In the end, the construction is substituted for the crude data and the specificity of enterprise is lost sight of.

Is logic simply problem-solving activity? We can only answer in the affirmative if we espouse the epistemological doctrine that logic is the process for attaining knowledge. Our objection is not so much that logic is reduced to the development of knowledge as that various kinds of clearly different intellectual enterprises are grouped together as one kind. Such generalization violates the specificity principle which demands that we consider the actual operations of what individuals do in particular situations. In this sense the problem-solving hypothesis is self-contradictory, since, though it starts out with an emphasis on actual enterprises, in finally reducing them all to a single kind it turns out to be a type of totalitarianism, even though the cosmos is interpreted as a naturalistic field of action.

Specifically, this theory places too much emphasis upon use and purpose and not enough upon formal system building which the sponsors of this hypothesis regard as useless and otiose. It is in this connection that the opponents of humanistic logic criticize it as too personal and as verging too much on private psychological interbehavior. Moreover, all complex human activity might be interpreted as problem solving or adjusting; thus all basis for differentiating between logic and other activities is dissipated.

On the whole, the particular process of system building is neglected except possibly the unintentional development of knowledge systems. A definite implication of specificity logic is that there are many system-building enterprises in which problem solving is involved to the same degree as imagining, believing, thinking, etc. On the other hand, there are undoubtedly just as many situations in which system building constitutes a part of problem solv-

ing. We are, therefore, obliged to consider system building and problem solving as two distinctive forms of enterprises with various interrelations between them as already suggested.

REASONING AND LOGICAL OPERATIONS

With the rejection of the prevalent notion that reasoning is a general faculty or power for achieving infallible intellectual ends, it becomes our task to describe the specific form of interbehavior isolated under this name.

Reasoning as Inferential Interbehavior. Briefly, reasoning may be described as inferential interbehavior with objects, events, and relations in a particular field. The action may be performed directly or through symbolic or other sorts of intermediation, the result of which is the achievement of an attitude, concept or general orientation. This achieved attitude or knowledge adds some novel factor to the person's intellectual equipment whether or not it issues in further action.

While the inferential or transitional process is the center of all rational fields we must not overlook the specificity of the operations. For one thing, the actual stimulus objects involved bring greater or lesser differences into the particular interbehavioral events. There are inferences concerning natural objects or contrived things. There are inferences involving concrete things in contrast to propositional or mathematical equational systems. These different situations may be simple or complex; they may imply immediate actions or remote attitudes, and also involve questions of means or ends, bases, and consequences.

All inferential fields reflect the actual circumstances of the inferring individual. Just as the number of decimals to which a calculation is carried depends upon the particular problem, so the character of an inferential activity is contingent upon time, place, economic and other circumstances. Similarly, just as scientific situations demand greater exactitude or rigorous measurement as compared with practical affairs, so will inferences vary. Such detailed factors are overlooked only because reasoning is traditionally interpreted in logic as an absolute power of some sort. Certainly traditional descriptions of inference are based upon a complete severance of reasoning from sense or perceptual processes. In this way a description involving completeness and finality is substi-

tuted for the actual adjustments of individuals. In other words a product or completed system substitutes for inferential processes. The halting, tentative, and partial interbehavioral operations are in turn replaced with a fixed and rigorous system. In this sense neither mathematicians nor natural scientists allow the observer to see the gropings and back-tracking involved in their attainment of results. They show, instead, finished products. Only when we find moment by moment recordings such as the note books of Faraday for instance, are we permitted to see the actual procedures. This unavailability of reasoning steps, however, implies no notion of absoluteness.

Reasoning as Ratiocination. Some among those who would freely eschew the doctrine that reasoning is a formal power or faculty still do not favor the inferential description. Though they accept the general notion of concrete action they regard it as ratiocination or rational discourse.¹⁰ By this they mean examination of an idea or suggestion as it is related to other ideas in a system. Such examination as actual interbehavior is a psychologically impeccable activity and moreover is certainly an essential factor in system building, but it is not reasoning, if we reserve that term for a distinctive form of interbehavior.

Reasoning and Thinking. When we explore the differences between inference and thinking operations we discover that the former is more abstractive than thinking activity, primarily because inferring as over against planning, judging, etc., stresses the attitude or position attained rather than the manipulation of a particular object.

Again, the results accruing from the performance of reasoning behavior are relatively more remote from the original stimulating objects or situations as compared with thinking or problem solving. While the most elementary forms of inference in the guise of simple prediction are intimately rooted in the properties of things and their relations, the more complex responses, especially those charged with cultural materials, attain the expansiveness of speculation. By contrast, when planning, judging, etc., the individual remains in closer contact with the original objects and conditions constituting his stimulus objects.

On the whole, too, thinking and problem solving are much

¹⁰ Cf. Dewey, *Logic*, p. 111.

more complicated interbehaviors than inferential action. A concrete problem-solving situation may involve many particular instances of inferring events. Thus reasoning interbehavior to be properly understood must be studied as it occurs by itself and again as a factor in intricate human enterprises. In interrelation with thinking and other intellectual activities inferring constitutes complex human situations. It is on the basis of this interrelationship that reasoning has erroneously been taken to be a general process of obtaining large-scale results in complex activities and other fields of action.

Inference and Discovery. Though inferential interbehavior results in the development of a new intellectual attitude or a new orientation or knowledge with respect to things or situations, only indirectly does it constitute the discovery of new events. The primary basis for this circumstance is that inferring interbehavior is for the most part implicit and hence more or less remote from the actual stimulus objects concerned.

To make discoveries, on the other hand, one must actually interbehave with the new objects, in other words, build up definite knowledge reaction systems in connection with them. True enough, inferential attitudes like assuming, guessing, and others may lead to the development of contacts with objects meriting the adjective discovery. In such instances it is necessary to apply and test one's inferences, an interaction we may describe as verifying if successful, or falsifying if the inference proves unsatisfactory.

Invariably, however, this potentiality of discovery is coupled with certain hazards. The very fact that inferential interbehavior is concerned with specific and detailed events means it may well miscarry. Such a flexible view of inference obviously is a decided departure from that of the syllogizing logician who regards inference as general, rigid, and necessary, but at the same time in its deductive form an agency of discovery.

The first, of the several misconceptions underlying the belief that deductive reasoning is productive of new knowledge is that it is taken to be synonymous with genuine inference. Inferential interbehavior may certainly lead to new knowledge, but such interbehavior is not to be confused with a syllogistic statement of the inferential activity. What we have in the syllogistic or deductive system is at best an organization or interrelationship of facts which

later permits one to move from one point of the system to another. When we declare that since all Frenchmen are Europeans this Frenchman is a European, we may certainly deny any discovery, even if we admit an inference.

At most, then, syllogistic inferences consist of the isolation of an instance from a class. We set up a law of generalization, after which we discover some slight knowledge concerning an instance because it is an instance of the system. Logicians insist that we may infer the new fact that the Martins' only child is a girl because she is in Devon College which is known to be a woman's institution. At most this argument signifies that generalizations can be made and are useful, albeit this hardly touches the problem of discovery in inferential situations. Important here is the observation that all actual knowledge is achieved by turning away from sentences or descriptions to actual contacts with things.

The problem of inferential discovery is focalized in the assertion that deduction is the invariable goal of science. This argument arises from the historical employment of geometry in building up a system in rational mechanics. By reducing mechanical events to geometric elements in relation, deduction became a useful procedure in this particular type of situation. When physical events form a closed system it is a decided advantage to be able to move readily from one point to another in the system. But when we are concerned with other types of physical events and with chemical, biological, and psychological happenings, no new discoveries can be made unless we first develop such systems.

When considerable knowledge of particular sorts of events is achieved we can organize it as a basis for genuine inferences. The discovery of Neptune through a knowledge of the interrelation of planets, the interrelation of the periodic law of elements with Moseley's law, and other such instances are classic examples. Only when we can justifiably employ processes of interpolation and extrapolation in addition to a general analogical situation can we speak of inferential discoveries. Here we are reminded of the accidents of human interbehavior in which false hypotheses and assumptions sometimes lead to valuable discoveries, but we can no more use this as a principle of discovery than celebrate the process of making several mistakes because it sometimes leads to a correct result.

We must therefore distinguish between actual interbehavioral processes and general impersonal or public systems that are products of such particular interbehaviors. It is a valid conception that both personal inferences and general systematic reasoning are grounded in natural events, but this fact favors rather than opposes the basic principle of interbehavior. What is called for is the elimination of the idea of generalized psychic or even psychological powers for accomplishing scientific and logical results.

Inference and Implication. Logicians guided by the notion of universality, certainty, and rigor have inevitably turned from inference to implication. Basically, this shift constitutes an acknowledgement of the tentative and specific character of inferential results. Implication, it is believed, is more objective and certain. Cohen and Nagel write: "It is essential to distinguish *inference* which is a temporal process from *implication* which is an objective relation between propositions."¹¹ Note the suggestion that objective relations are contrasted with temporality, although the contrast is really between (1) a static and established independent existence and (2) an ongoing process. When objective relations involve processes they comprise acts of discovery or selection. Certainly the behavior of selecting, discovering, and describing connections between things or propositions is different from the connections, but such connections between propositions neither constitute objective and independent existences nor imply universality and certainty.

Whenever propositions are in question it is obvious that operational processes are involved, for propositions can never be anything but behavioral products.¹² To be sure, they may be based upon observation of objective relations of things independent of observers. But in that case we cannot lose sight of the fact that whatever logical system we erect upon this basis involves operations upon the data, including examining and describing them and then organizing the relations between the propositions. Naturally it is legitimate to confine logical procedures to operations upon propositions. This is merely a permissible choice of materials with which to work.

¹¹ Introduction, p. 14.

¹² Whatever is true for propositions will be true in greater measure of sentences referring to or exhibiting propositions.

The operations involved in interacting with propositions emerge immediately when the question of significance arises. What kind of propositions are before us and what will be the significance of systematizing them? That the proponents of objective implication have had to struggle with this issue is manifest in the heart searching which results in the distinction between formal, material, and necessary implication.

Another item. Though striving for extreme formality and generality the symbolic logician cannot dispense with criteria such as truth and falsity which he uses for material implication. Defining "p implies q" as "p true and q false is in fact false," it follows that such propositions as "if anyone is just he is happy" and "if anyone is just he is unhappy" are both true when the hypothetical antecedents are false. The argument is that if there are no just men no one is just, whether happy or unhappy. It is in line with the operational principle to make plain that the present criterion of truth and falsity is merely one of conventional opposites. Similarly, the two paradoxical propositions which characterize material implication, viz., "a false proposition implies any proposition" and "a true proposition is implied by any proposition" bespeak a background of convention. All this means nothing less than going beyond the forms (verbal construction) and connections of propositions.

Eaton's¹⁸ intensional or meaning implication constitutes a search for definite content or, as we prefer to say, definite criteria for organizing propositions. Essentially this writer is searching for some sort of significant basis for interrelating propositions. We may add that the full force of implication comes out when we move away from propositions to actual objects and conditions interbehaved with.

The stimuli of implicational interbehavior are complexes of things connected with one another. Once the connection is discovered or set up one may proceed from the implicit to the implied. When many things are thus interrelated we may regard implication as activity affording rigid and certain attitudes. In no wise, however, do we avoid the particular and specific character of the objects and behavior involved.

Reasoning whether regarded as inferential or implicational can

¹⁸ General Logic, pp. 226ff.

never be considered as a labor-saving device to avoid the original manipulative and direct contact with things which is necessary for developing valid knowledge concerning them. Only when we have prior organizations of knowledge or propositions can we deal with actual things by this remote and roundabout procedure. There is a vast difference between the inferential and reasoning process when dealing with natural events and when dealing with systems of relationships, the fundamental basis of which has been previously established.

Interesting here is the distinction Dewey¹⁴ makes between inference as concerned with *involvement* as over against discourse which has to do with ascertainment of rigorous and productive implication. The point is that inference is concerned with "*what* conditions are involved with one another and *how* they are involved." This distinction is appealing in that it stresses the difference between actions with things and operations upon ascertained and stated relations. We must insist, however, that both are legitimate features of system building.

The Efficacy of Reasoning. Since reasoning or inferential interactions constitute specific fields and are subject to the risks of remote contacts with things, it is proper to ask what makes for effective reasoning. The question can be formulated as follows: Who can make effective inferences and on what basis?

First and foremost, good inference depends upon adequate acquaintance with inferential materials. To achieve satisfactory inferences in law, medicine, mathematics, or business one must develop necessary experience in those particular fields. This means a sufficient number of manipulations and observations in each domain to provide bases for efficacious inference.

Is it necessary to add that this situation precludes the conception of a force or power which is able to carry us from one situation to another. Anyone who has knowledge or a worthwhile opinion concerning events not directly observed—in other words, anyone who has any form of *a priori* knowledge, has only *a posteriori* knowledge. Moreover, the value of this inferential behavior or *a priori* knowledge depends directly upon the similarity of the situations inferred from and the situations inferred to.

In the second place, effective inference depends upon the charac-

¹⁴ Logic, p. 278.

ter of the inferential materials themselves. The certainty and reliability of the inferences follow from the stability of the objects reacted to. In mathematical situations there are advantages which are lacking in more dynamic and eventual fields. Note, for example, the effectiveness and scope of mathematical inference. For one thing, the mathematician dealing with abstracted relations is not hampered by problems of similarity and analogy, at least as long as he deals with a single system. Moreover, he confines himself to a postulate set which limits his inferential range and therefore produces effective results. The limit is the inference of complete or mathematical induction which turns out to be no genuine induction or inference at all.

The mathematical situation may be compared to the conditions confronted by the lawyer or physician. The uncertainty, flux, and complexity of events give scope for more authentic inferences, but at the same time make inference more difficult and challenge reasoners to make their inferences useful.

Variations in efficiency of reasoning inevitably bring us back to concrete interbehavioral situations and force us to distinguish all sorts of inferential events. In some instances there is a fixity and system character which become generally acknowledged and accepted, and many times conceal the underlying intimate psychological performances.

The Criteria of Rationality. Logicians who regard reasoning as fixed and formal techniques are not troubled by any problem concerning the qualities or grades of reasoning. On the other hand, when reasoning is treated as specific interbehavioral fields it is important to ask whether the latter are critical or uncritical. This brings to the surface the criteria of what is rational.

Whether or not one adopts the conception of an interbehavioral field it must be obvious that reasoning is relativistic. We reach this conclusion, perhaps, from the observation that even those who agree that reasoning consists of drawing necessary conclusions from premises do not agree upon the premises. How then are we to derive a criterion for rationality? A frequent answer is that what is rational depends upon the accepted conventions.¹⁵ Basic to such conventions is the principle of coherence, consistency or system,

¹⁵ Cf. Lesser, *Superstition*; Kantor, *Logic and Superstition*; Bush, *Superstition and Logic*.

which accounts for the tremendous place given to the principle of noncontradiction in every type of reasoning situation. On this ground, a rational action or product is one, then, that does no violence to the accepted premises. In other words, it fits in with a general scheme which is accepted because it prevails, as we saw in Chapter 6, when considering definitions of logic. For instance, in a cultural system in which demons are not accepted, sentences involving demons at once are labeled illogical and superstitious. On the other hand, in a situation in which demons are parts of the cultural system such statements are logical or rational. The criterion, therefore, is really adhesion or coherence. Such a conclusion appears binding in the absence of any sort of absolute criteria.

Still, we are left with the unsatisfactory result that no line of demarcation between the rational and irrational is possible. Nothing is more obvious than the fact that this view runs counter to both our everyday or abstract logical situations. What validity or significance then can we ascribe to the rational? Obviously we require a new basis for rational criteria.

The specificity principle offers itself. There is no such thing, of course, as general rationality. Only by an examination of particular circumstances, can we determine what is rational. If we are interested, for instance, in the rationality of demons we ask in what kind of situation they function. If demon materials are proposed to explain the curing of disease or the growing of plants then we have a very solid criterion in the interbehavior with things. Here clearly all reasoning not meeting the criterion is irrational no matter how inevitable from the standpoint of the cultural system. If, however, these demons are included in a mythological or ceremonial system they find their proper place and are rational components of the system. The problem, as our illustration shows, is simply to be familiar with different cultural systems and their various phases.

Similarly, we may distinguish between rationality in abstract systems as over against more concrete situations. What may be rational for a mathematical system need not be so for a natural-science system. In the abstract system sheer coherence and consistency are definite and sufficient criteria. In the natural-science situation we require, on the other hand, that our inferences be intimately integrated with natural events.

The assumption that criteria of rationality are unavailable or that such criteria must be abstract and invariable arises from a neglect of concrete facts. In other words, emphasis is placed upon the acts of the reasoner in isolation from the stimulus objects in the fields. This neglect, in part at least, may be attributed to the convention that in discussing reasoning or logical events in general we must deal with universals. In abstract logic the specific interbehaviors are masked by the necessity to deal with descriptions or propositions. Hence the inevitable trend toward formalized criteria which results in a departure from specific situations. To break through these barriers of propositions or formal descriptions provides us with ample and adequate criteria of rationality.

Rationalization and the Rational. How valuable it is to consider specific situations in determining criteria of rationality may be tested by invoking that sharpest of contrasts, namely, the authentic rational versus rationalization. Whoever makes this distinction must assume that there are definite criteria of the rational. The rationalizer, it is assumed, arbitrarily sets up criteria, supported by arguments and constructions which presumably are not intimately related with reasoning situations. The examination of specific fields in which we observe the rationalizer interbehaving with things yields a basis for determining whether or not his reasoning is adequate or faulty.

Interbehavioral versus Classical Inference. Universalistic logic has been limited to two types of inference, inductive and deductive. Not infrequently the former has been regarded as a type of the latter. Taken as an independent type of inferential process induction has been looked down upon as inferior and yielding only probability, not certainty. In rare cases classical logic has grudgingly allowed a type of inference from particular to particular, that is, analogical reasoning.

Interbehavioral psychology with its interests in the activities of individuals in particular situations not only adopts a different view concerning the general nature of inference, but also finds a larger number of authentic types of inferential interbehavior. From an interbehavioral standpoint any form of intellectual transition from one situation to another, yielding a new attitude or knowledge concerning things, constitutes inference. On this basis it is possible to isolate no less than five major forms of inferential interbehavior.

(1) *Deductive or Specializing Inference.* Essentially this is a process of achieving a result on the basis of a rule or general principle. The individual, guided by knowledge of some principle previously attained, reaches a conclusion. A may justifiably be assumed to have certain specialized knowledge because of having received an academic degree or having been an apprentice to a certain master, for instance. The general rule, which, of course, may or may not itself be justified, is that on the whole such a course of training or such an apprenticeship may be expected to result in mastership.

This notion of deductive inference certainly resembles that adopted by the classical logician. The differences, however, are great. The former is concerned with actual development of knowledge or attitudes through contact with things. The logician, on the other hand, has been interested in sentential systems. There is no doubt that if we go far enough back in the classical logical tradition, say, to Aristotle, the disparity between the two descriptions would not be so striking. Only after medieval verbal logic became established did deductive inference become simply an integrated system presumed to yield certainty of conclusion.

The more modern proponents of classical logic argue that the major premise, for example, is really a reference to some actual fact. Moreover, in discussion they illustrate their principles by considering specific events. In order, however, to attain the certitude so highly desired, deductive inference does become simply sets of propositions. As Joseph¹⁰ puts it, syllogistic reasoning consists of an argument in which a relation necessarily follows in the way of subject and predicate from the given relation of two terms in the way of subject and predicate to the same third term.

(2) *Inductive or Generalizing Inference.* The basic principle here is the achievement of an inferential result from an observation of similarities between things, since one concludes that further experience will yield other members of the same type. Inductive inference, then, is the discovery of a continuum on the basis of common factors. Observing the oncoming of one or more cars on a road one may infer that the road is open and there is a possibility that more cars will come from the same direction. Classical logic describes such inference as developing a general principle on the basis

¹⁰ Introduction, p. 248.

of particular instances. In other terms it has been asserted that inductive inference leads to the development of a general principle or class. This type of description possesses considerable merit, but only if one considers the background. This background is presumed to include the scientific field in which the observation of specific instances leads to a general principle in scientific work. The fault with this description is that it limits inductive inference to certain situations, and also it assumes that inference is exclusively an activity which leads to the building of formal systems.

(3) *Conductive or Analogistic Inference.* Because of the prevailing logical tradition that all rational processes must involve universal principles analogical or conductive inference has not been regarded as proper inference. From an interbehavioral standpoint, however, the development of an attitude concerning some object on the basis of a similarity to some other object is certainly an observed fact. When some thing or event shares characteristic A with another event, it is a frequent inference that the other object will also contain B in common with the first object. Only the universalizing tendency would lead us to deny the fact that many of our inferences, and these not the least useful, constitute inferences from one thing to another.

It is not, of course, legitimate to deny the occurrence or value of certain inferential activities because they do not fit in with particular preconceptions concerning logic and its inferences. Thus possibly the study of inferential behavior may itself add to our understanding of the nature of logic. Certainly the logician who regards the trivial *all men are mortal* as the ultraperfect example of certain inference needs to develop sympathy for the concrete inference that Dionysius of Syracuse intended to become a tyrant because he asked for a bodyguard, on the basis that Peisistratus at Athens made himself a tyrant when he obtained a bodyguard.

(4) *Constructive or Evaluative Inference.* To consider that some object possesses a unique character or merits the attribution of a distinct value illustrates a special inferential interbehavior. Evaluative inference is thus characterized by interaction with a single object. That such inferences are of logical importance mathematical history testifies. Whoever first inferred that a mathematical point could be regarded as a number triplet made such an inference. Similarly, it was a constructive form of inference which

resulted in the inclusion of irrational and imaginary numbers in the enlarged number system.

Whether or not we agree with the conclusion, the inference from the behavior of a delinquent boy that he will be an habitual criminal constitutes a genuine constructive inference. Predicative inference probably consists of a subclass of such constructive reasoning.

(5) *Reductive or Eliminative Inference.* Inferential operations upon things sometimes result in the conclusion that some characteristics believed to pertain to them are eliminated without any substitutes. The reasoner infers what a thing is not and in consequence finds himself in a sort of suspensive or doubting attitude. When a mathematician is unable to write an equation to cover an observed event he immediately infers that the data are incorrect or insufficient. In other cases inferences leave no doubt but rather the positive conclusion that something is lacking. When two judges disagree, the observer may infer positively that one lacks experience or acumen to operate properly in the situation concerned. On the whole, reductive inferences occur in situations in which the objects, conditions or evidence concerning them are inconclusive or indeterminate. Their importance is measured by the fact that such reasoning interbehavior brings us into close touch with actual events and behavioral situations, in contrast to descriptive generalizations which dissipate negative characteristics or absent properties of things that play essential roles in our actual rational adjustments.

REASONING AND SYSTEM BUILDING

Throughout logical history reasoning has been intimately interrelated with system-building procedures, albeit no entirely satisfactory description of the relationship has been available. For the most part, reasoning has been regarded as primarily an operation concerned with deductive or demonstrative and inductive or dialectic systems. More recently reasoning has been closely connected with the construction of hypothetico-deductive systems or doctrinal functions.¹⁷ Though we may agree that there are reasoning processes in every formal enterprise of logic, we disagree with the opinion that the inferential behavior actually present in deductive and

¹⁷ Cf. Carnap, *Logic*, chap. 11.

inductive situations covers all inferential activities. Nor can we agree that there are no system-building enterprises without inferential or reasoning interbehavior.

These observations are intended to make manifest the large place of inferential behavior in all complex system building. We repeat: All intricate logical work constitutes elaborate performances in which reasoning plays a distinctive and necessary role.

When, however, reasoning interbehavior belongs to a system-building field, it must be specific for the particular situation and conditions involved. In no sense are the reasoning or inferential processes generalized performances. In order to discover the place of reasoning among the other types of interbehavior and tools employed in carrying on system-building operations, the particular system-building situations must be analyzed. On the whole, the character and amount of the inferences performed depend upon the subject matter or materials being systematized.

Implied in our remarks is the warning not to begin our logical descriptions with a system as a finished product without regard to the actual operations by which the system is constructed. The striking case in which the latter procedure occurs is, of course, that in which the system consists of sentences or propositions in some sort of interrelationship. To begin with such finished products and then to look for implications which are later confused with inferences is to ignore that those implications are there only because of the way the system is constructed. The emphasis, then, must be placed upon the work of construction. Granted that such systems and the type of logic which they imply are valid and useful they still offer no warrant for misinterpreting the nature of reasoning and its place in system building.

CHAPTER VIII
LANGUAGE AND SYMBOLS IN LOGICAL
OPERATIONS

LANGUAGE AND LOGIC

LINGUISTIC events have always occupied a strategic position in logic. Aside from the obvious necessity for discursive and descriptive aids in any intellectual enterprise, logic has always been closely integrated with linguistics or grammar. This is the case not only in the classical systems based upon sentences, but also in the modern occupation of the logician with signs and symbols, as well as when logic is concerned with the implied relations between propositions. It has been aptly said that "it is not always easy to draw a sharp line between the grammatical and the logical writings of philosophers like Aristotle, Duns Scotus, and C. S. Peirce." How unsalutary has been the close affiliation of language and logic may be estimated by the resulting misinterpretation of both the related members.

In what sense is logic simply language? Actually, no logician who concerns himself with proper and improper word usage would identify himself with a lexicologist. Nor would he forego his more intense interest in the "content" of such words. Similarly, no logician concerned with the order and arrangement of sentences fails to go beyond these to what they assert. Whether one takes logic to be the method of science or the study of the laws of thought, both when thought is autonomous or when it is thought about things, there is certainly more to logic than mere language, no matter how necessary and useful the latter is in logical work. Again, can logic as system building be confined to language material, whether language is equated with symbols, statements or systems of propositions? Despite contrary appearances, it is fair to say that in ordinary logical work language operates as referential, discursive, or symbolic techniques rather than as material or subject matter.

As to language, since in logical work the descriptive and discursive functions are not highly valued, it is almost universally

¹ Cohen and Nagel, Introduction, p. 17.

reduced to symbols, presumably standing for either thoughts or things. Surely we have here an arbitrary transformation of linguistic events. For the purpose of assimilating language to logic grammar is made into a simple technique for manipulating symbols. Such manipulations of symbol things are confused with knowing, thinking, and reasoning. The peak of such procedures is attained when logicians transform language into mathematical symbols and propose to solve logical problems simply by operating correctly with the right language.

Studying actual linguistic events we immediately discover that language is not a process of manipulating symbol things. Nor is grammar concerned with intrinsic arrangement and rearrangements of symbolic elements which reveal profound truths and falsehoods.² Moreover, an unbiased study indicates that mathematics is a language only in the metaphorical sense which transforms architecture into frozen music, and music into speech. Neither is mathematics a set of statements which precede and follow each other irrespective of such specific facts as formal relations or natural events, periodicities, etc.

Though the identification of language with logic is hardly serviceable to either discipline, there has recently been a powerful move to reduce logic to syntax and semantics, as we have already seen.³ Behind this move is the creditable intention to improve logic; nor is it to be denied that some valuable results have been achieved. There is great force in the injunction to mend our language or it will mar our logic. Language here means, of course, not style but symbols or substitutes for proper reference. Unfortunately, the result has been so to formalize language and symbols as to make them completely empty, even so far as to disconnect them from the operations of the logicians who use them.

The fact that linguistic events are so thoroughly interrelated with all other human operations calls for an adequate analysis of the former in order to establish the nature of the interrelationship. The analysis for the most part is designed to answer the following questions. Are the symbols which comprise language formal and autonomous or are they interrelated with things? Among logicians there are those who regard the language of logic as sheer formal

² Cf. Kantor, *Objective Psychology*.

³ Chap. 4.

units to be arranged according to fixed laws, but there are others who insist that propositions and formulae go back to things. Unfortunately, some of the latter entertain the unsatisfactory notion that language is identical with the events to which it refers. Thus true propositions are asserted to be identical with facts.⁴ Doubtless such notions are connected with mentalistic psychology. Whoever identifies language with things must do so on the following bases: (1) events and notions of events are directly identifiable; (2) events and notions of events are indirectly identifiable because propositions, whether or not envisaged as *mental*, must be identical with events, since propositions are not the sentences which constitute only the means of referring to or *expressing* propositions.⁵ The difficulty following in the train of such identification suggests the necessity for studying language and symbols and their connection with system building, whether these systems are constituted of things, propositions or relations.

THE NATURE OF LINGUISTIC EVENTS

Whenever we deal with linguistic items, actions or things, we are inevitably concerned with interbehavioral events. When we study language events, speech, discursive acts, etc., we observe interbehavioral activities, whereas when we start with linguistic things we are concerned with products of interbehavior.⁶ Because this obvious fact is overlooked in practice it is expedient to point out the salient characteristics of different linguistic events and their interrelationships.

Linguistic activities and linguistic things. Probably because of the interbehavioral fact that linguistic acts and products are both indifferently subject matter for the student of language they are not sufficiently differentiated. Some students of language never concern themselves with actual linguistic interbehavior. Instead they are always occupied with things.

(A) *Referential linguistic activity.* Linguistic behavior may be divided into two general types: (1) referential language or speech

⁴ Ducasse, Propositions.

⁵ Keeton, On Defining; and Ducasse, Is a fact.

⁶ Linguistic things may be, of course, products of single persons, for example, sentences written down by an individual, or the combined activities of persons, the original writer and later editors or scribes.

exemplified by interpersonal conversation or intercommunicative performances, and (2) nonreferential types, illustrated by sign making and using. The essential process in referential interbehavior is referring and being referred to things. These events may be described on the basis of particular criteria.⁷

A says to B, "Your tire is flat." Here the flat tire is one of two stimulus objects in which inheres the adjustment stimulus function. This is so named because presumably the speech behavior operates primarily to refer to this event which we call the referent. B is the second stimulus object and in him inheres the auxiliary stimulus function. This function is called auxiliary because in many cases the person is acting primarily with respect to the referent stimulus object and only secondarily to the person spoken to.

It is of the utmost importance to distinguish between the various features of this complex field we are describing. We have here two salient interbehavioral activities. Because of the particular circumstances of our illustration we may designate A's action as referor behavior. Similarly, the interbehavioral performance of B may for convenience be designated as referee language, audient interbehavior if the action of the referor is articulated speech, or visual interbehavior if A's behavior is gestural. There are various styles of performance on the part of A, which, of course, correspond to different activities of B. In addition to the referential performances of referor and referee the field contains the activities of the objects or events referred to. These may be of any sort, actually existing or imagined, a thing, a relation, a language act, a symbol, etc.

What are called grammatical factors of speech constitute stylistic organizations of referor behavior, or point to its conventional or proper arrangement.

In such situations no one will confuse the referor and referee activities—that is, the reference and hearing acts with the adjustment stimulus things or objects. Note, however, that A can intercommunicate with B by transcribing his vocal behavior (writing out his reference) or by making various signs or symbols. In the latter case the product which appears necessary for the intercommunication is often confused with a genuine language interbehavior, and when the referent is nonexistent the language factor is confused with the referent. Among the most serious confusions

⁷ Cf. Kantor, *Principles*, chap. 23.

is identification of intercommunicative behavior with activities and products of record making and symbolizing things.

(B) *Nonreferential language activity.* Besides the referential forms of language interbehavior there are many sorts of activities of which naming, counting, recording, and symbolizing things are prominent examples.

(a) *Naming.* From a psychological standpoint naming interbehavior constitutes fundamentally an activity of associating a vocal reaction with some stimulus object. The effect is similar to that of placing a mark upon some object different from that put upon other objects. Such actions come within the general range of language simply because of the convention that all verbo-vocal action is called language.

To clarify the essential character of naming behavior we may contrast it with genuine referential language. Suppose that the namer actually converses with the named by saying, "I shall call you X," or "What objection would you have if I called you X?" etc. These are authentic referential reactions involving a definite bistimulational situation as might be the case in adult baptism.

The differences between such referential action and naming are striking when we observe that in naming things our interaction involves a single stimulus function, though this may inhere in two stimulus objects. To apply a tag to a thing whether vocally or manually is no different as a psychological interaction from putting one block upon another.

Furthermore, as a general adjustment naming operates as a simple identifying action no matter in how many different situations. Referential language, as we have seen, is of at least two distinct sorts—referor and referee language, each with a wide scope of adjustment. In general, too, there is greater probability that referential language will be connected with further action of all types than is the case with the relatively more restricted naming action.

(b) *Counting.* As a type of nonreferential language behavior counting differs from naming in that it involves an ordering process. Whereas naming is merely arbitrary attachment of identifying marks, counting is relational and results in some kind of arrangement or order. This counting behavior may proceed in two ways. In the first, the reactor combines elements by the one step of cumulative addition of simple units or by adding previously

related units as in counting by twos, fives, tens, and so forth. In the second, the counting or ordering is performed by asserting or declaring a correspondence between single units of differing things or groups of such varying units.

Though counting in ordinary practice when verbally performed in the manner specified may be classified as language (nonreferential) we differentiate between this form of counting action and two other kinds, on the basis of the different circumstances constituting the interbehavioral field or situation. First, we must mark off such counting behavior from other ordering and relating actions which comprise sheer manipulations of the objects or units concerned. The only similarity between these two consists of the ordering results obtained. In the latter and nonlanguage behavior verbal action is either not present at all or of no special significance. Next we consider ordering and counting behavior which is genuinely referential. As in the case of referential naming, so here the officer may perform genuine linguistic action, for instance, while ordering and counting his subordinates, the latter serving as auxiliary stimulus objects. When this is the case the genuine referential action must be sharply sundered from the nonreferential.

(c) *Recording*. The essential feature of recording is the registration or representation of objects or events. This action can be performed without any conventional linguistic responses, as when one merely sets down the existence of some thing, the occurrence of an event or a relationship between two or more objects. For instance, one records or notes that a certain number of trees grows in a particular forest, that the population of a town has increased or decreased by a certain number. The fact that such elementary forms of recording need hardly be listed in our series of nonreferential actions is of interest in suggesting that more complicated recording types achieved through complex graphic performances and by means of numerical or alphabetical marks are also to be distinguished from referential behavior.

(d) *Sign and Symbol Interbehavior*. In our discussion of symbolinguistic logic in Chapter 3, we spoke of the necessity to interbehave with objects and relations through the intermediation of substitute objects. Whenever stimulus objects are not immediately present in space and time we need to employ substitute stimuli. Substitute interbehavior takes innumerable forms. The emphasis may be upon: (a) the adjustment of the individual, (b) the present

object which operates as a substitute by its very presence, or (c) the absent object which requires to be substituted for. Another important differentiation concerns the question whether the interbehavior is casual and natural or deliberate and conventional. On the whole, sign substitutes may be connected with the former, symbolic substitutes with the latter, which also are more concerned with the construction of conventional logic.

Symbol behavior may be simple or complex. In the former case the person interacts with a couple, triad, etc. Thus when the person responds to the equation $x^2 + y^2 = z^2$, the various signs or symbols constitute a single psychological object, possess a single stimulus function. In complex symbol behavior the individual interacts within a symbolic situation, as when the substitute π stands for the number 3.1416. Here the response to the symbol leads to an interbehavior with the significant or symbolized.

Intellectual circumstances have resulted in the fact that we call sign and symbolic behavior linguistic, even though the common name obscures certain decided operational differences. Possibly we may account for this illegitimate identification because of the development of symbolic from referential interbehavior. Consider this development for a moment. In purely referential interbehavior the individual always gesticulates or verbalizes on the basis of a behavior development conditioned by specific cultural surroundings. In other words, how the person behaves depends upon certain conventionalities of communal life. These cultural auspices give form and pattern to his activity, which occurs because of a desire to speak of something or the need to have something done. Notice that these activities can be hypostatized through the evolution of writing. They can be translated into marks serving as substitute stimuli for the referee who must now visually interbehave with what he ordinarily would hear. The speaker himself can likewise translate his speech into writing, which may remain for him a record of what he previously did. It is this reification of the original referential behavior which has given rise to the conception that language consists of signs or symbols for things or for ideas. As we shall see, this identification of two altogether different types of operations and the consequent transformation of symbol interbehavior into the simple existence of symbol objects provide the source of many symbol-linguistic difficulties.

(C) *Linguistic things.* Typical linguistic things may be located

in definite referential interbehavioral fields. Specific instances are the referents or objects referred to or designated, and, we might add, the persons constituting the auxiliary stimulus objects. On the other hand, if we abide by the conventions we may also include scripts, printed texts, and even symbols standing alone.

(a) *Referential language things*. Quite as intimate as referents in referential interbehavior are the tools employed in performing it. Besides the objects used in communication such as writing and signalling instruments there are the actual scripts produced for purposes of distance communication.

(b) *Remote language things*. Remote language things comprise those objects which at one time were intimately concerned in actual referential fields, as well as things bearing some analogous relation to such objects. Among the best examples of this type of thing language are those contents of texts or monuments which like the Mesogothic Bible can be read but not spoken, or like the Etruscan or Cretan inscriptions which cannot even be deciphered. Such items of language are connected with linguistic behavior or speech inasmuch as they are transcriptions or reifications of the activities of referring to things. Besides these forms of language there are all sorts of symbols, inscriptions, marks, etc., which constitute substitute stimuli for the reader's reactions. A typical example is the Hic Jacet of the conventional tombstone or simply the name, family, relationship, and birth and death dates of the person interred.

(c) *Nonreferential language things*. Doubtless the most prominent type of nonreferential language thing is the set of marks, signs, and symbols used in calculative, recording, and marking enterprises. As we have already intimated such objects are intimately related to the other things manipulated in various situations. These things have become exceedingly significant in the numerous symbolic logics which have recently been so assiduously developed.

(d) *Nonlinguistic things*. Of the greatest prominence in current logical writings are the various marks, characters, and symbols called signs. In a genuine sense signs have come to absorb the whole field of logic and even everything that logic might be concerned with, whether abstract relations or related things. This process of absorption may be traced back to the close interrelation that modern logic has cultivated with mathematics. Since in mathematics symbols or signs are easily equated with or substituted for

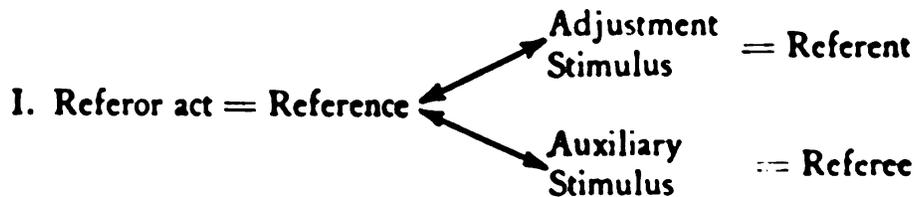
what they represent, the mathematician can confine himself to signs and forget their significant. Then, too, we have the popular view that mathematics is a language, and so formulae can absorb the things and events formulated.

But signs are endowed with an even greater potency when they are given causal efficiency, as in describing perceptual events. Things the individual interacts with are called signs which cause the reaction; thus, on the one hand, things become signs, and on the other, the describing language likewise. Consequently, everything is swallowed up by signs. The entire procedure is reminiscent of the Pythagoreans who, however, did not reduce everything to signs, but rather to magnitude or number.

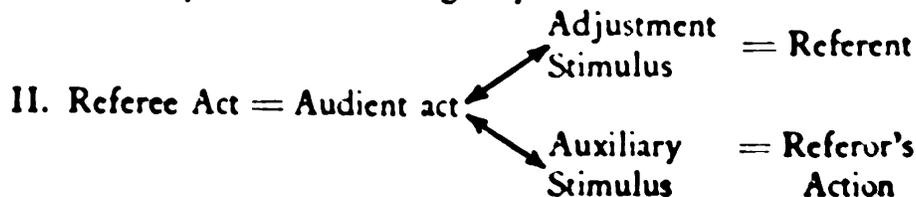
Though signs, marks, and symbols are things, just as are stones and trees, they are made into important features of logical discussion. This is achieved by the interpretational and constructional processes indicated. Stones and trees are signs for actions, words are signs for thought; and thinking, of course, is the essence of logic. In our next chapter we consider some prominent views concerning signs.

Field Analysis of Language. Properly to identify and describe linguistic events of whatever sort involves a field analysis of the interbehavior or things concerned. The accompanying diagrams and descriptions indicate the factors in each field.

(A) *Referential Language Interbehavior*

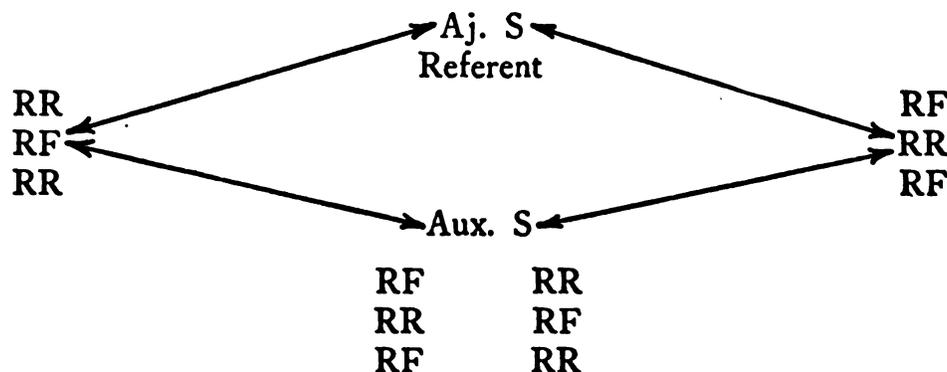


The adjustment stimulus or referent may be actions as well as things and may be the speaker's or listener's act or that of any other person or thing. Moreover, the object, event, act or relation referred to may be actual or imaginary.



The referee action or response whether auditory, visual or any other kind is psychologically, of course, as definite an action as the vocal or gestural action of the referor and may constitute an interbehavior with the same range of stimulus objects.

III. Referor and Referee Intercommunication.



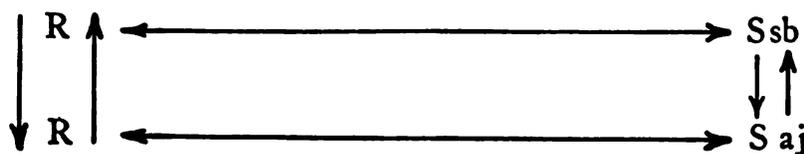
The best illustration of this mutuality of interbehavior is a lively conversation. The referent objects may be the same or a series of different ones in succession. The columns of RR and RF indicate the acts of Referor and Referee and their alternate roles as auxiliary stimuli in the various behavior segments.

The transcription of the reference if it is a vocal action, or a description if it is gestural or manual, may be regarded as a product of the linguistic response. The more subtle reference responses, describable as comprehension or appreciation, do not lend themselves to product making, but, on the other hand, they are more immediately interrelated with other interbehavioral fields, as when a referee act is followed by a compliance response.

(B) *Nonreferential Language Interbehavior.*

I. *Symbol Making.*

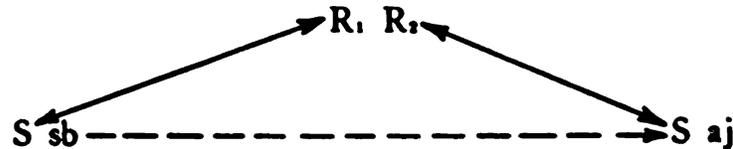
This is basically a process of closely adjoining two or more behavior segments, so that the occurrence of one is immediately followed by the occurrence of another. It is diagrammed as follows.



Specifically the process can be controlled by interrelating two stimulus objects, each of which can substitute for the other or in other words perform their functions in successive alternation.

II. Symbol-Using Interbehavior

Here two behavior segments are serially and successively performed. The first response is to the substitute stimulus and the second to the adjustment-stimulus object.



The substitute stimulus object as a symbol operates not only as a particular shape in a visual-perception field but also performs the function of pointing the designee toward the particular indicant or designant.

Transcriptive processes are not available in the case of non-referential language unless the symbol stimulus is orally presented and then we must regard the production action as a stimulus object. In other words, we have a scriptural description of a stimulus object. On the other hand, it is this fact that leads to the inept characterization of referential vocal responses as symbols. Designating stimuli are confused with responses to referents.

(C) *Linguistic Things*

I Referential Language Things	II Nonreferential Language Things	III Alleged Language Things
1. Referent 2. Referee 3. Tools employed in reference performance	1. Designant 2. Designee 3. Tools employed in recording and symbolizing	1. Signs 2. Characters 3. Marks, etc. 4. Natural or non conventional signs

The description *Alleged Language Things* is intended to indicate that such objects are stimuli in symbolic, nonreferential language fields or else are simply nonlinguistic things.

VARYING CONSTRUCTIONS CONCERNING LINGUISTICS
AND SEMIOTICS

Properly to relate linguistics, semiotics, and logic presupposes an appreciation of the varying views concerning these three types of events. Our discussion of language and symbols has already made plain the great difference between an interbehavioral and a noninterbehavioral view concerning these topics. Since certain decided advantages are claimed for the interbehavioral view we want to bring into sharp relief the essential principles underlying its approach to linguistics and semiotics.

Language and Symbols as Operational Factors. Interbehavioral linguistics treats both language and symbol actions and things as they arise from and operate in specific human enterprises whether trivial conversation or the most intricate work of mathematical discovery. Unfortunately this interbehavioral origin is frequently overlooked when one is deeply enmeshed in complex mathematical and logical problems. For example, logicians involve themselves in the belief that the referents of language and symbols somehow transcend the domain of human enterprise when they assume that logical or mathematical symbols stand for Platonic entities or existences of various sorts, numbers, infinities, infinitesimals, etc. Even if a logician is committed to Platonic existences his character as a logician constitutes a preoccupation with referents and symbols, the nature and interrelation of which he has himself constructed.

Not to mistake names for things, tautological systems for models of exact inferences, constructions for independent existences is simply to assimilate the postulation principles of mathematics and logic. We need only keep in mind the adjustmental and constructional character of the individual in his logical and mathematical work.

The full implication of this principle we may observe in the actual work of a logician or mathematician who finds himself concerned with problems taxing his capacities to solve them. The inability to prove Fermat's last theorem or to achieve complete freedom from contradiction affords a logician no occasion to fall back upon universes beyond his natural habitat or to endow himself with ultimate powers for transcending that habitat. We must, in other words, avoid the diseases of language and symbolism

which land us in mythologies that may not appear repulsive only because they are highly abstract.

Meaning in Symbols and Language. Very likely both the source and occasion for indulging in notions of transcendence and unmitigated powers lie in faulty views concerning meaning. For the most part the meaning of words and symbols is regarded as some psychic factor expressed by the speaker or maker of symbols and aroused in the hearer or interpreter. Whatever the constructional details of such notions they give rise to mystical ideas concerning linguistic events and make place for various imaginary powers of intuition and creation. Now for interbehavioral linguistics meaning is just as natural a factor as is the act of writing down a symbol. In no sense is it to be regarded as some sort of psychic factor basic to and behind the act of writing. Naturally the term *meaning* refers to various sorts of processes, sometimes to the intention of the logician or speaker and sometimes to the referent. In the latter case what a symbol or word means is simply the referent in a particular language field.

It is no serious difficulty to have to admit that frequently the term meaning when referring to the intention or the adjustment stimulus is not a necessary feature of a language or symbolic situation, but rather a descriptive construction of the by-stander. We may acknowledge the value or even the necessity for such a description as long as we do not confuse it with something described, and especially when we do not add some psychic cause or essence to the act of the speaker or hearer or to the object of which they speak.

The Field Principle in Linguistics. If we desire to keep before us the scope and possibilities of language as well as its limits we need only observe its field character. By this character is meant, of course, the actual field organization in which the speaker or the maker and user of symbols performs his action. It is from such field sources that we must derive our descriptions of what occurs in linguistic behavior and the nature of the resulting products.

When we persist in observing actual linguistic or symbolic events we not only keep in contact with what the speaker and hearer do in the way of referring to their adjustment stimuli, but we can criticize their style and appropriateness of action. By assessing the value of the reactions with respect to the existence and character

of the referents, we are able to judge the completeness, rigidity, certainty, and necessity of the assertions or symbolizations.

True enough, criticism of the speech and symbolization may require knowledge of the individual and his situation, but such knowledge is available. We can readily observe many of the events whereby an assertion or symbolization transcends an immediate situation. For instance, a particular logician relies upon statements because they have been repeatedly made by authorities or because they have been successively accepted by members of his school, as in the evolution of Inductive or Hegelian Cosmic Logic. To say that "nine times seven equals sixty three" or to write $9 \times 7 = 63$ may be possible only by the accumulative development of linguistic and symbolic products. Most of the certain and necessary items of our linguistic and symbolic fields represent a successive series of constructions by ourselves and our predecessors.

INTERRELATION AND CONFUSION OF LINGUISTIC AND NONLINGUISTIC THINGS AND EVENTS

Whether or not we agree that logic is coincident with linguistics and/or semiotics we cannot deny that all complex logical activities and situations not only involve linguistic interbehavior and linguistic things but also different varieties of each, all being intricately interrelated. It might be advantageous, therefore, to distinguish and separate the various factors, even if this involves an arbitrary process of isolation and analysis. For our present purpose we make these distinctions by indicating the confusion of: (a) various interbehaviors, (b) interbehaviors with things, either referents or products of action, and (c) products of linguistic interbehavior with things.

Confusion of various interbehaviors. A troublesome confusion consists of the improper identification of a language-describing activity with other kinds of activities. Linguistic and symbolic interbehaviors are wrongly identified with knowing, inferring, and even with investigational procedures of experimental science. When, however, we attend to the operations involved, nothing is plainer than that scientific manipulative activities, inferring, etc., are very different from the activities of referring to things or symbolizing them. How inept it should appear to confuse counting, measuring, calculating, recording, and interpreting with re-

ferring and symbolizing merely because in each case our operations include linguistic and symbolic procedures!

To illustrate the intricate interrelationship of interbehaviors in complex situations consider the activities of an individual when describing general or scientific things. Referential interbehavior is indeed involved, but it is just as apparent that referring to things does not exhaust the activity. In addition there are acts of pointing or indicating, as when we set down comparisons, ratios, and proportions; acts of recording, as in stating sizes, measures, etc. Studying actual interbehavior, we may differentiate between an act of pointing, whether performed verbally by gesturing or writing, and essentially referential behavior.

Confusion of interbehavior with things. First, descriptive behavior is confused with the events described. The description of an organism interbehaving with the event (dog swallowing a piece of meat) is not the event described. Because this is a simple case it is obvious. Actually, however, this type of confusion is one of the besetting sins of our intellectual life. For example, since we can describe the event by writing out words or drawing a graph, we tend to forget that our present interbehavior with words or graphs is not the original interbehavior with that event. Let us go further. Suppose we now focus our attention upon what has been written or drawn. Since the curves or formulae stand freed from the acts of producing them, they approach the free event and make confusion easy. When the event (stimulus object) is a thing instead of an action, the resemblance between the two makes confusion the rule. Mathematical formulae are identified with the events they only describe (analogize, represent). Here is the basis for the various idealisms, realisms, and their different mixed-breed offspring. When the basic interbehavior or operation is stressed, you have idealism; when the objects are emphasized, you have some form of realism.

The age-old confusion of the Euclidean descriptions or constructions with space is an excellent example in point. In this case the construction has been based upon a series of sublimated or attenuated contacts with spatial events. The mathematician calls this a construction based upon intuition. On the other hand, when non-Euclidean geometry is in question, especially with the Einstein-Minkowski addition of time as a fourth dimension, the con-

struction is projected as an independent existence, and we have the view that geometry is an empirical science—in other words, gives descriptions of things.

Two further types of confusion arise through the identification of linguistic and symbolic interbehavior with the products of such behavior and with the tools employed in them. The former is illustrated by misconstruing a logical system we set up by means of particular symbols and arbitrary operations as the model or exclusive processes of thinking. Certainly the study of symbolic logic convinces us that numerous postulational systems may be erected. If we endow any particular system with the generalized properties of all logic, we do so at our own risk and with the high probability that we shall fail to achieve the advantages desired from our system. According to the interbehavioral hypothesis we can not regard the deductive procedure of geometry as the essential type of reasoning, just as we can not regard any form of geometry as *the* geometry. Each form of reasoning and each form of geometry constitutes definite constructions for particular purposes based upon certain particular assumptions and conditions. Unless we are alive to these conditions, which may be cultural as well as immediately technical, and to the assumptions which are derived from particular situations or the summation of a number of situations, we shall inevitably confuse our activities with the products resulting from them.

Mathematicians who regard mathematics as symbols and functions of symbols, and logicians who look upon logic as sentences or propositions which imply other propositions illustrate the confusion of interbehavior with the tools employed in such interbehavior. In the former case it is overlooked that the symbols and their functions are simply tool substitutes for relations worked upon or representatives of the products achieved by working with symbols. In the identification of logic with implied propositions the system products of various operations are confused with the operations themselves. In both cases things are dealt with, but things must be separated from the activities performed upon them and the products resulting from such activity.

Observe how striking this sort of confusion appears when these same mathematicians and logicians are troubled by questions of relations between signs and their significant or meaning. The

choice and evaluation of signs amply testify to the interbehavioral or operational character of both logic and mathematics. As a matter of fact, the perennial occupation of thinkers with linguistic problems is no more than a just tribute paid to the activities of individuals in their interbehavior with events, even when this interbehavior constitutes abstruse logical or scientific work.

This particular difficulty is in all respects like that in which the symbols or other constructions, developed as substitute stimuli for contacts with things (events or relations) or imagined objects, are treated as the things substituted for. In this way arise the various formal (logical, verbal) and content (sensationistic) mysticisms.

Confusion of Constructional Products and Things. This type of confusion is excellently exemplified by the identification of description products (as in a protocol) with the things observed. Essentially, a protocol is a construction concerning what happened or a record product of an observation. Naturally in this situation it is difficult to separate the record of what happened from the record of how one obtained one's results. The protocol, then, records two kinds of events: (1) the process performed and (2) the behavior of the things operated upon. It is not too difficult to separate out the various factors, if only we distinguish between the constructional interbehavior, the product, and the original events which instituted the operations and motivated the product construction.

A more perplexing situation is that in which the things dealt with are themselves constructions or products. For example, when one sets up the identity $x^2 = x$, the relationship indicated is a construction for a certain purpose. The actual symbols are products of description used to describe a product. Still, by cleaving to the interbehavioral hypothesis one can keep these factors distinct. An excellent example of confusion here is the definition of a variable as a symbol which assumes various values. Is a variable a symbol or a sign? Or is a variable a factor in a functional field in which the symbol or sign is simply a transcriptional product standing for the functional field?

Note that *all* recordings and symbols constitute products or constructions. The same thing may be said of all linguistic interbehavior, even the most intimate referential sort from the stand-

point of the referents. In other words, all linguistic interbehavior is metaphorical. There is a constructional or product factor.

Is not such confusion witness to the fact that language is only one form of interbehavior among many others? Likewise that it is not a general or basic event which can control or validate all other complex activities, as, for example, the semantic analysts imply. Certainly, syntax or arrangement of symbols is not logic or mathematics, unless we arbitrarily make these disciplines identical with abstract system. It is perfectly clear that all who make symbolin-guism a basic enterprise in logic and science are capriciously apotheosizing thinking *ueberhaupt*. Applying the interbehavioral hypothesis, we discover many special types of interbehavior with particular kinds of stimulus objects.

More than once we have suggested that valid logics and knowledge systems can only be based upon concrete operations under the control of the investigating individual. Only when we inter-behave with natural happenings or with symbolic systems without too much prejudice imposed upon us by our general culture do we have valid criteria for our thinking and scientific constructions. In short, our linguistic constructions must all be made with a clear appreciation of our intellectual background. When we refer to certain objects and events—what is popularly called communication—or when we use marks or symbols for recording and manipulating them at a shorter or longer remove, notice that we are inter-behaving differently from when we operate with or manipulate things or relational systems. Science and logic obviously involve a large variety of different types of operation, but instead of this fact excusing misconstructions it places upon us the heavy obligation of avoiding such intellectual calamities.

LANGUAGE AND SYMBOLS IN UNIVERSAL AND SPECIFIC LOGICS

Although it is now sometimes asserted that there are variant logics and thus no unique logical laws binding on the universe or human reason⁸ it is true nevertheless that prevailing ideas concerning language and symbols imply a universalistic conception of logic. Certainly few accept the notion that logic is system building, much less, that the language and symbols involved in system building are as varied as the materials systematized and the cri-

⁸ Cf. Lewis, *Alternative Systems*.

teria governing the system construction. Possibly because the essential characteristic of system building is organization and unification the idea seems inescapable that logic is one, final, and transcendent.

To the universalistic attitude toward logic may be ascribed the fact that language and symbols are for the most part treated as though they were designed to serve the purposes of an absolute and total logic. Before the logician a standard language pattern seems to lie—one calling for the construction of unique terms or sentences capable of being integrated into complete and absolute logical systems. Such universal systems, of course, follow mathematical trends, which by abstracting from all concrete things and attaining to the most extreme generality yield a full and exhaustive organization of elements. Naturally, this pattern is in complete contrast with the specificity type of logic adhered to in the present treatise, according to which all systems are operational products, the abstract mathematical being only one type among innumerable others.

As long as a universalistic logician confines himself to natural numbers or conventional constructions he may achieve his universal goal without contradiction and without loss of rigor, but even here he must resort to hierarchies of formulation. Each level of organization must be justified by another more inclusive level.

As soon as system makers leave the special field of mathematics and become interested in concrete things rather than abstract relations difficulties constantly arise. In this case the aim to achieve universality leads to language or sentence systems which inevitably end in tautological trivialities. Accordingly it is highly important to contrast at least the most widely varying attitudes toward language and symbols as features of system construction. The opposing attitudes and methods of work adopted by the specific and universal logicians we will refer to as operational and symbolinguistic respectively. By reviewing some telling examples of their treatment both of symbols and syntax we may evaluate the nature of terms, symbols, propositions, and sentences in logical work and systems.

A. SYMBOLS

I. *Symbols and essences.* Current symbolinguism assumes that symbology implies essentiality. In other words, the symbolinguists,

building upon the ideas of historical rationalism, regard signs, marks, or characters as carrying within themselves certain essences, whether arithmetical operations, number or class values. Exemplifying this assumption is the popular conviction that number marks as against ordinary words are exclusively materials for mathematical processes.

Now turn to the interbehavioral or operational view. A symbol is a mark or character that operationally stands for something. Only through the work of mathematicians do mathematical symbols fit themselves into operational systems. We can not therefore regard these marks as "expressing" a reality independent of the particular systems.

True enough, certain marks or characters are better than others for particular purposes. Was it not a great achievement of Vieta and others to adopt letters instead of number signs for algebraic purposes and to use consonants for known and vowels for unknown quantities? Just as it was a step forward in mathematics in general to exchange Roman for Arabic notation. Yet no one will claim that mathematics itself—namely, numerical problems, reflection, and other operations—arises out of the particular notations. Though Leibniz's integral notation was regarded as superior to Newton's, is it not a fact that if we could override our cultural habits, we could invent a still better notation?

Concealed in the thinking of symbolinguists are certain assumptions concerning psychic elements. These elements called concepts are presumed to operate especially well in linkage with particular signs. But this merely amounts to an oblivescence concerning the actual operations performed. For example, it is undoubtedly true for number signs that when we have thoroughly abstractionized events—say, the relations between things—and set up conventional signs ($2 + 2 = 4$ or $a + b = c$) to stand for them, the association is well and effectively established. Furthermore, the great power of cultural tradition gives such associations a seeming finality and permanence.

II. *The Independence of Symbols.* Only Dean Swift's potent imagination could create autonomous word symbols which by mechanical operations combine themselves into philosophical, poetical, or mathematical books. When dealing with numbers—signs abstracted from numbered things—symbolinguists tend to regard

these signs as similarly independent. The great fallacy here is overlooking the relational background governing the operational functions of the signs. The fact that we can not add pigs to pencils keeps company with the principle that when we count significantly different objects, the symbols for the resulting numbers are in no sense independent of the objects.

That even the most abstractive symbols are not autonomous is exemplified by the following symbolinguistic quotation: "Mathematics is not an activity but a calculus, and . . . the activity comes in when we carry out investigations about this calculus within the physical science."⁹ Whenever the calculus is really used—in other words, regarded as an actual operational tool rather than as an ornamental thing—it loses its autonomy. Though this quotation stands upon logistic rather than constructional mathematical ground, Helmer's discussion indicates that a calculus can not be independent of the postulates and working methods of the construction.

The interdependence of numbers and things is manifest in considering the problems encountered by mathematicians and physicists when they are concerned with units. In a recent book Osgood¹⁰ objects to the equation $3 \text{ ft.} = 1 \text{ yd.}$ as indicating that the *same* line has a length of 6 when the foot is the unit and of 2 when the yard is the unit. He goes on to say: "But this makes confusion worse confounded; for $3 = 1$ is not true, while, on the other hand, to try to introduce 'concrete numbers' like 3 ft., 10 lbs., 5 secs., into mathematics is not feasible." Campbell¹¹ accepting the view that "a number can express a quantity only when the unit of measurement is stated or understood,"¹² counters:

From the equation $3 \text{ ft.} = 1 \text{ yd.}$ there is no more reason for writing $3 = 1$ than there is from the equation $3x = 2y$ for writing $3 = 2$.

He adds:

With Osgood a symbol or letter always signifies an arithmetical number. But why should it not be used to designate a *physical quantity*, or "concrete number" as he calls it? If s is the position of a point at time

⁹ Helmer, *The significance*.

¹⁰ *Mechanics*, p. 77.

¹¹ *Units*, p. 441.

¹² Love, *Theoretical Mechanics*, p. 362.

t (units included implicitly in both s and t) then ds/dt is the velocity (units included) at time t .

Campbell characteristically summarizes his view with the inevitable suggestion that the whole problem is solved by using homogeneous units.

By homogeneous system of units is meant: If in a general physical equation a set of corresponding values is substituted, units as well as measures, and if with the units deleted, the resulting equation (in measures alone) remains true, then the units used belong to a homogeneous system.

Howsoever agreed we may be upon the situation in which our symbols are employed, and howsoever unnecessary in consequence it may be to allude to that situation, the symbols are only valid and usable in the situation. In point here are the remarks of Bridgman¹⁸ concerning the need of a text for understanding equations.

Relevant here also are the comments of Dewey¹⁴ concerning the propositional function. Dewey criticizes the assertion that the propositional function "x is human" becomes a proposition by the substitution of "Socrates" for "x." He holds correctly that the term Socrates derives its meaning not from the propositional function but by the observation that there exists such an object as Socrates and this object is one of the class of human things.

III. *The Clarity of Symbols.* That symbols must be clear and unambiguous is an obvious necessity of exact thinking. These properties, then, are not only interbehavioral necessities, but also the results of interbehavior. Even number symbols, which Leibniz regarded as the paragons of clarity, derive their unambiguity from particular operational situations. Such situations contrast with symbolic context and they constitute the interbehavioral conditions under which symbols are developed and employed.

Unless we take account of the dependence of symbols upon the individual's concrete interbehavior with specific problems and circumstances, symbols are far from being infallible guides to accuracy and reality. Rationally conceived, they can and do persistently operate to befuddle the user. Rationalistic symbology has

¹⁴ Nature, chap. 6.

¹⁸ Logic, p. 378.

not only created havoc in the practical and philosophical thinking of workers in every field—economics, politics, religion, etc.—but it is also the basis for the mysticism of mathematics. To the prevalence of rationalistic symbology may be accounted the great space and effort devoted to the discussion of symbols in language and thought.

IV. *The Absoluteness of Symbols.* Wittgenstein (Tractatus 3.26) states: "The name can not be analyzed further by any definition. It is a primitive sign." This pronouncement points directly to the anomaly that, in an operational and relativistic culture like ours, Renaissance cosmotheological ways of thinking still prevail. Obviously the above proposition can only be accepted by taking into account the operations of the name giving. We may consider two such operations.

In the first, the naming process stresses the naming operation. This signifies that the name is a pure symbol for something. A word identifier is connected with a planet. Such a situation may be taken as absolute. A symbol stands for what it is made to stand and there is nothing more to be done about it. Anyone who will, may declare as senseless any tampering with this procedure, especially when situations involve formal symbols. But whatever strict connection there may be between the name and the object or relation arises from the particular operation; at all events, it is done first or exclusively by one person. In all likelihood, it is impossible to overlook the cultural influence upon even the most abstruse and exacting disciplines. The obligation is ours, therefore, to make certain that our linguistic practices keep pace with changing cultural conditions.

Such naming prompts no serious question of immediate propriety, except that we can never assume a cosmic connection between the name and the object. The case is quite otherwise when the naming concerns the nature or properties of objects, in which instance one may well question whether a thing has been properly named or not." Not infrequently, also, we can check any naming or valuing of a symbol if we are able to show that such an operation will interfere with further operations within the system.

* For an informing statement of the importance of names in science see Crane, *Words*.

V. *The Abstractness of Symbols.* Nowhere is the contrast between the symbolinguistic and operational views of symbolism greater than in the province of definitions. The symbolinguists' abstractionistic view influences them to look upon definition as the substitution of one symbol for one or more other symbols connected with some thing. To this rationalistic notion is traceable all the controversy centering around nominal or verbal and real or causal definition.

The interbehavioral view requires that questions of definition must always be based upon definite activity in an interbehavioral system. Prominent here is the interbehavior of the defining individual with objects of all sorts under innumerable conditions influencing the defining operations. As a rule, therefore, from the operational standpoint defining activities are much more complicated than from the rationalistic angle. Such operations, whether fundamentally descriptive or casually identifying, are, of course, necessarily abstractive. But this abstraction is itself operational and can not be confused with bare signs standing for existent or subsistent things. Only by neglecting the actual field of operations can we regard symbols as unlocalized and nonspecific. Moreover, problems involving symbols can only be solved when symbols are regarded as concrete objects, tools for accomplishing results.

An excellent illustration of the nature and significance of the abstractness of symbols is afforded us in the opposing views of the three current mathematical schools concerning the use of symbols.

a. *Logistic Mathematics.* Because they identify logic and mathematics the proponents of this school naturally emphasize the systems which they construct. Symbols then are evaluated and manipulated with reference to the systems. This is as true for the primitive or "undefined" terms as for any terms derived from them. We cannot overlook the fact that the development of the system as well as its deductive character involves a reciprocal movement between the center of the primitive terms or symbols and the circumference of the completed system. Whatever autonomy one ascribes to the system is a function of the frequency and effectiveness of such reciprocal construction acts.

b. *Formalistic Mathematics.* The formalists, quite contrariwise, stress their symbols in abstraction from the systems in which they fit. Naturally, symbols can not be handled except in terms of sys-

tems, but the formalists regard these systems as metamathematically determined and hence more or less divorced from their symbols.

c. *Constructionistic Mathematics.* Intuitionist or constructional mathematics stands primarily for the view that neither symbols nor operations with them can be altogether abstracted from such definite relations as are found in the sequence of natural numbers. As a consequence both symbols and the operations upon them are kept under manipulative control. Thus the excessive use of symbolism is disapproved of and constructional techniques are not surrendered to *a priori* axioms and such unavailable entities as the law of excluded middle decrees.

Basic to all three schools are definite operational principles governing mathematical work, though there is a stress of some one factor in the operation rather than another. Certainly the difference between the three schools is not merely what kind of language (symbol system and manipulation) is being used, as the symbolinguists imply.

VI. *The Arbitrary Character of Symbols.* As a final comparison of the symbolinguistic and interbehavioral views concerning language in mathematics and science we consider the problem of arbitrariness in the choice and use of symbols. The indefinable and the infinite serve well as illustrations.

The growing realization that an indefinable is always definable in terms of a particular operational situation adds weight to the interbehavioral view of arbitrariness in symbolological thinking. An increasing number of writers assert that an indefinable is a symbol or factor chosen as a starting point for particular operations. Taking the desired operations into account we can describe or define the factors with complete satisfaction. Arbitrariness is operationally only selection, as when Euclid adopted certain propositions as definitions, postulates, and axioms.

Actually, then, there is little disagreement between the symbolinguist and interbehaviorist. But the latter goes further. The adoption of the interbehavioral hypothesis forces him to consider the source of starting materials. It may be the (1) intuitive source, as in Euclid, (2) some arbitrarily chosen factor, as in the case of the mathematicians who build up arbitrary (logical) systems, or (3) some other culturally conditioned circumstance.

One of the best illustrations of operational caprice is the handling of the mathematical infinite. When it is said that only by discovering the infinite has the mathematician been able to reach his peak of capacity, we may take this to imply that he was forced to take over this symbol or tool, or we may regard the infinite as a heuristic principle suitable for accomplishing certain results. When we observe that the arbitrariness is concrete in the sense of adopting an analogy or a scaffolding to attain a desired result, we do not confuse the operational tools with some absolute theory which imposes itself upon us.

B. Symbol Configuration

Whether logicians are primarily interested in symbols or what they signify they inevitably enlarge their studies to take in complexes or configurations of symbols in the form of sentences, statements, "propositions," equations, etc. Whoever is interested in the linguistic or grammatical features of logic cannot stop with a study of the nature of signs and symbols or merely the character of language elements. After all, it is difficult to escape the conclusion that the complexes are prior to the elements or at least that logical work consists of the study of the relations between symbols if not the relations they symbolize. Moreover, if mathematics is considered to be language, then it seems to follow that equations or equation systems should be studied as complex organizations of symbol systems. The problems of symbol configuration are comprehended in the linguistic branches called syntax and semantics.

The contrast between symbols in universal and specific systems is paralleled, of course, in these two forms of symbol configuration. For the most part universalistic logicians occupy themselves more with syntax than with semantics. Syntax they define as the rules of formation and transformation which result in the organization and modification of the sentences constituting logic. Such symbol configurations or sentences make no reference to the meaning of the symbols constituting the sentences or to any sense of the expressions or sentences. Rather, such sentences are autonomous and derive their significance or value from the total configuration.¹⁶ That such formalism should be at all acceptable to anybody we can account for by the stimulation its proponents receive from

¹⁶ Cf. Carnap, *Logical Syntax*.

formal mathematical models. But even so, syntactic logicians overlook the fact that formalistic mathematics operates within a framework which consists of a hierarchy of mathematical systems.

It is only to be expected that such abstractionism should be found inadequate to set up a complete and universal logic. To withdraw so completely from all designata may yield impersonal and objective systems, but there is also nothing to hold them together. Before any fairly large structure is erected it inevitably crumbles. For example, linguologists attempt to make sentences and statements significant by ascribing to them definite and binding characteristics. They must be either true or false. For strictly mathematical sentences this amounts to a verification by the criterion of excluded middle. For more general logical statements it is asserted that unless they admit of a truth decision or verification¹⁷ they are metaphysical and void of sense. From this position, which already represents a retreat from pure syntax to semantics, a further step back had to be taken; so that verification amounted only to being theoretically imagined.¹⁸ A new term *confirmability* is substituted for verifiability to meet the new situation.

Probably the greatest blow to an abstract universal logic is the increasing realization that no formal system can be completely established. Stimulus for this view has been provided by: (1) the failure of formalistic mathematicians to reach a status of self-consistency, (2) the type of theorems that Gödel has brought forward to the effect that even evident arithmetical propositions cannot be deduced from the most favorably stated axioms, and (3) the general syntactical incompleteness of logic.¹⁹

For this reason we are not surprised when the most prominent syntactical logicians move over to semantics as a last stand.²⁰ Surely one of the most characteristic features of logical sentences is that the criteria for statements is enlarged from the simple classic dichotomy true and false to take in uncertainty, probability, and improbability. The result for linguistic logic is clear. It is no less

¹⁷ Cf. Wittgenstein, *Tractatus*, 4.06, 4.162, 4.2.

¹⁸ Cf. Carnap, *Testability*.

¹⁹ The relevant literature here is Gödel, *On Undecidable Propositions, Unentscheidbare Sätze, Ueber die Länge*; Tarski, *On Undecidable Statements, Der Wahrheitsbegriff, Einige Betrachtungen*; Church, *An Unsolvability Problem*; Rosser, *Informal Exposition, Extensions*; and Quine, *Mathematical Logic*.

²⁰ Cf. for example, Carnap, *Introduction*.

than a transformation to something nearer a probability theory than the intended all-comprehensive, certain system.

We must certainly conclude that we cannot step by step abstract from language to reach pure interrelatable forms out of which to construct a valid general logic. These steps are: (1) the equation of language with mathematics which deals only with relations, (2) forgetting that relations, after all, are genuine referents, and (3) cleaving to the references, but regarding them as autonomous connectable entities.

This idea of syntax is purely autistic and does violence to every workable notion of language. Syntax is determinable by situations, cultural groups, etc. No one can deny that every instance of syntax is a systematic organization of either acts or things. To build logic out of grammatical syntax is to make logic a precarious structure of fluid elements. This fact is, of course, recognized by logicians who thereupon proceed to distinguish between empirical and pure syntax. In constructing a pure syntax they risk the delusion that completeness and certainty can be obtained by arbitrarily fixing references and names. To avoid this dilemma, however, they can make their syntax a tight integration of pure signs. In this manner, indeed, an arbitrary form of genuine system can be achieved, but in no sense a *universal* or *necessary* logic. When the logician is interested in actual system building it is patent that systems can be built out of many kinds of materials, but by the same token none that can be universal and exclusive.

LANGUAGE AND SYMBOLS AS AND IN LOGIC

The excessive integration of language and logic not only involves a universalistic and therefore invalid notion of logic but it also implies an unsatisfactory idea of language. For the most part, as we have already seen, this misinterpretation makes of language static and autonomous things instead of activities and activity products. Thus we may contrast sharply the conception of symbol configurations with referring activities, assertion, and symbolization. That logicians fail to consider such contrasts is evidenced by the various ambiguous and even contradictory notions concerning the identity of and relationships between sentences, statements, propositions, formulae, and equations. Outstanding among the difficulties which these conflicts occasion is the failure to notice that

even if one finds abstract symbol configurations useful there is still the problem connecting such configurations with implicational or inferential systems. After all, the most characteristic thing about a universal logic is its systematic organization of implications or inferences.

In order to clarify the relationship between language and logic we consider the following problems typical of a large number.

(1) What is the relationship between assertions, sentences, and propositions?

(2) Must not sentences and systems of sentences be considered from the standpoint of the constructor of the system?

(3) A subsidiary problem here is: Does the emphasis on the constructor introduce a psychological or subjective factor in logic?

(4) What is the relation between sentences and things or referents?

Logical Acts and Logical Products. Those who regard logic as rigorous and normative must think it strange that no exact or standard definitions concerning sentences, statements, propositions, formulae, and equations are available. On the other hand, those who regard logic as specific system building can account for the lack of established definitions on the basis that logicians emphasize product and structure with a concomitant disinclination to consider the activities or processes of system construction. All students of logic may agree, however, that it is highly important to differentiate between these items, especially if arbitrary fixation of usages can be prevented. The writer submits that sentences, propositions, formulae, and equations constitute factors in interbehavioral fields which can be differentiated and described, once the notion of cosmic systems is rejected and the principle of specific enterprises adopted.

Sentences and Propositions. In classical logic, miscalled Aristotelian, sentences, of course, were regarded as the objective expressions of judgments. On the basis that the logician was interested in thought, in the sense of psychic states of various sorts, he did not differentiate between sentences and propositions. It was plain that the proposition or sentence was merely some manipulable factor representative of a mental state or process.

With the development of mathematical logic sentences became

regarded as simply the vehicles or carriers of propositions. In the first place, it was clear that a given proposition could be referred to or symbolized by different sentences or at least by sentences of different language systems. Again, it was not difficult to distinguish between formulae and the relations which formulae set forth.

Moreover, through various cultural circumstances it became increasingly realized that if sentences simply represented such impalpable processes as mental states or judgments logic could not be rigorous or powerful. This realization further fortified the conviction that propositions refer to things or relations. The extreme view here is to identify true propositions with events or things called facts.²¹ It is interesting to note the manner in which this identification is made. For example, Ducasse²² equates propositions and things on the basis that the expression "it is true that . . ." can always be substituted for "it is a fact that . . ." and vice versa.

From the interbehavioral standpoint propositions constitute products resulting from organizing relations discovered or assumed to exist between things, events, assertions or beliefs.²³ Now these propositions may be referred to, symbolized or recorded. In each case they comprise stimulus objects which belong to a particular operational field—in other words, referential-linguistic or thing-linguistic fields.²⁴

Sentences and Statements. Despite the fact that logicians strive to achieve objective systems without regard to processes of construction it is inevitable that they become involved in the problem of act and product. An example is the recent stress of the contrasts between statements and names of statements and the use and mention of statements.²⁵ Statements and use of statements in this development undoubtedly are assertions or expressions, whereas names and mention of statements constitute references or transcriptions, that is, thing or product sentences. Unfortunately the retreat from constructive acts results in confusing assertions or acts with linguistic things such as transcriptions or records. These

²¹ Cf. *Supra* p. 196.

²² *Propositions*, p. 710.

²³ Cf. Dewey, *Logic*, p. 287.

²⁴ Cf. Kantor, *Interbehavioral analysis*.

²⁵ Cf. Quine, *Mathematical Logic*.

confusions can probably be avoided by distinguishing between sentences as products or transcriptions of references, and sentences as the product or transcription resulting from symbolizing or recording actions.

Sentences and Symbol Configurations. On the basis of the interbehavioral field involved we can differentiate between sentences concerned in building systems out of things and sentences operating in mathematical systems in which the materials are relations. In the latter case the sentences consist of symbol configurations in the shape of formulae and equations. While formulae and equations are products of symbolization, the former may be concerned more with reference or act than with things (*designata*) formulated. Equations, on the whole, are usually concerned more with relations symbolized, though here we must differentiate between symbol configurations representing natural or discovered relations and equations standing for contrived or constructed relations.

Constructor and Construction. In what sense can we eliminate the constructor from the field in which a system is constructed? If we do so we forsake the basic discovery of modern logical and mathematical work—namely, the postulational principle. On the other hand, to keep constantly in mind the work of the constructor throws light on the nature of logical construction. Important here is the problem of the origin and significance of constructions whether of propositions or equations.

Origin of Symbols and Symbol Configurations. As to symbol origin, it is certainly clear that the process of setting up sentences goes back to referential interbehavior. (1) The first contact of an individual with a thing may be assumed to be a nonlinguistic form of event. (2) Next he may respond implicitly to (“thinks” of) that event. (3) A further step may be a referential interaction with it (speaks of it). In this situation the individual himself may be the referee, or the auxiliary function may reside in some other person. (4) Whether or not this speaking event occurs, the individual may set up or construct a notation or record of his first or subsequent contact with the original event. Examples of such a record are the materials of scientific protocols. When such recordings or symbolizations are formalized, we have the sentences of science and logic. Keeping these different stages distinct we observe that sentences and statements are definite products arising

out of particular kinds of interbehavior with an accumulative growth of factors and are employed in particular ways.

Significance and Operation of Symbols and Symbol Configurations. When we analyze sentences and statements we look upon them as products that now have particular functions and uses. They may have a referential function; in other words, sentences are transcribed references. For instance, they operate as the sentences in a letter which is an instrument for intercommunicating with another person at some distance. The important thing is that we can distinguish between the reference as an act of the referor and its relation to the referent and referee. Moreover, when we analyze sentences in this way we can observe their proper place in the situations in which they operate. We may differentiate between such materials as texts, as parts of systems set up or as instruments for various intellectual operations. It is only necessary that we keep before us the work of the individual in the various situations in which he operates, namely, produces sentences, manipulates, and transforms them or makes them stand more or less independently of immediate interbehavior.

Constructor Emphasis and System Validity. Despite the fact that every logical treatise shows not only the variant views and emphases of the constructor, a situation excellently illustrated by the schools of mathematics as well as the decided progressive improvement of logical and mathematical systems, it is still held that to consider the logician or constructor is to introduce a subjective factor or somehow to minimize the validity of logical work. We have already made plain that this situation is owing to the belief that logic somehow constitutes a complete and perfect process or product. We repeat, then, that since the evolution of objective psychology the consideration of the worker does not make the work less objective. It is a different thing to create systems autistically and to take account of the inescapable fields in which all logical work is located.

Sentences and Things. The study of the interbehavioral fields of logic forewarns us against confusing sentences, which constitute products based upon contacts with concrete objects and abstract relations, with those very things. Things comprise both the materials out of which or from which systems are constructed and the products of such construction. To keep these processes distinctly

before us would certainly prevent us from looking upon sentences, whether or not interpreted as linguistic material, as the exclusive logical subject matter.

THE ROLE OF LANGUAGE IN SYSTEM BUILDING

Granting that logic is system building, and no one observing the actual work of logicians (whatever type of systems, Aristotelian or symbolinguistic, they construct) can fail to do so, it is inevitable that referential and symbolinguistic activities occupy a large place in such constructional enterprises. It may not be supererogatory to mention once more that system builders inevitably resort to linguistic behavior as their essential and characteristic action.

It follows that whether and what kind of language and symbol behavior is employed depends upon the type of material worked upon and the kind of systems being constructed. For principle's sake we must include in logic such simple system building as requires no references or symbols. When more complex systems are constructed out of concrete thing elements, considerable referential behavior may be required. When relations and other more abstract materials are dealt with a higher ratio of symbolic to referential action is essential. In general there is a necessary substitutive process involved, so that the farther removed the materials are from things the more necessary and important become referential and symbolic substitutes.

Only by keeping constantly before us the central place of interbehavior in logic can we avoid the confusions and errors canvassed above. In analyzing the logical enterprise (chaps. 11, 12) we shall find that the attitude taken toward references and symbols as factors in logical work or as autonomous things or events is determined by the philosophical or other intellectual characteristics of the workers themselves. Knowing the logician we understand why he works with symbols at all and why he may regard particular sets of symbols as better than those favored by other workers.

Keeping in mind the essential system-building process of logic we may relate this process to language and symbols on the basis of two criteria. For example, we may: (1) begin with the particular enterprise and discover the function of references and symbols from the standpoint of materials, work, and product, or

(2) start with language and symbols and indicate their place as actions and things in the logical enterprise. The following schema illustrates the relationships.

(1) References and Symbols in logic.

(a) References:

1. To elements of system building.
2. To operations in system building.
3. To completed systems.
4. To justification of elements and systems.

(b) Symbolization:

1. Of elements.
2. Of operations.
3. Of completed systems.
4. Of justifications in the form of hierarchies of metalanguages, or symbol configurations.

(2) Language and Symbol actions in logic.

Language. It is obvious that simple system building, that is, such enterprises in which the constructor operates merely by ordering and relating things, requires very little language and few symbols. In more complicated constructions the system builder or logician must constantly perform referential language behavior in order to carry out his enterprise. This simply takes into consideration that referential forms of action are the more ubiquitous and probably the most effective. We are constantly aided in our activities by speaking of things, mentioning them to ourselves. In this sort of situation perhaps we discover the truth of the statement that thinking is conversation with oneself.

Symbols. When we perform complicated actions with respect to events it is hardly necessary to insist that we consider not only the operation of the events, but also the recording of them. This is true even in our interbehavior with things which are not systematic or do not involve system building. In the latter case perhaps the need for symbolization is intensified. Because of this fact we must keep distinct the different operations. System building is activity of a certain general type, comprising many special and specific instances and types of instances. Depending upon the complexity of the system building it may involve thinking, reasoning,

speaking, symbolizing, etc. It is fatal to regard logic or system building as nothing more than operations performed with symbols or sentences, or to reduce it completely to one type of construction such as a mathematical system.

SUMMARY

Briefly to summarize this chapter, it is clear that complex system building cannot proceed without language and symbols. At once, note, however, that this applies *only* to complex systems. In simpler cases the activity may proceed as a direct ordering and interrelating of objects. Language and symbols enter into the situation only as extra symbol-building features, that is, they may be employed in describing the materials, the constructional acts, and the nature of the product.

Let us note, too, that linguistic objects logically operated upon constitute the materials of system building. In such cases language and symbols comprise essential factors in the logical process. In other words, in linguistic logics the ordering of symbols, signs, and sentences comprises a special type of logical work. To observe these distinctions means to clarify the point that logic is neither language nor occupation with language. It means further that logic is an interbehavioral enterprise and that in the description and evaluation of such enterprises the materials worked upon, the types of processes or operation employed, and the resulting product must all be considered.

CHAPTER IX

LOGICAL OPERATIONS AND PRODUCTS

LOGICAL OPERATIONS ARE CONSTRUCTIVE ACTS

CONSTRUCTING syllogisms, interrelating implied propositions, or organizing postulates and theorems imply, as we have emphasized so often, the handling of various sorts of materials in order to organize them and build them into systems. Obvious as this fact is we still are forced to stress the constructional character of logical work for two reasons. In the first place, the constructive operations are frequently overlooked—for example, when it is held that logic consists simply of the manipulation of terms, propositions or symbol systems by way of equating them, substituting one for the other and so forth. Such manipulation is envisaged as the exercise of reason in the process of inference or the discovery of implications. Because the logical goal or system is stressed the actual constructive process, the production of a product, goes unnoticed.

Secondly, it is disregarded that not only do logical processes result in definite system products, but also that the traditional building materials are themselves constructional products. Obviously, logical construction should have definite materials or elements as building blocks, but it must also be recognized that these materials themselves are products of processes integrated with the system-building operations.

In Chapter 8 we indicated that in addition to conventions the basis for neglecting the constructive character of logic is the ubiquity of linguistic factors, whether terms, propositions or symbols. Because logic is so intimately involved with linguistic materials we forget the origin of our symbols as tools employed in our interbehavior. Consequently we hypostatize words, terms, symbols, and sentences into autonomous and definitive structures and in this way fail to do justice to the eventual and constructive character of the interbehavior resulting in the production of logical systems.

How important product construction is in logic may be seen by way of comparing logical and scientific operations. In the latter,

the primary emphasis is upon observations and manipulations which are basic to all interpretation and system construction. Logical operations, on the contrary, consist primarily of the construction of systems, whether of things, events, knowledge, statements, relations or any other sort of interrelatable materials. The essentially logical process operates with materials culturally given or selected. When logic and science are bracketed, therefore, the two are differentiated and interconnected upon the basis of observational and system-constructional features.

Similarly, when we compare the products of scientific and logical operations we find that the former consist of a series of equations or formulae which describe and sum up as well as indicate the consequences of certain interbehavioral operations. By contrast a logical system comprises a well-rounded structure or organization which may be described in verbal or symbolic form. Though in both cases the resulting products may be regarded as useful and valid they vary greatly in character. Whereas the characteristics of a scientific system are determined by the objects and events interbehaved with, in the case of a logical system, even if actual happenings or natural relations are organized, the stress is upon the systematizing procedure rather than upon the events. In logical constructions the products or systems to a great extent represent or substitute for the constructive process rather than the material worked upon.

MATERIALS AND PRODUCTS OF LOGICAL CONSTRUCTION

As we have seen, however, both the operational character of logic and the presence of products are concealed in conventional logic by linguistic or symbolic factors. When we keep before us the fact that words and symbols are themselves behavior products and therefore refer to or stand for certain kinds of events, we are better able to understand our logical materials and products. This point may be illustrated by the indefinable problem and the general primacy and ultimacy of "logic." A typical statement is that of Couturat¹ who asserts that it is impossible to give "rigorously logical exposition of the principles of logic," for such an exposition must result in a *petitio principii*. Sheffer² refers to this as the *logo-*

¹ Principles, p. 138.

² Sheffer, Review.

centric predicament. "In order to give an account of logic, we must presuppose and employ logic." When the existence and validity of logic are thus assumed it is clear that one is simply beginning with a prior construction and an accepted product. Logicians not too strongly committed to mathematical systems which build upon long series of prior constructions and products regard indefinables as so well understood as not to require explicit definition.³

From the standpoint of construction and product, "indefinables" may be looked upon as the most clearly defined elements in any system and in any logic itself built up as a system on a definite plan for a definite purpose. When we view a logical system as a product of interbehavior and not a cosmic structure what can be more definite and discussable than the materials out of which it is constructed and the canons governing that construction. For the understanding of logical procedures there is nothing more enlightening than the results of modern mathematics.

When Hilbert⁴ constructs Euclidian geometry out of five "undefined" factors or elements (point, straight line, plane, betweenness, congruency) and Veblen,⁵ Huntington,⁶ and Pieri⁷ construct geometries out of two each (point, order), (space, inclusion), (point, motion), they definitely construct systems of the best and most workable materials. The criteria of best and most workable, of course, are determined by the task at hand, but this is only a circularity when we overlook the larger setting of work to be done and think only of autonomous elements.

Logicians are able to concentrate upon the product aspect of words and symbols simply because products may be regarded as distinct and independent of the work of production. Yet it is strange that competent logicians revert to the intellectual status of the observatory visitor who is mystified by the astronomer's ability to discover the names of the stars.

Couturat throws light upon the present situation in declaring that "when we are dealing with the primary concepts of thought in general it is impossible to find any others by which they can be

³ Cf. Johnson, *Logic*, p. 106.

⁴ *Foundations*.

⁵ *System of Axioms*.

⁶ *Set of Postulates*.

⁷ *Della Geometria*; and Young, *Fundamental Concepts*, Lecture 15.

defined."⁸ We need only give up the mystical psychology which fosters such essences, and turn to an interbehavioral view. Whenever a logician declares, as does Pieri, "it will never be possible to afford deductive proof of the truth and consistency of the whole system of logical premisses"⁹ he is only referring to the fact that systems are constructed upon acceptable foundations.

A similar illustrative product is the sentential or propositional function. For example, it is said that in the statement "x is human," x has no specific meaning. Yet at once it is further asserted that x is a symbol for humanity in whatever instance it is found. Certainly the propositional function is a useful and important construction which does credit to the worker who first produced it, but obviously as in all cases of terms, symbols, and sentences, it is definitely a product of work in a particular kind of situation.

The same principle applies to the truth functions of sentences or propositions. The essential feature of a truth function is its acceptance in a certain form. There is a definite abstractionistic operation here, since the entire fact of false-true is derived from the basic correspondence of an assertion and an event. When we declare that "a statement is logically true if it is not only true but remains true when all but its logical skeleton is varied at will,"¹⁰ we simply adopt or set up a system in which certain relations are taken or "defined" as true.

In general, all propositions and syllogisms have definite characteristics which the producers thereof have given them. When products have been produced other individuals, of course, take them over, use them in their original form or modify them, and in general begin and continue a cultural tradition. We must never overlook the fact that all highly abstract work involves the creation of symbols and their designata, even though these symbols be zero, infinity, and the whole series of not-materials. When propositions imply one another or when syntactical transformations are possible, in both cases we have constructions for particular purposes. In some instances, as in dealing with series, we may never be in direct contact with each term, but are nevertheless handling a specific product system.

⁸ Principles, p. 138.

⁹ Sur la géométrie.

¹⁰ Quine, *Mathematical Logic*, p. 28.

The constructional process emphasized here with its resulting products must be interpreted as occurring in specific operational fields. Excluded from this discussion is any notion of world process or totalitarian universe. Equally unallowable is the conception that objects of knowledge do not exist independently of and prior to the activities of knowing. Such a construct, ultimately reducing the materials worked upon to the work performed, is part of an idealistic philosophical conception which has no place at all in a scientific logic. We stress this point because such a constructive theory can be so appealingly framed that it leads to an insidious acceptance of fallacious doctrine. An excellent example is Dewey's *Inquiry Logic*. In his basic principles he sets forth that logic is operational, that it has its matrix in biological and cultural facts. Yet it ultimately becomes resolved into a metaphysics of construction, and so aside from terms used it reverts back to a formalized universal system.¹¹

HOW OPERATIONS AND PRODUCTS ARE CONFUSED

For the most part logic is identified with linguistic materials or products because the descriptions of processes and results are confused with those processes themselves. Simply because we are obliged to record the descriptions of our operations in linguistic or symbolic form we tend to regard the products as exhausting the whole logical content. Note, however, that inferential and problem-solving interbehavior can occur in private situations which are never recorded. Let us stress nevertheless that even the private reflections of individuals concerning what happens in logical procedures are inseverably connected with linguistic events. In other words, all reflection may be regarded as an individual's conversation with himself. Still this fact in no wise necessitates the belief that logic consists essentially of symbol terms, statements or syllogistic materials.

An interbehavioral investigation of logical procedures illuminates the entire process of emphasizing words or terms or the more recent techniques of stressing numerical or logical symbols. Possibly the value of current symbolic logic lies in its vigorous employment of symbols, a procedure which has proved so fruitful in mathematics in avoiding the difficulties inherent in verbal ma-

¹¹ Cf., Chap. 6, p. 166 f.

terials and achieving a freedom in handling relations abstracted from content. As we have so frequently pointed out, however, the most formal relations are abstracted by individuals from situations, or projected on the basis of selectional operations. Our fundamental suggestion therefore is that all such activities are essentially forms of interbehavior. When these relations are simply recorded or organized into systems, such records and systems are interbehavioral products.

The situation becomes more complicated still when we have a cultural accumulation of logical materials or constructions, with the net result that prior products become the materials for present operations. It is culturally inevitable that the Aristotelian formulations of logic should set the pace and circumscribe the conditions for future work. Here once more we are reminded of the Kantian apotheosis of Aristotelian logic.

So important is this problem of construction and product that we may again consider how this situation operates in the field of mathematics. When Kroneker made his famous declaration that the integers alone are the work of God he overlooked that the integers too were products and that Xenophanes¹⁹ long ago knew that it was only because man had made the integers that they took the form they did. On the basis of the interbehavioral principle operations are always interactions with things or stimulus objects. Accordingly some operations result in products derived directly from natural materials. Thus the natural numbers are derived from operations upon natural things, their collection, separation, etc. The products then require recording or symbolizing, with the resulting creation of a representational or transcribed number system.

Later the observation of these products and the relations they sustain constitute materials for further productive operations and products. In this manner by abstractional constructions more and more complex number systems can be built up. To this process there is theoretically no limit, unless indeed one is influenced by consideration of time, usefulness or other operational criteria.

The error pointed out by Xenophanes is so ingrained a part of our culture that we find it at the heart of our most fundamental thinking. But recall all those scientists who after they construct

¹⁹ Fairbanks, *First Philosophers*, p. 67.

definitions, hypotheses, and laws mistake these products for the things and events from which they constructed them. A basic fallacy of great men is to mistake science for nature, even if that science is an effective and superior science *of* nature. Not even the enthusiastic assertions of Kepler, Galileo, Boyle, and others that the book of nature is written in mathematical language assure us that the book of nature is nature. Let us agree that science is mathematics but at the same time not forget that mathematics is the symbolization and calculation of events.

Never can we ignore the fact that all construction implies material and that in the final analysis such materials are natural events. Because we build upon prior building does not mean we can ever lose sight of the original starting point. Science and logic both constitute enormous towers, but both must stand upon a natural foundation.

It is most encouraging to notice that some faint recognition of these points is making its way into logical writings. An example is Quine's insistence upon the distinction between use and mention, between the concrete conditional and implications between propositions or statements. Without doubt this distinction makes for a better understanding of logic and its operational nature. This statement is deliberately set down. In spite of the fact that Quine¹³ himself is at a loss to say what the subject matter of logic is we may add that whether one deals with arrangements of things or sentences, in either case we have the building of a system.

Unfortunately Quine like other logicians is entwined in the meshes of language. While he sees that transcribed sentences or statements refer either to things or to other statements he does not consider both as products of special performances. When this is done, the system of products can then be arranged on the basis of what the constructor does, what sorts of referents or designata he is concerned with. The recognition of the operational foundation of all sentences and formulations would prevent such a statement as: "To be is to be the value of a variable."¹⁴ A valuable safeguard already stressed is the observation that while all complex logic operates with language it is not itself language. Quine does not go beyond the use and mention of language. Logic, however,

¹³ Mathematical Logic, p. 1.

¹⁴ Quine, Designation, p. 708.

is the building of structures, whether or not language constitutes the materials or the products.

RANGE AND TYPES OF LOGICAL PRODUCTS

Granting the organizational enterprise theory of logic it would obviously be impossible to exhibit an illustrative inventory of logical products. Even though an attempt to survey the field of such products is futile, it is, nevertheless, well to remind ourselves occasionally how wide is the range and how numerous the types. To do so is helpful in keeping before us the operational principle and in observing the continuity existing among technical logical systems and the organization of every sort and type of thing.

Regardless of purpose or interest, enterprises are undertaken to bring order and system into things of all varieties. Despite opposing tradition, we repeat, a filing system organized to arrange correspondence, catalogues of book titles or of minerals, and the organization and numbering of building stones requiring transportation constitute logical products quite as much as any form of mathematical system, for example an algebra or a geometry. Similarly, the ordered series of objects, persons or services in a political, military or religious organization comprise logical products, as well as the disposition of primacy and priority among the scientific disciplines or the organization of knowledge.

Is it anything more than the logic-glorifying tradition which militates against the continuity of process and product throughout the range of system building? To distinguish, as one must, between scientific and nonscientific system building fosters the view that system building is a specific form of enterprise yielding particular kinds of products depending upon the materials used and the situations in which the work goes on rather than the view that science and logic are identical or that logic is exclusively related to science.

Though traditional logicians have never properly isolated and evaluated the system-building features of logical work and certainly never were favorably inclined toward actual objects, they have been somewhat more sympathetic to the suggestion that the organization of a grammatical or semiotic system in general is close to logical activity and the work of producing a logical product. Influential here no doubt is the prejudice that a logical

order is concerned with abstractional terms and their interrelations which somehow represent exalted acts of thought. To emphasize the building of abstract systems by means of linguistic or mathematical symbols is certainly justifiable, but to do so certainly does not warrant the exclusion of any other form of product organization. Indeed we may assert that all those who regard logic as the science of order allow for this interpretation.

Though the operational and specific logician finds no basis for believing in any sort of reasoning so exalted as not to constitute an interbehavior with things directly or through substitution and hence does not hesitate to deal with simple material organization as satisfying completely the criteria of logic, he may still for illustrative purposes employ linguistic materials in order to demonstrate the salient features of logical processes and products.

As a preliminary essay in this direction we will consider some typical semiotic systems which in the present reign of symbolic and mathematical logic are regarded as close to logic's heart. Recall Peirce's remark, "Logic, in its general sense, is only another name for *semiotic*, the quasi-necessary, or formal doctrine of signs."¹⁸

The Semiotic of Peirce. Peirce's semiotic or doctrine of signs constitutes a theory concerning things (representamen) which stand to somebody for something. There are three basic types of representamen or sign, the Icon, Index, and Symbol.

A representamen is a subject of a triadic relation to a second, called its object, for a third, called its interpretant, this triadic relation being such that the representamen determines its interpretant to stand in the same triadic relation to the same object for some interpretant (C.P. 1.541).

The icon (picture, photograph, diagram) stands for something because of its resemblance to the thing. The index casually and naturally stands for something besides itself. As examples Peirce offers the barometer-level, weathercock, spirit-level, plumb-bob. The symbol is something conventionally made to stand for some thing, words, sentences, books, etc.

As is the rule with Peirce he does not limit himself to one characterization of signs. The above trichotomy may be described

¹⁸ C.P. 2.227. This term *semiotic* Peirce derived from Locke.

as based on the criteria of (1) the mode of origin of the denoting function and (2) the resemblance between the denoting and the denoted members of the couple. A second trivision of signs is based on the way they connote or represent their object. Signs may be (1) terms, (2) propositions or sentences, or (3) arguments or inferences (C.P. 2.243 ff.)

Aside from Peirce's own multiplication of sign classifications semiotic students assert that he described though he did not explicitly recognize others. For example, Moore¹⁶ suggests that Peirce really has two kinds of indices. One causes someone to think of something, whereas the other induces someone to act in a certain way.

It is important to point out that because of (1) the inclusion in Peirce's semiotic system of sentences and arguments as well as unit signs and (2) the triadic relationship of signs his semiotic is really a metaphysical as well as a logical system. Students of Peirce seriously disagree concerning his basic ideas,¹⁷ as indeed they might concerning so unsystematic a writer; still there is hardly any question that Peirce was a universalistic philosopher who attempted to build on a basis of irreducible principles primarily in the form of categories or signs.

While it is true that Peirce was scornful of philosophers nurtured in theological seminaries instead of the dissecting room or other laboratory (C.P. 1.620) and prided himself on his own early laboratory training he was not free from the influence of Kant and Hegel. Indeed, he regarded his philosophy as a resuscitation of that of Hegel (C.P. 1.142), whom he estimates to be in some respects the greatest philosopher who ever lived (C.P. 1.524).

What is of interest to us, however, is that Peirce basically regarded signs as the constituents of a universal logic and also asserted that this logic was ontological. His signs, then, in their various functions and combinations constitute excellent examples of constructions. There is hardly a doubt that the primary influences upon Peirce's thinking were the constructional processes

¹⁶ Moore, Indexical and presentative functions.

¹⁷ Cf. Weiss, *Emence*; Buchler, *Accidents*; also, Buchler, *Charles Peirce's Empiricism*; Nagel, *Charles Peirce's guesses*; and Dewey, *Development*.

of mathematics, for him the most fundamental of creational or constructional activities as the following quotations demonstrate.

Numbers are merely a system of names devised by men for the purpose of counting (C.P. 1.149). The system of numbers . . . is our own creation (C.P. 1.149). Mathematical reasoning holds because it relates only to the creations of the mind (C.P. 2.192).

Despite Peirce's leanings toward realism he does not get far from a close identification of things and thoughts especially if these thoughts are scientific thoughts experimentally determined. The crossing over from things to thoughts is easy by means of signs in triadic relations. He declares:

. . . the woof and warp of all thought and all research is symbols, and the life of thought and science is the life inherent in symbols; so that it is wrong to say that a good language is *important* to good thought, merely; for it is of the essence of it (C.P. 2.220).

Peirce is an outstanding example of a logician possessing excellent intentions, who, moreover, seems to have the sanest insights concerning a naturalistic and fallible view of science and the world of things, but who fails nevertheless because of an unsatisfactory psychological background. This comment is definitely not made in depreciatory criticism of the man, but rather as an exemplification of the cultural milieu in which he lived. For him psychology was, of course, not an objective interbehavioral science, as indeed it could not have been. For this reason he could so vigorously reject psychology in favor of a more ultimate criteriological discipline. The consequence for him as well as for the fields of logic, semiotic, and mathematics was and remains an inadequate notion of the nature of signs and their place in the disciplines mentioned, as well as system-building enterprises. Naturally the faulty ideas concerning the interrelationship between all these fields are intensified by the universalistic and exclusive notions concerning the character of signs. And so instead of envisaging signs and their systematization as instances of system building, as factors in one form of logic—semiotic logic—, signs become basic constituents of a totalitarian and relatively permanent ratiocosmological philosophy.

The Unifying Semiotic. An even better illustration of the proc-

ess of system building than Peirce's semiotic we find in the recent unifying semiotic system of Morris.¹⁶ This writer refines Peirce's semiotic by suppressing its parallelity to things. Whereas Peirce may have been guilty of constructing a metaphysic of signs, at times he did certainly no more than reduce thoughts to signs. Morris, on the other hand, absorbs the philosophic views both of the logical empiricists, who would reduce logical entities to formal language and objects to object language, and of the American pragmatists, who identify psychological processes with signs regarded as actions. On the basis of these refinements and reductions Morris constructs an all-inclusive system of things, acts, processes, etc., with signs as the exclusive materials.

Ostensibly this semiotic is part of a philosophical movement for the unification of science; however, the system becomes enlarged, quite in the manner of Hegelian cosmology, to unify everything. Semiotic is not only a special science of signs, but also an organon or instrument of all the sciences. "Logic, mathematics, and linguistics can be absorbed in their entirety within semiotic" (55). This inclusion extends also to the physical and humanistic sciences. Nor is this gigantic system limited to the propositions of science but also embraces the data whether things or events.

Signs are simply the objects studied by the biological and the physical sciences related in certain complex functional processes (2). Every empirical science is engaged in finding data which can serve as reliable signs (56).

The basis for this construction is the transformation of things into their description. Since all descriptions are reduced to signs, signs constitute everything. The fundamental constructional and transformational procedure consists of starting with knowledge which implies a polar relation to things. Since knowledge as recorded or referred to may be treated as acts or things standing for something it is reduced to signs. Thus is engendered the reduction of everything to signs as well as the glorification of signs and their theory.

This procedure operates in two steps. First, on the level of signs Morris stresses the triadic relation of signs to (a) the user (prag-

¹⁶ Foundations. The numbers in parentheses refer to pages.

matics), (b) the things designated (semantics), and (c) other signs (syntactics). This makes available the identification of sign users, things designated, and signs with signs. The sign user becomes reduced to something that takes account of something (4). Although in exposition the crude facts of activities basic to signs and sign operation are referred to, the general import of the whole unifying system is to liquidate everything but the signs and their functions. The liquidation is illustrated by reducing a dog fight to a series of signs and sign functions. In a dog fight the snarl of one is a sign for the attack, which is a designatum. The preparatory response of the other is the interpretant, and so the dog becomes an interpreter (36). Certainly anyone who likes may adopt this description. But surely there is not sufficient unifying or identifying power in such a description to make even one dog fight become another or to reduce the dogs and their actions to signs.

The expansion of signs and sign usage to a unifying metaphysic can only be achieved by performing a series of illicit transformations. To begin with, the method is to choose proper examples. The best for the purpose are reported acts of behavior. When describing behavior smoke can be treated as a stimulus for reacting to fire (37). Now in behavior situations things can stimulate in ways not based on natural properties. In other words, things are endowed with properties and also reacted to as substitutes for other things. On this basis one need not bother with the chemistry of smoke or fire. In descriptive metaphor, stimuli, and more frequently substitute stimuli, can be named signs. Moreover, since one is describing behavior and not performing the behavior described it is easy to slip over from the described behavior to an emphasis of the description. Descriptions, though acts, are made into signs and sign operations. Signs, then, swallow up things and actions. Mead whom Morris follows supplies an excellent paradigm for such transformation. He argues that "things are what they are in the relationship between the individual and his environment, and this relationship is that of conduct."¹⁹ A favorite example is food, which is not food unless and until eaten.²⁰ By this appealing device he bypasses all the natural

¹⁹ Mead, *Philosophy*, p. 218.

²⁰ Mead, *Behavioristic account*.

properties of food objects and gets food reduced to the name or symbol *food*.²¹ The difficulty here is that objects are reduced to what things mean to the individual who interbehaves with them. Behavior or act becomes enlarged from a specific interbehavior with a concrete object to a general metaphysical device.²²

What might be a satisfactory statement of signs and sign relations if kept within semiotic bounds thus becomes an arbitrary and gratuitous metaphysic. Not even referential language or logic can be reduced to such a sign system, let alone all of science. Morris observes that the subject of meaning requires confinement to specific situations (45); yet he elaborates the sign system into a universal discipline without regard to the specificities involved. Certainly his system is remote from any genuine investigational enterprise. Rather it constitutes a type of semiotic Berkeleyanism. The argument that because discourse and records involve signs (words) then all thought and science can be unified by semiotics departs radically from critical enterprises. The sciences, architecture, and accounting may just as validly be unified because those who engage in these enterprises use pencils to make signs or marks.

One final comment. The psychology presupposed by the notion that since mental (psychological) happenings can be equated with sign responses, consciousness (awareness) with reference by signs, and rational behavior with control by signs (42), psychology and social science can be unified is certainly misleading. On the surface this notion appears to bypass the psychic factors of unscientific psychology and as such has been attacked by a philosopher of dualistic persuasion,²³ though at bottom it reveals itself as a radical departure from objective psychology. Indeed, many psychological activities involve signs, but such signs and sign usage do not exhaust even such behavior. Signs are made and fashioned by behavior and are not behavior; nor are they exclusively the things interbehaved with. Signs are employed to mark things, to help enunciate, etc. A chemist does not seek data (litmus paper, chemical reagents, etc.) to be used as signs, but instead such sign usages, if any, are for the purpose of solving a chemical problem.

- Kantor, Philosophical implications.
- Murphy, Concerning Mead's philosophy.
- Cf., Ducasse, Some comments.

The same may be said for a mathematician or logician, even though their preoccupation with relations makes their signs superficially appear as the most readily observable things they are interacting with.

*The Semiotics of Ducasse.*²⁴ As a final instance of a semiotic system we choose Ducasse's construction concerning the nature of symbols, signs, and signals. Though Ducasse writes primarily to clarify his notion of symbols as factors in logic and mathematics we may assume that he regards his remarks as applicable to any situation in which signs and symbols are employed.

On the surface at least Ducasse's system has the merit of treating signs and symbols as items localizable in particular situations. Although there is a general metaphysics behind the system the semiotic itself is not essentially a metaphysics or an all-inclusive construction. This does not mean to say that Ducasse does not regard his simple formulation as holding in all situations. As we shall presently point out, it is a failing of his formulation to disregard variations in the nature and operation of signs and symbols under various auspices. Furthermore, we shall find that the exclusiveness of description harks back to a psychic doctrine by virtue of which essential specificities of all sorts are made completely dispensable.

Another appealing feature of Ducasse's semiotic is that it is superficially at least situational, as announced in his assertion "that nothing is intrinsically a symbol, but that anything is a symbol if and only if it symbolizes" (S.S.S. 41). But here again the psychic viewpoint intrudes to neutralize the effect of the formulation.

Proceeding to some of the details of Ducasse's semiotic system we note that his elements constitute tetradic relations. In order for A to be a symbol for B there must be C, a mind trained in a certain way, and D, a certain matter with which that mind is occupied at the time (41). Actually there is no reason why we cannot regard symbols as factors in dyadic relations, namely, a symbol connected with a thing symbolized. An objective psychological view must take account of such facts because behaving with symbols includes the discovery of such relationships. Also, when setting up such a relationship we may have a triadic relation, by adding to the symbol

²⁴ Symbols, signs and signals. Hereafter referred to as S.S.S.

and symbolized the act of connecting them. It is the psychic viewpoint of Ducasse that prompts him to add a mind to the symbol situation. So ingrained is the psychic in his thinking that physical events can only be defined for him in terms of an interpretation of certain states of consciousness (S.S.S. 42).

This notion of interpretation as an essential feature in semiotics is only the reverse side of Ducasse's psychic obverse. It stimulates him to overlook the process of setting up symbols, an activity which by no means carries any interpretative construction. Revealed here too is the influence of starting with signs as things which require interpretation.

Another detail is the distinction between signs and symbols and the further introduction of signals. A sign, Ducasse thinks of, as a semiotic interpretandum that begets opinion. It causes us to think of a proposition and inclines us to believe the proposition it makes us think of. A symbol, on the other hand, does not cause us to become conscious of a proposition or if it does it does not cause us to believe it. Signals are particular kinds of signs or discursive symbols put into the way of another person to cause him to think of or to think of and believe what the similar sign signifies for the utterer.

In all this description it is clear that an emphasis is placed upon things and their interpretation. Several important and valid distinctions are made. For example, among the various signs and symbols some are discursive or verbal, whereas others are things or conditions. Notice, however, that when things and words are regarded as factors in an interbehavioral field there is room for all sorts of signs and symbols; also they can be discriminated on the basis of the interbehavior involved without assuming some sort of mind whether or not called habit which accounts for the differentiation. Moreover, an interbehavioral approach to semiotics allows for the observation and description of the details of semiotic situations, and especially the differentiation between ordinary language situations, logical and mathematical symbols, and other semiotic factors found in various interbehavioral fields.

An interbehavioral approach obviates not only the suffusion of semiotic situations with an innermost psychic but also makes unnecessary the notion of causes. The cause construction follows from

the injection of things plus "meanings" into the semiotic situation. No doubt the use of the causal construct is to put a plating of objectivity on the symbolic situation. Not only is this unnecessary if we cleave to actual interbehavior of persons with things, but without taking account of the actual situation the dodge is worthless.

It is only natural that so well thought out a semiotic should at various points make contact with actual interbehavioral situations. An instance of this is Ducasse's distinction between indicative (or demonstrative or denotative) and quiddative (or descriptive or denotative) symbols. The former he describes as orienting to a place in an order-system, whereas the latter stand for certain kinds of things, a certain *what* (S.S.S. 46). Quite necessary here is an elaboration of these processes and functions to include many other types, an elaboration which would lead to the actual construction of symbols and their use in many specific situations.

To illustrate only from the field of symbolic logic. In a recent tilt between Ushenko²⁵ and Quine²⁶ the question is debated whether P and curl-P are to symbolize occurrences, statements, or names of statements. Then, too, the question arises whether symbol usage is to substitute for or to replace statements or quotations.

Probably one of the most objectionable features of Ducasse's semiotic is the implication that postulating, defining, calculating, and in general the work of symbolic logic and mathematics are concerned exclusively with verbo-verbal interpretation. It is hardly possible that such important enterprises should be concerned merely with interrelating words and symbols. We have repeatedly pointed out that logical and mathematical words and symbols are elaborate constructions occurring in complex situations and influenced by various intricate problems and interests. All these factors, of course, are absorbed by the psychic in Ducasse's system, but this certainly cannot satisfy anyone really interested in events. The suggestion, then, is once more to penetrate to the actual enterprises involved. In these interbehaviors we can isolate the constructive results in the symbol products and thus relate them to the things symbolized.

²⁵ Dr. Quine's theory.

²⁶ Reply.

TYPICAL CONSTRUCTIONAL PRODUCTS

If logical enterprises are system-construction pursuits, then a proper sampling of the products requires the consideration of every variety of system. To exhibit the nature of such products and the producing techniques we need only examine some standard elements developed in mathematical and technical logical enterprises.

Number. When we consider as we must, the cultural evolution and character of number, we are struck not only with the product character of such an entity, but also with the cumulative and accretional process in its development. This constructional character of number applies both to reactions and to materials as stimulus objects. Whoever attempts to incorporate number in a logical system is interacting with a thing which not only has been worked upon for ages, but which has been modified, corrected, and presumably improved. As to relations the various successions of number constructions suggest a variety of manipulations of number, whether regarded as natural quantity, magnitude or relation (similarity or diversity) or definition and description creations.

Since logicians and mathematicians deal with number only as defined and described it is indeed remarkable that the question whether number is a product of constructing operations should be raised at all. That such a problem arises can probably be ascribed to two circumstances. In the first place, however number is accounted for, there exists the tendency to accept products as unguaranteed entities simply discovered. The long reign of "axioms" is witness to this tendency. In the second place, those who work most with symbols, with abstractions, with construction reveal the greatest repugnance toward genetic investigations. Students of number as seekers of continuity, validity, and consistency easily slip into Platonic realism. Because the constructional feature of numbers is so generally overlooked no apology is required to stress it, even though the results are merely reminders of obvious events.

Before offering demonstrative instances of the constructional and product character of number we must be reminded of the enormous scope of the number field. It is well to keep alive to the fact that the term *number* designates many widely different sorts

of events. Hence we must be specific in our discussion. We have already mentioned the difference between number reactions and number stimuli. Further, number as element and number as system should be kept distinct. Not only must we observe that there are all sorts of number events, but also be specific in our descriptions and evaluations.

Whatever may be the connection between the natural numbers and all the other kinds, in the case of the former the basic field of operations can be plausibly constructed. Individual numbers consist of definite products of number operations. We may assume, despite our ignorance of the detailed genesis of our number culture, that the natural numbers were few at the beginning and were produced in the attempt to ascertain elementary plurality—how many pots or sheep one had.

Various instances of such processes may have given rise to number systems. Thus the latter may be regarded as products of operations designed to achieve or simply to result in an organization of a scheme for handling various pluralities. It is not unreasonable to suppose that such intimate and fixed plurality systems as anatomical numbers should provide scales or criteria for limits and ranges of counting, tallying or enumerating things.

Numbers as products are excellently illustrated in the cultural evolution of number tools and representations, such as stick notches, sand marks, pebble heaps, various symbols and designations, and the many matching operations with fingers, etc., in counting and calculation. At this point we may be reminded that basically the difference between ordinal and cardinal numbers is in the closer connection of the former with number operations. An ordinal number is intimately set in the number operation, or we may say it is the product *in potentia* or partial product as it arises out of a process of counting or ordering, whereas the cardinal is a product of a full or completed process. Here as everywhere in the mathematical and logical fields we must obviously be on guard against confusing the actual factors in operational fields with descriptive or designative references or symbolization of the factors treated.

If natural numbers constitute products of interbehavior with natural things the other parts of our number system comprise in the same sense products based upon the former products. In other

words, natural numbers result from primary interbehavioral operations and provide the foundation of a hierarchical set of interbehaviors terminating in a further development of the number system.

Because cultural development does not constitute a straight-line evolution and because the actual developmental steps are not available, we cannot hope to trace out the precise details of number evolution. Yet much of this development is satisfactorily demonstrated in the evolution of our technical number system. As the mathematical historian points out, the irrational numbers were inevitably discovered by the Greeks in their mensurational operations. And so the observation that the diagonal segment of a square was incommensurable with a side led to the number $\sqrt{2}$. The assimilation of negative numbers and fractions clearly is the next step in the development of numbers and is based on operations (progressive subtraction and division) upon positive numbers. In this sense we may regard the operations involved as on a more abstract level than the development of irrational numbers.

Imaginary numbers, developed as they were in the formal operations of calculation, that is, as the "false" roots of such equations as $x^2 + 1 = 3$, $x^2 + 3 = 1$, and $x^2 + a = b$, certainly constitute products of a more abstract and remote form of interbehavior than do real numbers. Moreover, the process of their assimilation in the modern number system through the constructive labors of Wessel (1797), Argand (1806), and Gauss (1831) reveals operations with geometric constructions. The history of the development of such numbers shows at every step the complex manipulations necessary to make them workable in various forms of calculation. Additional indications of their constructive character and the nature of their construction are found in the facts (1) that they became established through acceptable interpretation and (2) were later demonstrated as useful in applied mathematics and physics. The further extension of the complex number interpretation to three dimensions (vector analysis, Hamilton's quaternions) and to spaces of n-dimension (Grassmann's *Ausdehnungslehre*) excellently illustrates the hierarchical construction of numbers.

Whether or not one wishes to accept the notion of a single

number system developed on such a hierarchical basis as we have indicated or whether one prefers to consider the various mathematical numbers as more or less isolated developments it is still possible to argue that the evolution of the transfinite numbers proves the general constructional character of number. In a sense, when we put together all these instances of number development, despite the historians' discoveries concerning independent duplications of particular number constructions, we are ourselves exhibiting the constructional process.

Perhaps it is well to be reminded once more that the constructional aspect is not confined to the names, signs or marks of numbers, but to the various number objects, beginning with identities (similarities), discreteness, and relatedness of things as natural stimulus objects, and extending to the products of interactions with those natural objects, such as the elements dealt with by the mathematician. From the psychological standpoint, of course, we are also interested in the reactions located in the same fields as the stimulus objects mentioned.

Those, however, who regard numbers as Platonic entities and who as a consequence really deal with names also resort to interactions. To say that "a number is anything which is the number of some class," and "the number of a class is the class of all those classes that are similar to it" is to come more or less inevitably to the position that "number is a way of bringing together certain collections, namely, those that have a given number of terms."²⁷ Certainly the operation indicated here suggests the product character of number.

The Variable. Both mathematicians and logicians differentiate themselves sharply into those looking upon the variable as a number or a quantity and those defining a variable as a symbol. In the former case the inclination is toward an object, some mathematical element or series of elements sustaining certain relations to other elements. In the latter instance the emphasis is upon the index or representation of the mathematical element or elements. In both cases it is clear, however, that the variable thing or symbol is decidedly isolated from a larger complex involving functions, relations, and other factors which for expository purposes are also treated in isolation. When we consider the locus and range of

²⁷ Russell, Introduction, chap. 2.

variables, their constructional and product characters are obvious.

As to locus of variables it is clear that they are either mathematical elements, that is, factors in mathematical systematization or calculation, or else simply items which are measured and calculated. As such, variables belong to an operational enterprise in which the minimal actions are isolation and attention. As to range, there is immediately suggested a system definitely organized for some observational or calculational process.

In brief, the study of variables implies all sorts of levels of action, the lowest being simply the observation of variations in nature, the magnitude and range of which one organizes into a system. When we consider the historical development of conceptions concerning variables, for example the acceptance and rejection of the notion that variables are things which change, and the later development of the complex notion of class of items with a range, the fact that variables are products is amply demonstrated.

On the interbehavioral basis it is easy to differentiate between various views concerning variables and to formulate descriptions of the activities involved.

When the relational objects interacted with are noncontrived, then the variable and its range exist prior to the interaction, and the construction involves only the descriptive features of both. We do not underrate the importance of the units nor the constructive work involved in their development. Aside from the relevance of the system set up it is not unimportant which is selected as an independent or dependent variable, even though the observed relations fix the value of the variable. But we must consider the great difference between such an interbehavior and one in which the relationship itself is also contrived.

In more formalistic situations when the terms of the relationship involved are themselves constructed, both the variable and its range of values are constructive products in two respects. The objects as well as the descriptions are produced by the interbehaving individual. Not only does he create the entire system in the sense of choosing a relationship, establishing whether the variable series is continuous or discontinuous, determining the unit value of variables, etc., but he also controls the way in which the system is observed. We may be reminded here of Peirce's distinction between mathematics as the practice of reasoning or deduc-

tion and logic as the description of the processes involved (C.P. 2.192).²⁸

For the most part we have been discussing interbehavior with variables on the basis of established mathematical procedures. For us it is important, however, to revert to the question of the original construction of variables, to the development of variables as interbehavioral objects. Here we start from the fact that the whole variable construction is based on contacts with both natural things and the generalizations achieved in symbolic arithmetic or algebra. In this latter connection we observe the development of variable elements such as in obtaining negative roots of numbers, which in practical situations are regarded as meaningless and therefore discarded. Here is a rich vein of natural resources for the development of variables as general interbehavioral products.

The entire interbehavioral idea of products rests on the distinction between products and descriptive references to products. Products imply manipulation. These are performed on various levels. Variables derived from natural events, as in the relationship of pressure and volume in gases, are generated on the basis of actual manipulations, the instrumental handling of gases in cylinders. On a more formal level, as we have indicated, the products are derived from the manipulation of abstract relations in calculation. Such products must be distinguished from descriptions which may vary in the style of recording, as for example $y = f(x)$, $y = F(x)$, and $y = g(x)$, as well as from the theory of or attitude toward the nature of variables.

Functions. Despite the intrinsic interrelatedness of functions with variables, relations, transformations, and other features of mathematical systems and their constructions, they afford us unique opportunities for the investigation of operations and products as well as the relation between both of these and their descriptions and representations.

Frequently it is said that the mathematical use of the term *function* has been adopted in common life.²⁹ It is hardly true that the various uses of the term have all been borrowed from the mathematician after Leibniz first introduced it. When we compare the mathematician's function as a relation of variables, the psy-

²⁸ Cf. Buchler, Charles Peirce's Empiricism, pp. 221-230.

²⁹ Cf. Whitehead, Introduction, p. 145.

chologist's as the action or operation of anatomical structures, and the humanist's as use, operation, service, and purpose, we observe that the referential function of the term in each instance is quite different. Even when the humanist employs the term to refer to a relation between one concrete item and another the differences are apparent. Whether or not one insists that the written or spoken word is the same or different because of its usage it is clear that its significance depends upon the interbehavioral situation in which it occurs.

Of great importance to us is the question whether the actual usages of the term have a common origin, either in mathematics or elsewhere. For our part, as usual we stress the fields in which the term and its referents occur. Accordingly we should insist that the referent of the term *function* is a product of interbehavioral fields quite distinct from the formal operations of mathematics. It would be the least of our evidences that the lack of a sharp separation between mathematical and other enterprises in Leibniz's time might have yielded both a mathematical and non-mathematical derivation of the function product or construction, or even that Leibniz brought the product and term into mathematics from some coordinate domain.

To turn, however, to the mathematical usage, which on the whole is based upon products or symbols for products, we find that though formulations differ they assume accomplished facts. For example, whether a function is said to be a correlation between two variables or a law governing the interdependence of varying quantities, there is little suggestion of how the functions are derived or the processes of their evolution. These factors are of extreme importance, for without them one not only misses the full appreciation of the nature of functions but also is likely to adopt a Platonic attitude toward them.

The formal mathematical function as represented by a graph product must by all means be distinguished from the operational products of observation and selection of interrelated factors in actual events. Courant and Robbins²⁰ compare the interests of the mathematician in computing the values of u from those of x when he has the formula $u = f(x)$, and the physicist who is bent upon

²⁰ What is Mathematics? p. 1-6.

experimentally ascertaining the resistance u of air as it is affected by the velocity v , whether or not he has in hand a formula for computing $u = f(v)$.

We have here simply another illustration of the important distinction between functional systems, the elements of which are natural events, and calculative constructions. In the former case the operations are discoveries or observations, whereas in the latter they represent more comprehensive constructions in the sense that the elements themselves are contrived. An instructive example consists of the alternative relationships set up by Galileo to determine the rate of falling bodies. Before he became satisfied that the velocity a falling body acquired is proportional to the time of the descent he assumed that it was proportional to the distance traversed.²¹

Those who remain unconvinced of the general orientational value to be derived from distinguishing between function processes and products may be moved by the suggestion that possibly an appreciation of the difference may add to one's capacity to develop and make use of formal functions as hypotheses to further investigation. To give adequate consideration to the interbehavioral operations involved adds a flexibility and utility to functions, which upon a basis of Platonic essences or simple, closed systems they lack entirely.

Relations. Since no subject matter or topic in the domain of logic is more fundamental or prominent than that of relations it is extremely important to follow through the processes of constructing and developing relation products. Such a study uncovers the original isolation of relations and at the same time the series of variations in handling them. It is fair to say that the interest in and isolation of relations as compared with things is a comparatively late cultural development. Even such elementary relations as fractions, ratios, and properties have been derived from original interests in things and integers.

Turning to the more technical occupation with relations we see even better the interbehavioral processes involved in dealing with them. It is highly probable that Gauss hit upon the fundamental definition of mathematics when he declared that "Mathematics is

²¹ Mach, *Science*, p. 130f.

concerned with only the enumeration and comparison of relations."²² This statement is an admirable report of work on a specific and unique level. The emphasis is upon a technique with the materials provided and at hand. The work of enumeration begins with the completion of the work of abstraction. Relations even in applied mathematics have previously been isolated from all the factors in the relational situation, from the relata, from the concourse of events which make the relations available and significant, and finally from the workers' interests and manipulations which, even in the case of natural relations, where only observation is the process, certainly have to be taken into account. It is significant to recall that Gauss was an enemy of pure formalism. But in opposing pure formalism the mathematician risks the danger of minimizing the creative emergence of new relations. Here is the profit to be derived from the interbehavioral theory. To restrain the wild flight into pure formality, such as Euler was charged with, and not discourage the freedom so often claimed for the student of relations²³ it is only necessary to keep alive the notion that relations are stimulus objects to interact with, whether such relations are parts of systems of natural events or of systems developed by pyramiding results obtained from an original recording of such relations. What is required is the control provided by the specificity principle, resulting in a differentiation between autistic operations, a concern for natural or at least rigidly bounded relations, and in general an appropriate appreciation of the enterprise at hand.

Traditional logic as differentiated from mathematics has increasingly become involved with relations and concordantly with the same advantages and disadvantages. It is not unlikely that most logicians basically regard relations as their proper field of work.²⁴ This is as true of those who are especially concerned with the relations of propositions as of those who describe logic as the general science of order. Neither logicians nor mathematicians, however, inquire whether relations are exclusive elements or can

²² Bell, *Development*, p. 195.

²³ Recall Cantor's "The essence of mathematics is its freedom" (*Grundlagen*, p. 10).

²⁴ A suggestion not wholly missing the mark is that the mathematician unlike the logician would never ask whether relations can be disengaged from relata, as the logical external relationist would.

sometimes be systems. From an interbehavioral standpoint it is not difficult to determine in each instance which case the worker is handling.

To keep the enterprise in view enables us also to distinguish between relations as things, such as dealt with by the mathematician on the whole, and as descriptions which are favored by logicians. It has been asserted that logicians agree that the proximate subject matter of logic consists of the relations of propositions to one another. These relations are affirmative negative, inclusion exclusion, particular general, etc.³⁵ Insofar as this is true the logician deals with secondary relations if not with descriptions. In the former case we need only point out the specific type of relations dealt with. In the latter, certain necessary emendations must be indicated. First, of course, besides propositions, there are relations also between related stimulus objects, whether or not these relations are properties. Again, it is difficult to reduce all systems to systems of relations. Obviously for specificity logic it is better not to restrict the great variety of system-building situations to the organization of relations. And finally all relations whether they are merely isolated and selected or altogether constructed must be studied on the basis of the proper place they occupy in the enterprises in which they are located.

Series. Because series consist essentially of systems which are comparatively simple in their construction they definitely illustrate the development of products through specific stages. Without distinguishing between technical and nontechnical series and disregarding any exact steps in cultural evolution we may assume that the basic development of series arises from the observation of sequences, recurrences, and periodicities in nature. Bell traces back Fibonacci's famous recurring series which is defined by

$$u_{n+2} = u_{n+1} + u_n, \quad n = 0, 1, \dots, \quad u_0 = 0, \quad u_1 = 1,$$

and which gives the sequence 0, 1, 1, 2, 3, 5, 8, 13, . . . to a problem concerning the progeny of rabbits.³⁶

The first step in the construction of series is to develop a formula which simply records the sequence as observed. This describing or recording formula constitutes a product resulting from observing the sequence. A more advanced step is the construction of the

³⁵ Dewey, *Logic*, p. 1.

³⁶ Bell, *Development*, p. 107.

rule of the sequence. Here the constructive behavior is more apparent because it is somewhat more remote from observed events; still it is after all a simple procedure involving only such operations as elementary addition and subtraction. The product results from a discovery of the relations between the items of the sequence record.

As the history of mathematics indicates, research on the questions concerning convergence, divergence, and the denumerability or nondenumerability of series may well be considered the most intricate development of mathematical analysis. Accordingly such work not only departs widely from the selection and arrangements of natural occurrences, but also involves more elaborate constructional procedures. The products too are much more intricate than the original formulae or laws of the series concerned. Despite all these elaborations and complexities the behavioral procedures as well as the materials worked upon are still discernible.

Classes. One of the most interesting as well as important features of classes is that no matter how directly derived from operations upon events they are definitely products traceable to constructive procedures. Unlike sequences or series they are not in their elementary forms simply products of isolating and recording operations, but unmistakably constructions. No matter how obvious the events from which they are derived they presuppose acts of classifying. Nor is any diligent search required to discover the criteria for classificatory action. Such criteria may be founded upon needs and utilitarian occasions, but just as often upon arbitrary procedures.

Needless to say, the creative character of classes does not put them into competition with: (1) classes as Platonic entities, (2) the names of classes abstracted from the products they designate or (3) the creative processes which produce them. Indeed, the Platonic essences as well as their names are likewise constructed in specific interbehavioral enterprises. We need only consider three levels of class interbehavior.

As the first level we choose the construction of classes based upon natural things. Surely the metal class is a result of the selection of certain natural properties existing prior to and independent of any human being. Upon the basis of natural interbehavior between organisms and objects still more fundamental classes can be established, when chemical and technological properties are

discovered, for instance. Furthermore, by investigating such behavior of particular individuals we are able to determine whether classes are constructed upon a natural-property foundation, by sheer assertion, analogy or some other method.

Secondly, the construction of name classes or class names ordinarily follows the interbehavior resulting in the acquaintance with natural properties and use of things. When we designate or refer to classes, as when we say "Iron is a metal" and "Gold is a metal," we apply the names which we have previously constructed. The words, signs, numbers or other symbols for designating classes are products of interbehavior and cannot be separated from the constructive behavior which produced them. The question whether the designations or names are useful or valuable depends upon some particular activity of persons. On the whole, names which are reminiscent of the activities producing both the classes and the names are more important than those not known to be related to both or either one.

On the whole, the third level of classes, that of the essence type, is constructed from more tenuous materials³⁷ such as relations, order, or mathematical series, sets, and groups, if indeed they are not more autistically developed from words or names.

An excellent example of a tenuous class construction is the type of product dealt with in the logic of classes. As is the case on the two other levels it seems plausible to regard a class as some sort of aggregation or collection. Yet such a view is unsatisfactory to the symbolic or mathematical logician since it would clash with the fact that there is a null class or a class of one member. In the latter class, too, there is the difficulty of avoiding the identification of the one member of a one-member class with the class itself.³⁸ Again, to regard a class as a collection implies that it is based simply upon an extensional rather than an intensional foundation. That is to say, the members would be enumerated by the employment of the term *and*. The objection is that certainly infinite collections would be impossible or impractical.

Classes are then regarded as "all the entities satisfying some propositional function," or "all the entities to which some predi-

³⁷ Tenuous from the standpoint of the conventional substantive way of thinking based upon contacts with crude objects and especially those visually presented. Relations, sets, groups, and even imagined things are from the interbehavioral standpoint equally substitutional stimulus objects.

³⁸ Cf. Russell, Introduction, chap. 17.

cate, or relation, can be truly assigned."³⁹ The characterization of classes on this level appears to satisfy the assumption that classes are products of operations with particular sorts of rather abstruse materials. They are certainly products produced from products. This view is not satisfactory to the exclusive formal logician. To take one example, Russell⁴⁰ began with the idea that classes are objects, reals in a medieval sense. Later he denied that there were any classes. Classes became for him symbolic or linguistic conveniences, not genuine objects.⁴¹ The same position was assumed with respect to cardinal numbers, since they were defined as classes of classes. Of considerable influence upon the change was the consideration that while a class could not be an individual neither is it a universal because of its dependence upon individuals. For though Russell appears regretfully to have veered away from medieval realism⁴² he has hardly departed from the position that: "There is a certain lordliness which the logician should preserve; he must not condescend to derive arguments from the things he sees about him."⁴³

It is hardly to be doubted that classes are products, in fact particular kinds of systems. Surely they do not exist as medieval entities. But neither are they simply words or fictions. From an interbehavioral standpoint they are definitely constructions and they must be differentiated on the basis of the particular enterprises in which they are employed. In a sense foreign to the symbolic logicians classes can be collections of words or symbols, but they may also be collections of things or people. As we have indicated, there are classes in a general sense of natural things. The emphasis, then, we have to place on the enterprise, the particular interbehavior, not upon something regarded as independent in every sense of the activities and interests of the classifier.

Axioms. The view that axioms are postulates or explicit assumptions is now so widely prevalent that their constructive character may be regarded as established. Indeed this is the case when we reject the notion of an axiom as a condition or relation forced upon us by the very nature of things. When intellectual culture was simpler, as in the case of the Greek geometer, he could accept

³⁹ Eaton, *General Logic*, p. 446.

⁴⁰ *Principles*.

⁴¹ Whitehead and Russell, *Principia Mathematica*, vol. 1, p. 75.

⁴² Russell, *Principles*, p. viii.

⁴³ Russell, *Introduction*, p. 192.

axioms as indicating the basic character of things. With the development of complex mathematics and different types of geometry it is no longer possible to regard axioms as anything but accepted assumptions.

Still, the basic constructive character of axioms or postulates requires elaboration. For one thing, it must be stressed that axioms are unique to systems; they are assumptions set up for specific enterprises. Accordingly they are not, as some hold, free constructions which may appear to be arbitrary. On the contrary, they are themselves constructed products set up as guides or criteria for the subsequent construction of a particular kind of system. Because of their function in system construction they are not simply adopted propositions without modification, but must be constantly reconstructed if not found suitable for the development of a projected type of system.

Since all enterprises are unique forms of interbehavior they, of course, differ in the matter of things operated upon and the techniques of operation. When axioms are employed in mathematical enterprises in which more or less fixed relations are worked upon, postulates maintain their original properties unless found to lead to inconsistencies or contradictions. In general there is a limited freedom in postulate construction. For example, while it is notorious that the commutative law can be dispensed with, the associative and distributive laws are much more resistant to change.

Postulates and assumptions in natural-science enterprises are by comparison much more easily altered on the basis of the facts uncovered in investigation. What interferes with necessary modification are the established conventions concerning the events dealt with or the proper forms of technique for interbehaving with them.

Limits. In a number of particulars the construction of limits represents all of the typical features of the development of systems as products of distinct operations. For one thing, the limits construction has definitely evolved against a background of cultural evolution in several stages. Again, in the development of limits we find the influence of various general philosophical attitudes as well as a definite indication that the constructors were thinking in terms of things, actions, processes, and even the language of symbols involved in the construction of the various items.

Although the limits construction is treated as a single product it is, of course, necessary to consider that it is a specific product

derived from particular situations and belongs to particular systems or fields. It may be admitted that as mathematical products limits should be generalized. For example, a theory of limits should cover limits of series and limits of functions, as well as various operational limits. Still, the history of the process of constructing limits indicates the various sources of the construction and the diverse operations by which the product was achieved. For this reason we may consider the development of the construction as more or less unified, though we refer to different time periods and different forms of the products.

As is true for even the most abstract and complicated logical mathematical product there is a direct connection between it and natural things. Accordingly, we may trace the origin of the limits idea to the exhaustion method of Archimedes when the limit was present and the problem was to develop a method of achieving it. All this, of course, proceeded in the form of equating or matching relations. It is significant that the first work on limits was geometric and that among the Greeks the matching of curves and lines appeared to be operations with various definite things.

Probably a distinctive phase in the development of limits is the investigation concerning the presence or absence of a limit, as in the case of convergent and divergent series. The poignancy of this problem arose from the fact that workers thought in terms of ordinary things and assumed that an end point had to be reached. What was lacking was the consideration that the limit might be an abstruse construction or concerned merely with actions such as pronouncing or writing down integers and fractions, etc.

Both the problem and process of limits and limit construction are clarified by considering the place of infinitesimals and infinites in the limits problems. Essentially the construction of actually infinitely small quantities and infinitely small numbers corresponded to a period in which the work of logic and mathematics was concerned with entities which could have an existence in a spiritual domain even if they did not exist among natural objects. The acts of the individual were to a great extent submerged in the face of things of whatever sort, or the acts were regarded as references and representations of such actual things. Hence it was not possible at that time to think in terms of concrete constructions of specific logical systems for particular purposes.

CHAPTER X

THE OPERATIONAL VERSUS THE TRADITIONAL TREATMENT OF LOGICAL PRODUCTS

LINGUISTIC FACTORS AS LOGICAL PRODUCTS

DESPITE the importance of products in logical work, logicians have neglected them. For the conditions leading to such neglect we may undoubtedly look to the traditions of universality and ultimacy.

While considering the role of language in logical evolution (chap. 3) we observed that words and symbols not only serve to blur the distinction between logical operations and products, but also constitute the means whereby universal, transcendent, and ultimate logical systems are established. The interpolation of language and symbol products between logical operations and the materials worked upon makes possible the attitude that logical systems are comprehensive, independent, and final. Descriptions and symbolizations always play a double role. On the one hand, they are indispensable for interacting with things, especially complex ones. On the other, their role can be played to such advantage in handling abstract materials that the descriptions or symbolic systems replace the operations and the materials. Because we can substitute number or symbol couplets or triplets for concrete space units, we are prone to believe that the space we started with has become transformed or that the operation upon that space has been so increased in power as to transcend both itself and the space in question.¹

So important in system building is this hierarchy of materials, products, product materials, etc., as to warrant an investigation of referents and designata products of conventional logical systems. This investigation may be carried out by a study of linguistic factors in classical, universalistic logic. The general problem may be subdivided into three subordinate problems: A. The nature and meaning of words and symbols, how as products they serve as materials for later products, and in general how words and sym-

¹ In this connection see McGilvary's protest (Lorentz transformation) against assimilating time to space by adopting the velocity of light as unit of velocity and writing it "1," or by using $\text{—}1$ in a similar procedure.

bols achieve fixity; B. The nature and operation of sentences and symbol structures in logic; and finally, C. The problem of syllogistic and implication system products.

A. *Words and Symbols as Product Materials*

The Fixity of Words and Symbols. The process by which words and symbols achieve fixity and independence may be well described as simply personal declarations. This is essentially the procedure whereby logicians presume to march on from the immediacy and casualness of psychological events to the permanence of logic. The whole process is illustrated by the remark that "Logic deals with timeless propositions and their timeless logical relations."³ Similarly Dewey asserts that the terms of a proposition do not sustain a temporal relation to one another.⁴

Probably both writers intend to point out that once products have been produced they are independent of the producing processes. But products no less than processes have definite spatio-temporal localization. It is a distinct error so far to separate the products from the processes which engendered them as to deprive the products of all habitation in a spatiotemporal frame of reference.

Look into the history of thought for the many instances in which words or symbols are the means of establishing (really asserting) the existence of permanent and independent entities (designata). While historical logic was more concerned with the creation of substantive characters, present-day logic is mainly interested in relations or in the authority and validity of the creative processes. Thus writers attempt to establish a domain of the unchangeable, permanent, and necessary besides that of the actual and modifiable existing things and events.

This domain of permanence and necessity is erected on the basis of words and symbols organized into hypothetical or universal sentences or statements. Such statements are valid and independent of existing things since they express necessary relations of abstract characters.⁵

The only warrant for relative fixity of language is the fact that

³ Braithwaite, *Relevance*.

⁴ *Logic*, p. 45.

⁵ Cf. Dewey, *Logic*, pp. 45, 117, et passim.

one can refer to various things and events some of which have longer endurance than others. Here, however, the endurance is of things, not of sentences or words; hence the careful asserter will not say anything about timeless things. Again, one may refer to references about anything at all; one can talk about the eternal, the timeless, etc. This is undoubtedly what is meant by connecting logic so closely with language. But how valuable such a logic is can be measured by its variance from the value it is believed to have by its protagonists. As we have said so often, the analysis and value of linguistic performances and products can easily be assessed by comparing them with other sorts of activities.

The Nature of Words and Symbols. The nature of words and the source of their meanings are classic philosophical problems. But even today as in ages past the same difficulties are present and for the identical reason, namely, that words and symbols are approached as simply existing or occurring objects or acts, without considering them as processes in or products of definite interbehavioral situations. An informing example we find in Peirce's⁵ differentiation between Type-words and Token-words. These two classes of words Peirce constructs in an effort to divide off the 15 to 25 "the's" occurring on a printed page from the "the" as part of the English Vocabulary. The twenty printed words as single objects in single places at specific instances he calls Tokens. The English vocabulary word "the" he declares cannot appear on a page or be heard in any voice for the reason that it is not a single thing or single event. Peirce names this a Type-word and asserts that though it is a definitely significant Form it does not exist; it only determines things which exist and it can be used by being embodied in a Token which is an instance and a sign of the Type and of the object the Type signifies.⁶

What a solution for a simple problem! One of the most common elements in English speech and writing does not exist, but this nonexistent Form can determine existing things and also can be embodied in Tokens. Elsewhere⁷ a Type-word or symbol is called a Law or Legisign which acts through a Replica.

Fitting these language things into an interbehavioral reference

⁵ C.P. 4.537.

⁶ C.P. 4.537.

⁷ C.P. 2.244ff.

frame we can treat them naturally and reasonably without resort to ghostly Types or Laws. The word "the" of which there are 20 on a printed page are typed or symbolic transcriptions of an item abstracted from a complex speech action. We know further that the abstracted speech element is a stylistic factor called an article which occurs in English among other reference systems, but not in Chinese among still other languages. Both the written or printed words as well as the spoken words are products of action performed in specific interbehavioral situations.

So important is the language situation for current logic that it is desirable to consider at some length the struggles of logical writers with such language problems as Peirce discussed. For this purpose there is fortunately available a paper by Professor Stebbing⁹ who has been concerned with what words and symbols are and how single speech sounds occurring at different times and places are related. Adopting Peirce's type-token distinction one naturally runs into the problem of how one token-word is related to another and how all token-words are related to their type-word.

Concerning the relation between token-words Professor Stebbing has two answers: (1) similarity and (2) meaning. Similarity, however, makes trouble since token-words as defined in the type-token distinction consist of both shapes and sounds. Shapes and sounds may be more or less alike, but any particular shape is very different from any sound. To resolve this difficulty Professor Stebbing resorts to the view that shape-tokens represent sound-tokens. This is highly questionable when both are regarded as autonomous things. What is implied here, especially when Professor Stebbing declares that sound-tokens are more fundamental, is that both sound and shape-tokens are behavior products and that sounds are historically and culturally prior because more clearly connected with speech behavior. This is only one of a number of instances when Professor Stebbing has been forced to turn to activities. When the question arises how sounds and shapes are connected she answers that they are associated by convention. She means that people used shapes as they used sounds. If we add that both usages go back to referential behavior the action feature comes strongly to the front.

To relate token-words by meaning occasions further difficulty.

⁹ Sounds.

It is clear that unless a sound or shape has a meaning attached to it, it is not a word; yet if we relate sounds or shapes because of meaning we run the risk of calling entirely different sounds or shapes the same word. For example, table, Tisch, and mensa would then be the same word. On the other hand, meaning must be employed as criteria, since the problem arises whether on the basis of shape the word "fair" in "He made a fair bargain" and in "He is a fair man" is the same word, since clearly in "He is a fair man" fair may mean either impartial or blond—that is, has two meanings. Professor Stebbing sees no way of admitting that there is only one word. Obviously the trouble here is starting out with independently existing words. Always on this basis the troublesome question arises how meanings are attached to these words. But when we consider these sentences as merely transcriptions of referential action or as the actions themselves performed in particular situations, such perplexities never occur.

Another difficulty arising from the neglect to consider words as products Professor Stebbing finds in the general connection between words and meanings. Though she once believed tokens—namely, shapes and sounds—had no meaning, she now believes, that unless they did they would not be words. A word, then, is precisely a shape or sound to which meaning is attached. Now strangely enough this process certainly indicates that there is something wrong about the notion of dealing with words (meanings, sounds, and shapes) as unproduced things. Its ineptness is intensified when Professor Stebbing correctly sees that attaching "a meaning to a sound (or shape) is to use the sound (or shape) to refer to something." To see a little further that the meaning depends upon the thing in connection with which it is used would be describing perfectly a type of interbehavior.

How things become stumbling blocks for logicians is again illustrated by the problem of the meanings of type-words. On a thing basis Professor Stebbing says that because type and token-words are of different logical types nothing can be significantly said of one that can be said of the other. And so although type-words are logically constructed out of token-words such words simply *have* meanings and do not have meanings attached to them. Yet she contradicts herself when she says that having meaning must be defined in terms of attaching a meaning, for

otherwise where would a type-word get a meaning? All this confusion can be cleared up by describing a type-word as an action in the original reference situation which can be transcribed or described to yield a series of action products.

The same device can be used to clear up the problem of how various token-words are related to their word types. Professor Stebbing finds difficulty with Peirce's statement that token-words are instances of type-words. She asserts that there are three senses of "an instance of P": (1) "A certain piece of paper colored scarlet may be said to be an instance of red," (2) "Scarlet may be said to be an instance of red," (3) "A proposition is an instance of a propositional function," none of which covers the relation of token and type-words. Her own view certainly amounts to a retreat from the word-thing conception, since she asserts that the type-word is not some *one thing* to which the token is related as its sign or as its copy. It is even fair to say that Professor Stebbing approaches a behavioral view when she declares that a type-word cannot even be mentioned without using a token-word, while its meaning has to be defined by reference to the tokens.

Actually, however, a type-word is a less acceptable something than a sound or shape thing. It is apparently a nonexistent, many-existent entity, though Professor Stebbing rejects the notion that a type-word is a class, even though it is a logical construction. If one must have type-words they can best be described as schema originated to account for the recurrence of certain features of linguistic activities of persons in similar situations with corresponding action products—for example, the word *type-word*.

As troublesome as it is to deal exclusively with word things or products the procedure precipitates an intense crisis when sentences, expressions, and propositions are concerned. This is illustrated by Professor Stebbing's puzzled inability to differentiate between expression sentences and propositions, though such a differentiation is of extreme importance. She exemplifies her point by writing "There is a book on this table," and saying "That is a proposition," "That is an expression," and "That is a sentence." She adds, "It is clear that 'is' has a different meaning in each of the three cases mentioned above. But what exactly these different meanings are I do not know."

Probably because of the conventional preoccupation with sen-

tences Professor Stebbing does not regard her difficulties as very serious, although the sailing is far from smooth. Type-sentences are presumed to be composed of type-words and token-sentences of token-words. That there are type-sentences is obvious because the same sentences may be spoken or written many times—that is, there may be different tokens of the type. But are sentences merely complex things made up of simpler elements? Apparently not, since Professor Stebbing says that the word *composed* has a different meaning in each case. What those different meanings are, of course, she does not say. As to the meaning of a type-sentence, that is determined by the meaning of the words and their arrangements. Since Professor Stebbing deals with sounds as well as shapes it does not escape her that spoken sentences may be new and unique, so they are not types and in direct contradiction with theory should have no meaning.

Inscribed sentences also present a type difficulty. Suppose you start with a single inscribed sentence; then you have a token without a type. But since type implies repeatability, you reproduce another token. Still, how is each of these related to a type? The way out is to deny that the type is a thing. “It is a law, as Peirce calls it, or a rule in accordance with which token-sentences could be so used, as to have meanings attached to them.”⁹

When Professor Stebbing turns to expressions she confronts what she regards an insuperable difficulty, namely, the inapplicability of the type-token distinction. To say “lions are mammals” twice does not give her two expressions, though she can write the same expression twice. In facing the question whether by analogy with “same words” expressions might always be types she accepts the negative, since types are only relative to tokens and she believes there is no such thing as a token-expression. Actually the trouble here is merely that in the case of written materials she is able to overlook the action of which the transcribed words are products, while in the case of expressions the action will not do. The type-token distinction is, of course, a scheme for classifying things and miscarries when used for other purposes. As we have seen, the distinction will not even apply to meanings, which always involve action with respect to things.

Let us consider next the interrelation of sentences, expressions,

⁹ *Sounds*, p. 15.

and propositions. According to Professor Stebbing, a sentence may express a proposition or in other words set forth a proposition. Were she not prevented by the thing bias she might see that an expression is always the name for an action of formulating a proposition which may or may not be involved or translated in sentence form.

When we consider the above written sentence as an action product we can indicate in what sense it represents a sentence, a proposition, and an expression. When the sentence is an act-product we inquire of what sort of act it is a product. The sentence may be: (1) a set of words put together as a model for penmanship, in which case it is an object-or-thing sentence contrived independently of referential language; (2) a grammatical or accepted form rather than a word or phrase; (3) a graphic reference to what is on the table; (4) a word picture or formalized description of a vocal reference; (5) a transcription or symbol for an assertion or reference to an intellectual attitude (though we must now distinguish between such a trivial item and anything that might properly belong to a logical system); and (6) a product or symbol of an asserting or expressing act.

Granted that our terms and descriptions are not standardized or accepted, at the same time only by considering operations and their descriptive products can distinctions be made. By taking interbehavioral situations into account—that is, by inquiring what the person is doing when he performs each action—we can decide whether or not our terms are synonymously used and can hope for a standardization of terms as well as for a possible agreement in usage. One point is certain—namely, words in any form come to us as evolution products from referential linguistic behavior.

When we say we must observe what the person does, we mean literally his interbehavior with things, whether they are immediately present concrete objects, absent objects substituted for, his own actions, concrete (observed) or abstract (constructed) relations, etc. In this way we can determine whether we are dealing with things or with conventional, formal sentences or other sorts of language. We can also judge whether a type-token description applies altogether in a given instance and what variation these terms allow in different situations. We may conclude that the type-token distinction can be contrived only for “words and what

is composed of words," that is to say, autonomous things, and that when meanings, expressions, and sentences are involved we must start with interbehavioral situations and not with shapes and sounds.

The interrelation between sounds, shapes, meanings, sentences, etc., may be illustrated by two series of events beginning with the same interbehavior. In the first the logician develops an attitude or judgment toward logical problems and then sets up the proposition that all valid thinking is syllogistic. This action he formulates vocally in the statement: "Only syllogisms are valid modes of thinking." Then he writes it down, thus producing a written expression or statement. The last as written or as read may be regarded as an autonomous series of shapes.

Functions of Words and Symbols. Words or terms under the grammatical influence of traditional logic not only constitute the elements or materials of sentences or propositions but also provide them with their functions and values. In addition to their general function of symbolizing things and expressing ideas logicians have assumed that words or terms perform specialized functions or carry particular meanings. These specialized meanings are presumed to concern: (1) the extent of symbolization, (2) the profundity or depth of symbolization, as well as (3) the character of the symbolized things. To indicate these functions terms have been classified in various ways, for instance, as singular or general, concrete or abstract, connotative or denotative, and extensive or intensive.

On the whole, the notion that words or terms constitute simple independent objects with autonomous functions is traceable to the logicogrammatical way of thinking, a notion that reveals a lack of appreciation of the relationship between words, things, and logical acts. Not only is there no uniformity in the treatment of these items, but the items themselves are confused, as illustrated by the following four differing views concerning the functions of terms based on different philosophical and psychological backgrounds.

a. *Nominalism.* For the nominalist, of whom we take Mill¹⁰ as the example, terms are names of things and not of ideas or concepts. The logical functions of names are derived from the process of bringing two names together, so that something is affirmed or denied (predicated) of something. As a sensationist who does not

¹⁰ Mill, *System*.

allow any existence except of particular things he cannot distinguish between names and universals, so that all functions of terms are presumed to be direct acts of conversing about either things themselves, their attributes or characteristics, or both at once.

Now on this basis terms as the principal instruments or aids to thought function or operate to stand for things, their properties or attributes, or their number. These terms or names of which Mill offers five pairs may be listed as follows:

- 1a, General terms, capable of being affirmed of an indefinite number of things, e.g., man.
- 1b, Individual or singular terms, capable of being affirmed of only one thing, John.
- 2a, Concrete terms, stand for a thing. Old, the sea, this table, white.
- 2b, Abstract terms, stand for an attribute of a thing, whiteness, humidity, old age.
- 3a, Connotative terms denote a subject and imply, involve or indicate an attribute. Examples, white, long, virtuous, in fact, all concrete general names and some abstract names, for example, fault, equivalent to bad or hurtful quality. Subjects are directly signified, attributes indirectly.
- 3b, Nonconnotative terms signify a subject only, John, London, England, or an attribute only, e.g., whiteness, length, virtue; proper names are nonconnotative.
- 4a, Positive names, man, tree, good.
- 4b, Negative names, not-man, not-tree, not-good.
- 4c, Privative names, equivalent to positive and negative names taken together, e.g., blind.
- 5a, Relative names, given in pairs, when predicated of objects suppose other objects with some or correlative name. Examples, father, son; ruler, subject; like, unlike; equal, unequal; longer, shorter; cause, effect.
- 5b, Nonrelative names (water, gas, tree).¹¹

Mill assumes considerable substantiality for his terms, as evident from his classical limitations of names to categorematic elements such as substantives, adjectives, and certain verbal words. In this respect he differs from more recent symbolic logicians who have assimilated such syncategorematic terms as prepositions, adverbs, and conjunctions. Basically he regards logical terms as sustaining definite relationships to things. For example, though

¹¹ Examples from Jevons, *Elementary Lessons*.

such words as sun or God "may be in fact predicable only of one object, there is nothing in the meaning of the words themselves which implies this."¹² Again he asserts that "names which are positive in form are often negative in reality" and vice versa.

b. *Neo-Classicism*. For the proponents of this type of logic, of whom we take Joseph as an example, a distinct difference exists between words and terms.¹³ Terms are not words standing for things directly as they would if they were names, because logic is concerned with thought about things and not with things themselves. The objects dealt with are objects of thought. Yet words are indispensable in logic, since without signs or language thoughts about things cannot be studied. Logic must deal with properties which are expressed in words or judgments or units of thought.

Terms, then, are of two sorts: (a) terms of thought comprise whatever can be thought of as the subject or predicate of a proposition, whereas (b) terms verbal are words or combinations of words capable of standing as the subject or predicate of a proposition. Now inasmuch as the neoclassicist regards objects of thought as existing equally with things the primary difference between the functions of terms in such a logic, as compared with a nominalistic one, centers about the connection with universals.

A more momentous difference between the terms of these two types of logic concerns the question whether intension can be equated with connotation, while extension can be made equivalent to denotation. In contrast to the nominalist who makes no distinction between intension and connotation or between extension and denotation, since he is concerned only with the connection between names and things, the neoclassicist, who also includes in his system objects of thought, severs these two. Because the neoclassical logician still maintains a core of the ontological content of Aristotelian logic he revolves his intension-extension distinction around the genus-species differentiation. Intension or meaning thus is a function of terms signifying something or everything concerning the subjects which they denote. In sum, what these characteristics are may be ascertained by defining the terms. Now these characteristics indicated may be distributed in a number of forms or species which are regarded as the extension of the term. Connotation, on

¹² Mill, *System*, p. 43.

¹³ Joseph, *Introduction*, chap. 2.

the other hand, consists of a general meaning function in the sense that a term indicates what character a subject possesses to warrant the ascription of the term. Denotation, then, is simply the function of pointing out or marking the subjects of which the terms can be predicated.

c. *Formalism.* Because of the prominent place that words or language have in all conventional logical systems and because of the fairly large agreement of logicians concerning the nature of language we should expect an ample measure of uniformity in the formalistic logician's treatment of the functions of words or symbols. As we have already indicated, however, formalists place a much greater emphasis upon syncategorematic words. To quote Eaton whom we may take as our representative of formalistic logic: "The most important elements of propositions from a logical point of view are expressed by syncategorematic words."¹⁴ It is these words which best represent the logical as against the general referential form of propositions.

One of the most essential differences concerning functions marking off formalistic from other types of logic concerns the intension and extension features of terms. Since formalistic logicians regard universals as real things, they look upon the function of extension and intension as involving the relationship between universals and particulars. By intension of a term is meant its universal character, by extension the spread or coverage of the universals over individuals or instances. Intension concerns the universal taken in abstraction from its instances, while extension refers to universals considered with respect to the variety of its instances. The fundamental idea here is class membership. According to the formalist the members of a class need not be specific individuals, but may be classes or universals, the primary requirement being simply predication or attribution. In other words, the subject must be genuinely an instance of the adjective which appears as predicate. Extension is genuinely exemplified by the terms of the sentence "Men are numerous," since the class of men is a member of the class of numerous classes, just as in the sentence "Socrates is a man," Socrates is a member of the class men. The criterion here is simply to avoid confusing class membership which is a nontransitive relation with class inclusion which is transitive.

¹⁴ General Logic, p. 308.

d. *Instrumentalism*. Since instrumental logic as fostered by Dewey¹⁵ makes logic a mode of inquiry or action, yet still considers language a basic feature, it shows some unique departures from the traditional notion of terms. The instrumentalist objects to: (1) sharply sundering names from terms on the ground that only terms belong to logic—since this would exclude concrete terms and existential propositions, and (2) the notion that a name can only refer to or designate something concrete. The following schema represents, then, the functions of terms assigned by the instrumentalist.

(1) Abstract terms designate conceptions including relations taken without actual application to things—examples, sweetness, redness, solidity, angularity, presence, absence, position, etc.

(2a) Concrete terms designate immediately experienced qualities, e.g. sweet, hard, red, loud. They also comprise (2b) demonstrative words, for example, this, that, now, then, here, there, (2c) common names designating kinds, and adjectives designating characteristics by which kinds are identified and discriminated.

General terms are resolvable into: (3a) universal terms which are conceptual, since they designate operations possible of reference, and (3b) generic and singular terms which are existential in reference.

(4a) Denotative terms refer directly or indirectly to existence, while (4b) connotative terms refer to abstract characters.

(5a) Intensional terms are the descriptive features of denotative or existential terms, whereas (5b) existential terms display the range of applicability of denotative terms.

(6a) Definitional terms constitute the factors of connotative or abstract terms, and (6b) comprehensional terms indicate the scope of the conceptual contents of an abstract universal or connotative term.

Common to all the views concerning the function of terms is that these functions are inherent in the words used and consequently are more or less independently established. It is true that in given situations terms have certain values and functions, but these are always established as results or products of language circumstances. These circumstances belong both to long-time cultural evolutions and to immediate and current situations. In either case

¹⁵ Logic.

the functions of words are clearly conventional usages and reveal the human circumstances and activities to which they owe their development.

The preoccupation of logicians with words and terms certainly suggests the conventional and constructive notion of logic. It is conventional to build logic upon grammatical things and processes, and likewise a matter of convention that the different versions of the nature and function of terms correspond to different logical notions. To illustrate, for the extreme nominalist terms are only names of things, because his philosophy only comprises psychic states and names corresponding to these states regarded as elements or complex things. Similarly, the realist who harbors universals among his realities must have words and functions to deal with them as well as with things and qualities.

Although logicians of all persuasions write as though isolated words possessed grammatical and logical functions, really none of them fails to say that actually terms do not function except in connection with other terms either in the form of sentences, statements, or formal implication structures. In general, the tradition prevails that terms have no meaning, value or function except as they form parts of larger units—namely, sentences. This view is concurred in by such widely differing logicians as Bosanquet of the Hegelian tradition, Russell of the logistic tradition, and Dewey as an instrumentalist. In each case, of course, the reasons are somewhat different. With Russell, for instance, the emphasis of mathematical relations implies connectivity between the terms. For Dewey terms are logically conditioned by sentences arranged to yield judgments.

To make the functions of terms dependent upon the sentences in which they are found suggests the interests and work basic to setting up the sentences. While stressing sentences we not only depart from the autonomous appearance of words but also see that the meaning of terms, the question of their denotation or connotation are all derived from the operations performed by logicians in particular kinds of system building. The types of terms used, whether concrete or abstract, extensional or intensional, the question whether they are descriptive words or symbolic representations depend upon the activity engaged in by the individual, be it logical or some other kind. In other words, terms are products of system-building enterprises.

Not that it is intended to deny that the realm of logic includes systems of words according to criteria of usage. Obviously grammars and dictionaries constitute such systems. To allow grammars, dictionaries, and other organizations of words or terms a place among logical systems at the same time acknowledges the system-building character of logic and disavows the view that the conventional logics of terms constitute significant systems of thought and reasoning. Moreover, whether or not we accept conventional linguistic systems as satisfactory or exclusive items of the logical domain we can only regard words or terms as factors in referential and symbolic actions and as products of such actions.

B. *Sentences as Materials and Products of Logic*

In consonance with the prevalent linguistic and formal views of logic, sentences are defined as complex compounds made up of terms. In other words, like terms, sentences are things, materials ready at hand, or products, without due regard to the various actual operations of specific system construction.

As may be expected, of course, the assumption that sentences constitute organizations of terms or symbols assumes various guises in the logical treatises of writers rooted in contrasting philosophical backgrounds. Certainly logicians do not agree upon the essential nature and function of sentences. In some instances sentences are simply taken as the raw materials for system organization; in others they are regarded as expressions of judgments, whereas in still other cases they are looked upon as assertions culminating from generalized patterns of inquiry. Again, some logicians identify sentences and propositions, while others do not. In the latter case we rarely if ever find a satisfactory description of either sentences or propositions or the relations between them. To illustrate, we consider briefly some representative treatments of sentences.

Sentences as Materials of Inquiry. An important view concerning sentences is Dewey's theory that they are tools for carrying on inquiry. In this connection Dewey makes a distinction between generic and universal propositions; the former are defined as having existential import, that is, they refer to spatiotemporal connections between existents. Their properties, validity, usefulness, etc., depend upon actual happenings. Universal propositions, on

the other hand, express necessary relations between characters or possibilities, and are valid without regard to existent things. In other words, they are autonomous and purely relational. Consequently, universal sentences may hardly be considered as genuine constructions for specific system-building purposes.

The instrumental doctrine embraces two incompatible views, namely, that logic is exclusively concerned with formal materials, that is, sentences or statements, and at the same time is uniquely an inquiry enterprise. This doctrine ignores the fact that propositions or sentences do not necessarily constitute the subject matter of logic. In other words, logic is not exclusively concerned with sentences which symbolize or record results of either actions or observed events. Furthermore, logical systems may be constructed out of sentences, which constitute formal organizations or systems of terms, or symbols unrelated to inquiry, unless of course, one defines inquiry as the occupation with such formal objects and systems.

At this point we are confronted with the relationship between logical forms and functions. Are not forms, which the instrumentalist envisages as sentences or statements, also tools or products which can have many functions independent of any logical enterprise? Notice, however, that when propositions or sentences are taken to be an exclusive concern of logic, and logic is regarded as exclusively the theory of inquiry, they assume a thing character hardly reconcilable with actual inquiry. Propositions as sentences can, of course, be employed as instruments for actual investigative interbehavior, but such actual inquiry or investigation has to do with events and not only with sentences. We must not confuse the logical process of building up systems of propositions *about* inquiry with *inquiry* itself. Moreover, we should not assume that logical propositions are exclusively concerned with inquiry, in the sense of resolving indeterminate situations, since they are also materials for building mathematical and other systems.

When we inquire concerning the functions of propositions as logical forms we run into the conventional treatment of the relationship between forms and their meanings. What are such meanings, and how do sentences attain them, are questions frequently asked. From the standpoint of the specificity principle the mean-

ings of sentences consist entirely in their uses in logical enterprises, no matter how we define logic. Sentences may be employed in carrying on various investigative enterprises, but they also constitute the elements of syntactical systems. These are only two uses of language as instruments, out of a class containing many members.¹⁶

Sentences as Judgments. Logicians who are thoroughly ensconced in the grammatical tradition assert that a sentence or proposition is the same thing as a judgment,¹⁷ though this assertion alternates to read that propositions express judgments.¹⁸ The basis for identifying judgments and sentences called propositions is revealed in such statements as "every judgment makes an assertion, which must be either true or false." Again, the grammatical tradition fosters this identification by the principle that to judge logically is to affirm or deny a predicate of a subject.

Aside from the question whether logic consists of judgments we suggest that: (1) it is improper to confuse things such as sentences or propositions with acts such as judging, and (2) both judgments and the sentences which refer to or symbolize the former are products of definite and complex actions.

Both of these points may be illustrated by differentiating between sentences and other things as follows:

(1) Vocal acts called sentences which constitute particular references to some thing, act, event or situation.

(2) Conventional marks resulting from the transcription or representation of the vocal reference acts referred to in (1).

(3) Formal organization of the transcription in (2) according to various grammatical rules.

Now, so far, the acts and things conventionally called sentences, and which we have listed, have no connection with judgments. Notice, however, that the reference in (1) may concern: (a) an act of judging, (b) a verbal formulation of the result of judging, or either of the representations of this reference as in (2) or (3).

When language acts or things concern judging we may draw up

¹⁶ In this connection Hofstadter (Logical form) has suggested an interesting distinction between logical forms and systemic functions with various relations between them.

¹⁷ Joseph, Introduction, chap. 7.

¹⁸ Ibid. pp. 160, 171.

a series of objects and events as follows:

- (1) Judging behavior in a judging situation
- (2) A judgment as a product of the judging act.
- (3) A verbal formulation of (2).
- (4) The acts of composing and printing (3).

In addition to the language factors indicated we must take account of language acts and things which may be necessary for achieving a judgment. Throughout our analysis and survey it is clear that by judging and speaking we mean definite interactions of persons with things. For the most part the stress of a connection between words and judgments or thought constitutes an attempt to anchor psychic processes in the more substantial ground of word things or actions.

The Production and Product Character of Sentences. Logical studies make plain that sentences are not organizations of given terms, but rather products of particular constructional procedures. Logical sentences may be characterized as intellectual products constructed for systematically handling objects, events, and relations.¹⁹ Essentially they are intellectual or conceptual organizations of things and relations for some purpose. Depending upon the logical situations for which they are produced the sentences may be either (1) tools for referring to or designating, representing, or symbolizing things, or (2) the referred-to or designated things themselves. In the former case, sentences may operate as statements of hypotheses or postulates, in other words, tools for organizing a system, whereas in the latter the system is built of sentences as the prefabricated materials.

As in the case of terms, when logicians consider sentences as things immediately given or found ready to hand instead of products, they ascribe varying characteristics to them. This is done by defining sentences on the basis of various prior assumptions. These variant characteristics are regarded as inherently existing in the sentences instead of being differences in function resulting from actions that produce them. To illustrate this point we consider three sentential variants.

a. *Sentences as Syntactic Structures.* Sentences like "All centaurs are Greek" or "All centaurs are x" typify what are called syntactic structures. They are described as symbol structures or

¹⁹ Cf. chap. 8.

empty forms which can be manipulated in various ways by techniques of formation and transformation. Our examples are obviously produced on the pattern of "All men are mortal." Once formed such symbol structures can be subjected to such transformations as "No centaurs are non-Greeks." Such analogical constructions must be derived from some sort of transcriptional pattern or model, and their use or value directed toward purely metaphysical description or representation. Moreover, the systems which they form or into which they fit are rigidly closed and complete. In fact, it is proper to say that not only is a model the sole basis for constructing such propositions, but also the preconstructed system dominates every operation that can be performed upon the component members. Since the general framework of such systems, whether single sentences or sets, consists of a grammatical universe of discourse, such elements or sets are closely integrated with the words, signs, or symbols from which they are constructed.

Apparently syntactic sentences can be best employed in mathematical work in which abstract relations constitute the materials dealt with. In such situations once rules in the sense of a system are developed the various relations can be symbolized by formal structures. Aside from the literal representation of relations, for example, no elaborate mathematical system could be developed without such tools. In other words, the manipulation of such relations including their modification, coordination, subordination, etc., can only be performed by the transformation of organizations of structures constituting such sentences. Notice, however, that whereas we are dealing with abstract materials as stimulus objects, interbehaviorally they are very definite and concrete. It is not going too far to regard such interbehaviors or operations as are indicated by symbols as definite referents of verbal or symbolic propositions.

(1) *Sentential Functions*. Among the best illustrations of syntactic or formal sentences are the sentential functions "a is b," "x is rich," etc. Unfortunately sentential functions are regarded simply as syntactic structures, whereas it is clear that they are abstract tools necessary for certain sorts of activities, and, as most logicians indicate, are instrumental for setting up potential sentences which can be actualized whenever necessary. Here once more arises the question whether there really are empty forms or meaningless symbols. Only by forgetting the system with which one works and

the operations one performs can one describe mathematical symbols as simple counters.²⁰

(2) *Transpositional Inference*. What classical logicians call propositions immediately inferred from others constitute another type of syntactic structure. The emphasis is upon the intrinsic interrelationship between organizations of terms rather than any kind of interrelationship between sentences and implied content or things designated or referred to. Transpositional inference comprises essentially a manipulation of sentences on the basis of their so-called quantity and quality. Actually we have here a set of alternative modes of reference. Historically the criterion for the manipulations and the actual transformative constructions have been systematically named conversion, obversion, contraposition, and inversion. Their variants belong to a system showing how a linguistic status quo may be maintained, while the system organized into the square of opposing sentences constitutes a schema of interrelationships between varying sentences.

Is it not plain that whereas the quality, quantity, truth, and falsity of statements may be regarded as inhering in their structure, actually their independence as elements is derived from setting them up in that manner on the basis of conventional referential criteria or on that of even more explicit system construction?

(3) *Sentences as Referential*. Sentences characterized as referential are set up to refer to or symbolize something. The emphasis here is upon things, events, and relations, though the reference to them is regarded as firmly established either because the referents or designata are stable and tightly integrated or because the views or attitudes toward them are well established and fixed. On the whole, sentences of this sort can be set up with words as well as more abstract signs, though this depends upon the type of referents involved. On the level of abstract referents we may distinguish between referential or semantic and syntactic sentences by comparing sentential functions before and after values have been ascribed to their variables. An essential feature of these sentences is that they are produced to mediate between the individual, who uses them as instruments for dealing with things, and things themselves. Sentences or symbol structures may serve to register and record actions or events; they may be employed to organize and

²⁰ Lewis and Langford, *Symbolic Logic*, p. 127.

systematize things or acts by separating and combining them on the basis of such criteria as true or false or super- and sub-ordination, etc. Sentence products of this type may be regarded as basic to other types, since their systematizing functions are more versatile and comprehensive and in consequence presupposed by other formulations.

Because of their referential character sentences of this class have a large range. The number of systems in which they can play a part is limited only by the interests of the logician or system constructor. To illustrate this range we need only differentiate between sentences for thing organization and for the systematization of relations. Risking the displeasure of the conventional logician we propose the organization of a system of proverbs, adages, maxims, etc., as illustrating thing-organization sentences. We do not regard as objectionable the fact that all the sentences are presumed to be true. No doubt systems of postulational sentences would be more acceptable to logicians as illustrations of this type of sentence. On the whole, however, the terms of such sentences are as effectively constructed in connection with words as with more abstract signs.

Sentences referring to relations are, of course, best illustrated by scientific formulae such as $S = \frac{1}{2}gt^2$, $P_1V_1 = P_2V_2$, $T = 2\pi\sqrt{\frac{l}{g}}$,

$M = \frac{M_0}{\sqrt{1 - (\frac{v}{c})^2}}$, etc., and mathematical identities, for example, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$, $(a + b)^2 = a^2 + 2ab + b^2$, $A^n \cdot A^m = A^{m+n}$, etc.

(4) *Sentences as Calculative.* For the most part we shall take as illustrative of calculative sentences such structures as are produced for carrying on calculative operations. The simplest illustrations are the hypothetical sentences of the entrepreneur who wants to calculate the results of a financial operation. Such sentences are, of course, constructed on the basis of known or assumed data concerning things and conditions in economic situations. A second field of sentence construction involves the production of systems of terms in order to calculate the results of observed events. An example is setting up equations of conditions for solving problems within situations affording certainties when conditions are known, or probabilities when they are not determined. These sentences we call calculative or algebraic because they are primarily manipulative

or operational, designed to achieve results not already given or known. On the other hand, sentences may be constructed for the purpose simply of ascertaining or exhibiting the variability within a given system. The rules and operations are set up to keep the system intact. In the calculative sentences the rules and operations are established techniques and are applicable to situations in which the final results or systems are not established prior to the employment of the equations.

In each of the three cases indicated the sentences and their manipulation constitute tools for interbehaving with particular kinds of stimulus objects in specified situations. A similar statement may be made concerning the series of syntactic, referential, and calculative sentences. The emphasis in each instance is, of course, upon the constructive work in the production of the sentence. We must note too that such constructive interbehavior is not confined to system building or logic. Furthermore, when such statements are produced for systems they do not circumscribe the domains of logic, whether or not logic is regarded as an organon or a manifestation of reason.

Sentences and Propositions. So far in this chapter we have not attempted to distinguish between sentences and propositions. But such a distinction is imperative not only because this differentiation has become a live topic among logicians but also because from an interbehavioral standpoint it is necessary to separate entirely different things.

Though we cannot pause to consider the various problems which have concerned logicians relative to the separation or identification of propositions and sentences²¹ we must indicate that whereas sentences are linguistic products, propositions constitute products of interbehavior not only in system-building procedures but also in scientific and everyday enterprises.

Probably most writers on logic differentiate propositions and sentences on the ground that sentences express propositions. This is both an acceptable and an unacceptable view. It is acceptable in the sense that we may distinguish between a proposition as a constructed referent and the sentence embodying it or symbolizing or referring to it. For example, the proposition that the area of no square can be exactly equal to that of a circle is separated from the

²¹ Cf. Kantor, *Interbehavioral Analysis*.

discussion of that mathematical product. This discrimination is based upon the observation of two interbehavioral fields. On the one hand, there is a complex interbehavior with points, lines, surfaces, and relations, as the units of geometric figures (squares, circles) within the domain of Euclidean geometry. On the other, there are many fields, including the linguistic, in which the propositional product is the thing interacted with. Granted that the work of constructing this proposition involves, besides thinking and imagination action, also the use of pencil and paper, and that therefore words and symbols constitute processes and tools with which we construct a proposition, there is still no occasion to overlook the differences in the factors involved. From an analytical standpoint the use of such tools is accidental and not absolutely essential for the production of such a product. On the other hand, once we have developed this product we refer to it and record it. In both these instances it is metaphorically correct to say that the linguistic activities or the symbols recording them constitute expressions of the proposition.

Those who wish to ignore the work of creating the proposition and deal only with the reference to or recording of it as a product need not distinguish between propositions and sentences. It is important, however, to be clear concerning the two situations involved, in other words, to recognize two operational fields. In the above exposition and especially while discussing calculative sentences it is apparent that the term *sentence* refers to genuine propositions as well as to recording sentences. The writer has proposed²² the transcriptive scheme of writing p(energy radiation is discontinuous and is measurable in definite quantity) for propositions (in physics) as over against the use of quotation marks or straight text for sentences.

Propositions constitute products of human action in precisely the same sense as a musical composition, a painting or drawing or a table. In each case the individual operates upon specific materials with a more or less definitely organized plan. While in making a table the manipulations involved are the most obvious, the performances in the other cases mentioned are not one whit different in principle. Since this is so, we need not hesitate to include the three kinds of products in the same class.

²² Interbehavioral Analysis, p. 334.

As we have indicated, the materials interacted with in the production of propositions include every type and variety of object. They range from the most crassly manipulable objects to imagined (created) things or the materials of scientific operations. Whenever we have an authentic field of human interbehavior we have a potential location for the process of proposition production.

C. Complex Systems as Products

That logical systems in the sense of the various historical organons and modern symbolic structures are products is manifested from the psychanthropic conditions under which they evolve. Howsoever much the mathematical logician may regard himself as a simple discoverer and manipulator of absolute relations instead of a constructor he does not escape the annals and records of his own discipline. In this section, however, we are not directly concerned with such large-scale systems, but rather with subordinate unit products making up the larger comprehensive system.

All comprehensive logics as ultimate system products constitute nests of subordinate units of which some are themselves systems of smaller scope. Lower order symbolic or mathematical systems comprise at least two propositions or subsystems, which are regarded as interrelated by some sort of necessary or binding principle, for instance, implication or if-then relations. Syllogistic systems consist of three subunits in intimate relationship; while sorite systems involve four or more smaller systems or products tightly knit together.

Strata of Complex System Construction. The construction of complex systems proceeds in steps or upon different levels or strata. A basic question at every level is: Of what degree of prefabrication are the present materials? Do the system builders start with objects and their relations, as when a system of natural numbers is culturally developed, or do they build upon this number system as a basis in diverse ways? The next steps may be calculated systems on a binary, decimal, sexagesimal, or some other plan. How far we can go in this process of regarding products as basic materials is evidenced by the denial of the evolution of complex number systems from more elementary systems. Strangely enough the basis for such a denial is simply overlooking the cultural evolution involved, with its transmission from generation to generation of

products achieved by earlier generations. In the case of more specialized logical systems the constructive procedures are more evident. In such instances too we can observe the amount of originality necessary for productive processes. These differences come into view by considering various types of stratification.

a. *Local and Temporal Stratification.* System construction may proceed on the basis of a local enterprise in which the logician organizes materials on a new plan and basis without obvious historical antecedents. An example is Frege's development of a logical system of arithmetic. Although here there is undoubtedly an historical forecast in the growing recognition of the systematization of geometry, we must grant Frege's originality in attempting a comparable procedure in what may be regarded as a new field with new materials. Such system building contains sufficient novel elements to characterize a fresh procedure and enterprise.

By contrast the temporal stratification implies a closer connection between the different strata. When a system builder follows definitely an earlier selection of material, aim, and technique, the strata he lays down definitely have a basis in former work of the same nature. In general all of Frege's successors in the enterprise of logicizing arithmetic and mathematics illustrate successive or temporal stratification. In other words, they build on products developed from prior basic materials.

b. *Individual and Cultural Stratification.* In Chapter 2 it was pointed out that in classic logic, even as early as Aristotle, formulation was based on a series of successive cultural establishments of particular institutions. This hierarchical evolution of products constitutes a definite stratification of materials and products. On the other hand, whatever achievements individuals attain, either in the modification of earlier products or the application of systemic procedures to new materials, comprise personal contributions. In either case the stratification of enterprise and products leads to specific comprehensive logics.

c. *Material and Description Stratification.* From the standpoint of the continuum which includes the organization of systems of things and the most elaborate formal systems we may indicate another stratification. At the bottom are systems contrived of objects of any sort that constitute organizational material for such systematizing enterprises. The naming and describing of such or-

ganization products form another stratum. This second level constitutes a linguistic system or product the materials of which are statements or propositions. On this stratum as a basis can be laid down layer upon layer of system-building products as successive materials for more and more elaborate descriptive or formal products. Despite the conventional views to the contrary the most formal mathematical logic undoubtedly consists of an elaborate stratification of materials and products beginning with actual identification and organization of relations. Upon this level we can point to all sorts of products such as logical primitives and constants, classes, relations, and every variety of interrelation of propositions. Taking the various stratifications as they are connected with each other we find that they comprise system fabrications beginning with the relating of elements and proceeding to the relating of the relations in a progressive construction of intricate systems.

Varying Functions of System Products. Not only are complex logical systems specialized products with respect to materials, cultural auspices, and mode of construction, but they also display the use or purpose for which they are designed. To illustrate this point we suggest the following three-fold division of complex, conventional system products: (1) technique systems developed for achieving various calculation operations, for example, algorithms and calculi, (2) classic syllogistic and implicational systems constructed to demonstrate, in the sense of exhibiting, necessary relations, and (3) basically descriptive systems operating for the most part to designate and exhibit relations and processes. Examples here are doctrinal functions and particular formalizations of mathematical systems.

a. *Algorithms and Calculi.* Algorithms, such as (a) Euclid's technique for finding common factors of any two numbers, (b) the process of extracting square roots, (c) the technique for expansion of series, (d) the method of developing continued fractions, and (e) the Heronian algorithm for obtaining a sequence of rational numbers approaching the limit of \sqrt{n} , all illustrate simple or complex tools constructed to carry on certain calculative operations.

Similarly, every sort of calculus constitutes a more or less complicated tool system for performing manipulative actions of some sort or for organizing relations. The differences between calculi

are determined, of course, by the different ends they are to subserve. In the case of both types of systems the constructive action involved consists in building upward or outward on the basis of prior operations upon included elements. For example, Euclid's algorithm is based upon the achieved system that a divided by b yields a quotient and a remainder as indicated in the following symbolic statement:

$$a = q \cdot b + r_1, \text{ when } r_1 < b$$

To stress the fact of construction is to observe that the stratification of factors implies bringing together subprocesses developed for the immediate occasion or organized by previous cultural accumulation.

Calculi and algorithms may themselves be differentiated on the ground that algorithms generally subserve a more specific sort of end. In a genuine sense the algorithm is a tool product or tool machine for dealing with mathematical relations themselves, whereas calculi are designed to go beyond the exclusive manipulation of mathematical relations for the purpose of organizing objects of various sorts.

Operational systems or products of operations may be generalized and described by formulae or equational statements. It is improper, however, to limit them to this manner of description. Various organizations of statements directing the assembling or operation of a machine likewise may be regarded as algorithms. From an operational point of view, the fact that things, in the one case, and numbers or relations, in the other, are handled is an insignificant difference.

b. *Syllogisms and Implications.* Whether one believes that syllogisms originated as complete systems designed as such or evolved as an elaborated product based upon a series of component prefabrications,²⁸ each view presupposes the underlying constructional activities. Certainly an analysis of syllogisms uncovers a series of constructs—term, minor, middle, premise, conclusion, relation, and necessary relation. All these may be regarded as unit features which are integrated into a large complex structure. Conceding that the syllogism is a unit and the subunits are all products of intellectual manufacturing processes it matters little whether

²⁸ See chap. 3.

one differentiates the larger from the smaller units. The important thing is that an object, call it tool or instrument, has been produced for some specific purpose. Syllogisms in all their moods and figures displaying their qualitative and quantitative interrelations of subjects and predicates must be taken as such products and for the accomplishment of such purposes as they can handle. To be avoided is the notion that a nonanthropomorphic method or a thing has been discovered to achieve logical results.

Every use or function of syllogistic systems clearly illustrates their constructional background no matter how complex the systems may be. Whether a syllogism operates simply to classify things by subsuming them under others or by inevitably relating propositions, in each case the syllogistic system must be designed for the purpose. The argument that the deductive function of syllogisms yields new information does not militate against this view. Assuming it is true that the syllogism, "No man lighter than 100 pounds is in the army": "John is in the army": "Hence John weighs at least 100 pounds" yields information concerning John's weight, the novelty of the information is no more than the location of an item in a previously constructed system. To know anything about John's actual weight a weighing operation is required.

The various discussions concerning the relationship between deductive and inductive processes likewise emphasize the product character of syllogistic systems. To set up the major premise of our illustrative syllogism it is necessary to achieve a product based upon ascertaining proper weights of soldiers or at least to discover what rules have been set up by army authorities. That inductive processes are basic to deductive ones or at least are fundamentally interrelated with them points to the constructive processes which yield syllogisms as products. To deny that the nature and uses of syllogisms are the *products* of purpose or design to achieve classification or various relations is simply to overlook their constructional history.

Similar observations concerning the product character of implied relations and necessary statements are inevitably made when we study such logical systems. The invariability of relations and the articulation of propositions in implication systems both reveal the work of setting up the rules and methods by which the elements are interrelated. For example, in material implication it is said

that by definition propositions are related; so that "p true and q false is false" is established. Definition here means construction, whether or not such construction is justified. Actually there is no firmer basis for the necessity or interrelation of statements than various sorts of constructive procedures either of a personal or cultural sort.²⁴

The argument of the preceding paragraph is strictly reinforced by the following points: first, the distinction between logical and investigative facts, and secondly, the relationship between the two. As to the former, all logical materials and results are, of course, constructions. As to the second point, it is true that descriptions and propositions are developed on the basis of contacts with natural and contrived things. Necessity, then, may arise from the nature of things dealt with. In both cases, that is, whether necessary and implied propositions have necessary factors, the necessity arises from interactions and not from any *a priori* sources, either considered as the nature of objective relations or the necessary operations of the mind.

c. *Mathematical Systems and Proofs.* In no field of human endeavor can better examples of constructive procedures and products be found than in pure mathematics. This is true because pure mathematics is an essentially constructive discipline that begins with certain relations and proceeds to work out all sorts of implications. The emphasis centers upon the work of organization and transformation. To illustrate the creative and product factors in mathematics we choose three items—namely, mathematical systems, proofs, and doctrinal functions.

(1) *Mathematical Systems.* One of the distinct achievements of modern mathematical thinking is a growing emphasis upon the constructive procedures, whether exhibiting basic relations or indicating transformations or invariances in those relations. The evaluation of mathematics has led to the progressive realization of the process resulting in the organization of systems of relations. This realization has itself resulted in an intellectual product which we may call the basic understanding of the material, the work, and the products of relation ordering or system building.

As a capstone product in mathematical construction we have the mathematical system described as an organization of axioms or

²⁴ On this point see Black, *Conventionalism*; also *Analysis*.

postulates and a set of theorems fundamentally interrelated with these axioms to form a unified product. Such systems yield particular products for algebra, geometry or perhaps a more generalized mathematical system. It is only necessary here to refer to such final products of mathematical system building as Huntington's.²⁸

(2) *Mathematical Proofs.* Mathematical proofs of the indirect or *reductio ad absurdum* variety and the construction of denumeration models such as the Cantor diagonals admirably exemplify not only the construction but also the function of complex mathematical products. Each instance plainly shows the projection of a design and the perfection of a structure or system to carry it out.

For our purposes the proofs involved in infinity problems are by far the most important, as they exhibit the essential creativity basic to and characteristic of any product. To consider first Galileo's proof that one infinite quantity can not be greater, less than, or equal to another, we find that it consists of constructing a matching scheme. Pointing out first that in the natural sequences there are more numbers than squares—in fact, among the first hundred numbers there are only ten squares, that is, one tenth the numbers are squares, among the first ten thousand numbers only one hundredth, and among the first million only one thousandth part are squares—he proceeds to the paradox that since every number is a unit of a square and every square is a product of roots, when we consider the totality or infinity of all numbers there are really as many squares as numbers.²⁹

The basis for this construction, of course, is the recurrent operation of squaring the numbers of the natural sequences following the lead suggested by the square-root relation of that sequence. Whereas the creative process is based upon contact with legitimately established relations, its unique feature is the verbal transcendence of those relations. It is interesting to note that Galileo regarded infinity problems as difficulties arising when men with their finite minds attempted to deal with the infinite by assigning to it properties belonging to the finite and limited. Yet such handling of the infinite was necessary, since only by taking an infinite

²⁸ Fundamental Propositions, and Continuum.

²⁹ Galileo, *Dialogues*, p. 31f.

number of indivisible or dimensionless points could one build up a line.

Galileo's proof that there are no differences between infinite quantities certainly provides a striking illustration of a proof construction, but it is greatly surpassed by Cantor's proof of the opposite proposition.²⁷ By an exemplification of his notion that the essence of mathematics is freedom Cantor proceeded to prove what Bolzano had already suggested, that in the domain of the infinite the part has the power of the whole, and further that there are different powers in the various partitions of the infinite. Thus the nondenumerable infinity of the continuum or series of real numbers has a greater cardinal number than the denumerable infinity of rational numbers. The result of all this is the development of a new number system, the transfinite cardinals and ordinals as follows:

$$1, 2, 3, 4, \dots n; \dots \aleph_0, \aleph_0 + 1, \aleph_0 + 2, \dots \aleph_1, \dots \aleph_2, \dots$$

$$1, 2, 3, 4, \dots, \omega, \omega + 1, \omega + 2, \dots \omega + \omega = \omega \cdot 2, \dots, \omega \cdot \omega = \omega^2, \dots$$

The basic features of the proof system which Cantor has created are, of course, the matching processes by which he can differentiate and enumerate the different powers of infinite aggregates. The fundamental steps in this proof construction may be characterized as the progression from the creation of dimensionless points to the creation by transformation of sets or collections from numbers or relations represented by number signs.

With these creations in hand Cantor can prove that two unequal line segments contain the same number of points; any line segment no matter how small contains as many points as an infinite straight line; a segment contains as many points as there are in an entire plane or in the whole of three dimensional space or even in the whole of an n dimensional space; or finally in a space of a denumerable infinite number of dimensions.

In addition to the proof based on the one-one denumeration and the auxiliary diagonal technique there is also the analogy of mathematical operations derived from finite numbers to show their properties of addition, multiplication, and potentiation, as well as

²⁷ Contributions.

their mode of combination among themselves and the finite numbers, as in the following illustrations.

$$\aleph_0 + \aleph_0 = \aleph_0$$

$$\aleph_0 + n = \aleph_0, n = \text{any finite number}$$

$$\aleph_0 = \aleph_0^2 = \aleph_0^3, \text{ and in general } \aleph_0 = \aleph_0^n$$

If on the face of such a proof construction is stamped the impress of error and dubiety it might be ascribed to the fact that the construction is too elaborate for the materials at hand. The result must be, if not fog, as Weyl puts it, which will not support anybody who earnestly desires to stand upon it, at least a grasp which exceeds any possible reach.²⁸ Cantorians forget the mathematician who is doing this creative work and the materials with which he starts. It is true that there are processes and operations which are recurrent and may be continued for a long time, but there is always a limit. There is a difference, then, between constructive work of a mathematician and the creativity which finally goes no further than the series of verbal or symbolic tools out of which the great structures are made. Inevitably such creativity is found to be based upon an unwitting assumption that what is symbolically constructed contains something essential which it represents.

The opposition which such creativity has met at the hands of Kroecker, Brouwer, and Weyl, who want to base mathematical construction on such palpable materials as the natural numbers, was only to be expected. Such proof constructions as indulged in by the Cantorians certainly illustrate the piling up of constructions as material to the point at which the process of construction itself comes under suspicion.

Mathematical writers²⁹ have pointed out the similarity between mathematical discussions of infinity and theological arguments for the existence of supernatural things. Indeed there is considerable similarity in the psychological processes involved. It must not be overlooked, nevertheless, that the mathematicians are simply extrapolating from systems of relationships and can actually justify

- Ueber die neue Grundlagenkrise, p. 20.
- Bell, Development. Dantzig, Number.

their creations by pragmatic applications in calculation and interpretation of mathematical processes.

(3) *Doctrinal functions.* Another telling illustration of the function of system products is afforded by the doctrinal function proposed by Keyser.³⁰ The doctrinal function may be described as a generalized system designed to exhibit or describe any mathematical system-building procedure. Starting with the sentential function which offers a basis for developing sentences Keyser proposes a set of blanks to be filled in with particular postulates and theorems. The distinctive function of a doctrinal-function system is to provide a linguistic or descriptive index to mathematical system construction. By contrast the function of mathematical systems consists of the exhibition of the interrelationships of mathematical relations.

On the whole, the functions of mathematical systems and proofs constitute exhibitions and descriptions of hierarchies of operations and products. Each level consists of results derived from interbehavior with prior products in an ascending series. Such being the case there is clearly a breakdown of the distinction between the genetic and validity aspects of mathematical and logical theory. To support the setting aside of this distinction as a fundamental argument one may urge that the notion of rigor and the acceptance of a criterion of rigor have consequently undergone radical changes in the history of thought.

THE SPECIFICITY OF LOGICAL OPERATIONS AND LOGICAL PRODUCTS

From the view that logic is essentially a system-building enterprise, an enterprise for organizing, arranging, and ordering things for particular purposes, certain consequences follow.

First, with the elimination of all absolute, *a priori*, and conventional factors there is a salutary stress of materials, procedures, tools, and products. In other words, the actual situation and the exigencies of work receive their full consideration. Not only is there proper recognition of the one general factor of logic, namely, system building, but also a fitting appreciation of every instance

³⁰ Keyser, *Mathematical Philosophy*.

and type of system. The advantage for general logic is that conventional products of traditional system building need not be allowed to dominate present logical activities, but can be critically evaluated for their relevance and pertinence in present situations. Accordingly, such established products as syllogistic or implicational systems may or may not be brought into close juxtaposition with immediate system-building enterprises.

In the second place, when construction is stressed as a genuine enterprise not only can the work be separated from the product, but each can be given its proper emphasis. In actual system-building enterprises clearly there are all kinds of constructions. There can be no single and exclusive form of construction because materials differ and the organization of a system depends upon the particular materials operated upon. Similarly, there can be no single or ideal type of product, since the latter depends upon the sort of material and interest which prompts the system development. Certainly the construction of a formal system irrespective of the so-called content goes counter to the notion of a definite enterprise.

Incidentally, the imposition upon logical work of a type of construction to yield a formal product, whether by abstracting from all special characteristics of related things or by reducing things to referring sentences, illustrates the objectionable process of making logic into a universal and absolute procedure. Actual constructions and products deviate widely from conventional conceptions of form and content.

Finally, stress of the specificity of construction and products fosters the distinctions between construction, tools, procedure, and materials. Probably the most important result of such a critical separation of the phases of system construction is the segregation of grammar from things, and the processes of organizing sentences from the activities of systematizing objects and relations. It is probably the attitude toward and the handling of language which mark the greatest differences between traditional logic and the view of specific system building.

As to language, we are able to separate the function of words and symbols as aids in organization and system building from expressions or manifestations of reason or other profound intellectual powers. Again, we can distinguish between symbols and words as

aids and as materials. In the latter case we have systems of syntax or grammar instead of records and inscribed arrangements of things referred to as in system. Thus by keeping distinct various types of system we can avoid confusing original data or materials and different resulting products. On this basis we can understand thoroughly the process by means of which things and events can be transformed by sentences, equations, and other referential or symbolic formulae. For example, we observe how motion can be made into such products as space points and time instants, and in general how dynamic events can be reduced to static entities.³¹

Paying proper regard to the specific character of system-building enterprises we are able to see how the attempt to identify logic and mathematics appears to transform logic into a single, universalistic system. We know, however, that mathematics is in no sense a logical system, except by the identification of the systematization of mathematics and the complex processes systematized. It is a highly improper identification to equate a product of systematizing mathematical materials with those materials themselves, whether they are relations or procedures of work on these relations. Obviously the work of mathematical discovery is not simply the development of abstract systems. As Young points out, "Imagination, geometric intuition, experimentation, analogies sometimes of the vaguest sort and judicious guessing . . . are the instruments continually employed in mathematical research."³² Certainly the results obtained by whatever method must be capable of abstract formulation, but this formulation product need not, as is often the case, be confused with what is formulated.

³¹ As exemplified by d'Alembert's principle.

³² Young, *Lectures*, p. 221.

CHAPTER XI

LOGICAL THEORY AS INTERBEHAVIORAL ANALYSIS (I)

LOGICAL THEORY CONTINUOUS WITH PRACTICE

CONVENTIONAL logical theorizing is continuous with logical practice. The theorist begins with some type of logic and builds up a system of propositions concerning the nature of the factors within that system. For example, the theorist of classical logic includes in his system statements or propositions concerning the nature and significance of terms, propositions, and systems of propositions constituting all types of syllogisms. The modern inductive theorist similarly constructs a system expounding and explaining how comprehensive and universal propositions are generated from specific statements. In both these cases as well as in recent mathematical logic the theorist simply assumes the pre-suppositions basic to the comprehensive system espoused.

From the interbehavioral standpoint another continuity apparent in conventional logical theorizing is that every instance of such theorizing constitutes an interbehavioral enterprise, though on a little different level from that of primary system building. The procedure is somewhat enlarged and elevated by the addition of statements which are expected to justify the specific system-building processes. This is especially true when the theorist differs from others on particular issues within the framework of a particular conventional system; the amplification is even more apparent when a competing type of conventional system is sponsored.

For the interbehavioral logician there is still another feature of continuity between theory and practice. What the theorist aims at is a system of propositions concerning the nature of logical interbehavior based upon an investigation of the activities performed by (1) logicians as builders of particular conventional systems and (2) individuals actually constructing systems in all sorts of situations. While in a sense theorizing appears to be a more general procedure than specific system building it consists after all of interbehaving with a particular class of stimulus objects. Logical theorizing is the observation of what logicians actually do,

whether they are professional logicians, scientists employing logical techniques or laymen constructing systems of propositions, beliefs or traditions. Incidentally, all accretions and extrapolations from these concrete activities of individuals can also be observed, analyzed, and interpreted by the investigator who studies the logician and his interbehavior in the way a naturalist observes the adjustments of an organism to its environment.

What appears to be the more general character of the theorist's operations is derived, of course, from the relatively greater amplitude of his scope of action. Whereas the primary system builder adopts certain premises on the basis of patent conditions and immediate interests, the theorist is concerned with an understanding and comparison of all the different premises and reference frames employed by particular system builders, whether they build in a particular (legal, scientific, commercial, mathematical) or general (some form of traditional logic) field. Specifically the logical theorist is interested in comparing the operations and products or results of a biologist developing a system of biology, a mathematician organizing a postulate-theorem structure, a logician creating a local scientific or symbolic system of propositions or a structure of sentences to achieve a cosmic order whether it includes or excludes metaphysical factors.

Though logical theorizing and simple logical construction or practices are continuous as interbehavioral operations, there are specific differences in the operational conditions. The latter impose certain interbehavioral procedures upon the logical theorist. A fundamental check upon him is that he may not generalize or extrapolate his premises to constitute totipotent and universal principles of investigation. For example, logic cannot be made into a particular kind of system such as a comprehensive tautology, in which even scientific propositions are reduced to language without reference to objects or events spoken of or referred to. On the other hand, the theorist cannot, by arbitrarily adopting a definition of logic instead of deriving it from what system builders actually do, eliminate absolutes or cosmic systems from the domain of logic. To avoid such mistakes and to achieve proper checks upon our logical theories it is only necessary to respect logic's operational character and thereby achieve an adequate criterion for evaluating logical constructions.

On the other hand, to overlook the constructive character of logic is to risk the danger which besets a scientist who lets his apparatus dominate him in choosing and solving a problem. Logical processes must themselves be developed or adopted for solving particular problems. Essentially, then, both the processes and the logical situation must be regarded as unique and specific. There are no general forms of thinking, reasoning, proof, inquiry or valid inference. All such notions go back to cosmological or theological ideas. We make no exception for those who limit their holistic procedures and systems to tautological or limited mathematical structures.

An important difference between the interbehavioral and the conventional logical theorist is that the former does not limit himself to simple system building. He is an investigator who looks into the basis and conditions of particular logical enterprises connecting them with the cultural and school auspices under which the work is begun and carried out. In emphasizing the specific and particular characters of systems the large number and great differences between systems can be taken into account. Obviously this is not merely a process of justifying an accepted system, as is the case in conventional logical theory when one argues that there is a general and unique theory of inquiry, a set of valid and reliable canons of science, or an inevitable dialectic of thought and things. Rather, the interbehavioral attitude is simply an extension of what takes place in the case of any complex system construction. Only the simpler systems consist of elementary and immediate organization of items.

LOGICAL THEORY AS ANALYSIS OF LOGICAL ENTERPRISES

Logical theory constitutes an investigation and analysis of logical enterprises. The theorist investigates particular interbehaviors of individuals when they perform activities falling within the domain of system building. On the other hand, the theorist may be called upon to study alleged logical activity which may turn out to be something else than system building, for example, scientific method or mathematical calculation. In either case the theorist observes and reports the actual operations performed with particular kinds of material, under certain conditions,

and with the employment of particular criteria of organization, validity, etc.

It follows naturally that the theorist is engaged upon an evaluative enterprise, one of assigning values and determining the significance of the work the logician performs. In this way he is able to estimate the difference between the logician's asserted belief that he is occupied with transcendent materials and what he actually does. From an interbehavioral standpoint all occupation with total and transcendent systems and all commerce with transcendental universals and classes constitute constructions of verbal materials and attitudes on the basis of extrapolations from actual interbehavior with local and tangible materials. Similarly, the determination of logical laws has its starting point from specific investigative interbehavior. The universals which traditionally were regarded as the refuge of nonhuman propositions turn out to be much more intimately related to psychological events than other sorts of logical materials. On the whole, all such generals, universals, and classes constitute constructions usually of verbal materials on the basis of immediate or remote interbehavior. By remote interbehavior is meant, of course, contacts with materials extrapolated from direct interbehavior.

We must, however, eschew any notion that all such remote interbehavior is performed exclusively for some socially significant purpose. Those who hold such a view inject a particular bias into their assumptions—a bias antagonistic to formal systems. For us it is sufficient that such interbehavior with formal objects occurs. It makes no difference whether it is instrumental to any other sort of event.

When we analyze any particular logical enterprise from the interbehavioral standpoint we discover a series of necessary aspects or dimensions, some of which are basic and inevitable, whereas others may or may not be important, depending upon the particular logician or type of logic. In the following paragraphs we list six factors which play a part in an interbehavioral logical field and indicate some essential facts concerning them.

1. *The Actor or Logician.* Since the interbehavior consists of a worker operating upon particular materials it is necessary to consider the person engaged in logical work. Psychologically

speaking we are concerned with the logician's personality. Relevant here are questions concerning his general orientation. Does he work with sufficient regard to his historical and cultural place in logic? Is he a philosopher who invariably places his work in a comprehensive framework, a scientific investigator concerned with definite natural systems, or a mathematician standing against a backdrop of abstract relations? Important here also are such questions as personal and professional interests, aims, and motives. These factors, though related to the general orientational background of the worker, provide necessary material for the understanding of particular logics as well as the logical enterprise in general. On the whole, when we consider the worker and his orientation we discover the basis for his fundamental presuppositions as well as the plan and character of his work.

2. *Work or Action.* In investigating this factor which covers general operations or particular techniques of system building, we discover whether the worker is operating on the basis of a definite postulation system or is simply assuming that there are fundamental canons and procedures of logic. An important question here is whether his work may be classified as organonic, that is, whether he assumes logic is a tool or organon for operating on various sorts of materials, or energonic in the sense of performances either for attaining a certain goal or simply adapting himself to certain situations.

3. *Materials.* Out of what materials is the logician building his system? Is he concerned with things or language, that is, sentences? Again, does he distinguish between things and propositions or does he regard these as common and interchangeable materials? In either case are these materials classifiable as existential or substitential? Are they formal or contentful?

4. *Auspices and Conditions.* The setting (auspices) under which a worker operates or interbehaves with materials constitutes an indispensable feature of every interbehavioral situation. Not only do settings contribute to the vigor with which logic is cultivated, but they also affect the types and trends of system building. The presence or absence of materials furthers or hinders organization. The availability of much or little material favors or discourages formal or contentful systems, linguistic or thing systems. Certainly the patterns of the general-cultural or local-school influences

throw light upon particular systems and the techniques and operations of constructing them.

Similarly, the particular conditions under which logics are constructed play a large part in the types of systems developed. The history of logic indicates how new discoveries, fresh movements of population, and, in general, changes of human circumstances fertilize logical soil for the practice and modification of logic. Again, logics constructed for defense of belief and dogma differ materially from those designed to attack such prevalent attitudes or to establish new ones.

5. *Criteria or Canons.* For the investigation of the nature of logic the topic of canons is of the utmost importance. Upon whatever theoretical basis logical work is carried on, the criteria or canons of system building determine the general character of logical systems and the work of logicians. Among the questions asked concerning canons are the following. Is a logical enterprise influenced by the notion that there exist absolute and indisputable canons, or are canons relative, arising out of the particular enterprise at hand? Do logical criteria or canons issue from the possibilities, limits, and general necessities of reason or mind, in other words, from the laws of thought, or do they stem from the discovery of ultimate relations in the nature of things? Other problems of criteria concern their number, stability, generality, applicability, and so forth.

6. *Products.* Of tremendous importance are the identification, comparison, and evaluation of logical products. Whether such products are studied in conjunction with other factors in system-building enterprises or as independent products the results are valuable for the light thrown upon the nature and workings of logic. Are logical enterprises autonomous and ends in themselves, or do they answer to some specific need and arise under certain circumstances? Do they become what they are because of some intrinsic necessity of thought or things, or are they conditioned by specific collocations of circumstances?

Variations in logical theory and disagreements among logicians concerning the character and potency of logic are based upon either (1) the occupation with some of these factors to the exclusion of others or (2) the overemphasis of one or several of these factors.

For example, those logicians who argue that logic has nothing to do with any subject matter but only with pure forms aim at rigor and consistency rather than organization of facts; they likewise minimize materials in favor of work technique. On the whole, such logical theory bespeaks a lack of appreciation of the interbehavioral operations involved in all logical work, as well as an oblivion to the fact that the relations with which logicians are vitally concerned are definite subject matters. Such logicians remind us of the linguists who looked upon Chinese as a formless language, because they overlooked the point that variations in speech are only stylistic, since every speaker and every language system operate exactly as any other from the essential intercommunicative standpoint.

A corrective for this view is to consider that stimulus objects consist of relations as well as trees or stones. As we have seen, the interbehavioral process is the interconnection of response and stimulus functions. Whereas the former always inhere in the specific and concrete activities of organisms, the latter may reside in absent or nonexistent objects. In the latter case, the individual interbehavior is concerned with substitute stimulus objects, especially with words or symbols. Abstruseness constitutes only an interbehavioral incident.

In a similar way, some logicians stress the outcome or goal. The classical dialectical logicians are interested in achieving a unification and comprehensiveness of the universe. Whether one or the other of the dimensions is stressed, the result is a serious truncation of the logical enterprise. The intellectual penalty, of course, is a failure to achieve a proper intellectual orientation toward logic. Probably one of the most serious instances is to adopt a theory such as that of the extreme formalists, namely, that in logic one is preoccupied with rules and processes without regard to their nature and significance.

THE LOGICIAN AS SOURCE OF LOGICAL THEORY

Eschewing as we do, all *a priori* notions concerning the nature of logic we derive our theory of logic from an examination of what logicians do and the basis upon which they work in particular enterprises. As soon as we start such an investigation we confront the problem of individual differences. Despite the common

cultural factors leading to the logician's interest in and work at his craft, there are still large variations in aim and intention. The study of these likenesses and differences found in general logic and among members of particular schools is important for the understanding of logical theory. Such items may be placed under the headings of aims, capacities, and orientation.

A. *The Logician's Aims and Aspirations*

The varying aims and ambitions of logicians constitute a rich source of information for logical theory. It is hardly an unfair generalization to say that logicians of all schools and persuasions regard logic as some powerful organon or instrument to achieve some aim or purpose not possible by other means, say, scientific investigation or technological invention. Recall the Hegelian formulation of this aim. "To purify those Categories and to raise the mind through them to Freedom and Truth, this it is which is the loftier task of logic."¹

Precisely how these ambitions are to be realized depends upon the logician's particular philosophical views, since logic has always been regarded as a philosophical discipline. Theories of logic accordingly are necessarily consonant with the philosophical view of the logical theorist. Need we add that all the different theories based on the common aim to achieve comprehensiveness, transcendence, truth, rigor, etc., are formulated by means of a specialized treatment of language, whether language is presumed to be intimately interrelated with thought or capable of expressing general existence.

I. *Comprehensiveness.* Though logicians differ in particular ways they do not escape the general ambition to achieve comprehensive and all-inclusive systems. These aspects of inclusiveness may concern the whole of reality, all processes of reasoning in interconnection, or the methods for attaining truth or summing up the modes of inquiry. Avowedly metaphysical logicians such as Hegel, Bradley, Bosanquet, and others, frankly proclaim that "every inference involves a judgment based on the whole of reality, though referring only to a partial system which need not even be actual."² For such logicians logic is a system which exhibits

¹ Hegel, *Science*, p. 46.

² Bosanquet, *Implication*, p. 4.

the interconnections of propositions or judgments all of which cohere to form a cosmos. Such logics imply the possibility of the general efficiency of thinking or reasoning and/or the theory of a generalized existence or being.

But even those logicians who recoil from such comprehensive systems exhibit the same ambition of achieving inclusiveness. In a former chapter we considered how Dewey, who repudiates traditional metaphysics, regards logic as a philosophical discipline which shows the way to general achievement of philosophical results. Logic as the theory of inquiry becomes a general process with a general pattern. The theory that inquiry aims at warranted assertibility still maintains the view that logic constitutes the organization of sentences or propositions to form a general system. Though Dewey does not fail to assert that logic is investigative, his inclination toward universalization actually removes his logic from specific problems and specific inquiries. For him logic is "the generalized account of the means by which sound beliefs on any subject are attained and tested."⁸

It may not be an unfair question to ask whether this comprehensive theory can yield actual results in any particular inquiry. What if the descriptions of how investigations go on are fairly made! Will this produce any result in the actual investigation? Hardly. It may be said, then, that the theory of inquiry results only in a detached system of description without actually penetrating into the investigative work or the particular systems employed.

II. *Data Transcendence.* A prominent ambition of logicians is to set up schemes for transcending data, for getting beyond immediate knowledge. When this is achieved in concrete situations it is done on the basis of (1) investigative procedures, (2) inferential processes, or (3) predictive and probability hypotheses. Conventional logical theory, however, relies on fixed systems. Syllogisms, it is argued, for example, yield new information on the basis of known premises or principles. Unless logicians are willing to accept this argument on faith they offer only trivial examples. From the rule premise that "policemen shall be five feet nine inches in height," one may know that if Brown is on the police force he is that tall.⁹ Though not much new knowledge is

⁸ Logic, p. 535.

⁹ Jeffries, *Scientific Inference*, p. 3f.

claimed there is still the obligation to find out whether the rule is enforced.

The most radical aim of the logician is to set up a transcending system in order to attain some culturally determined ideal, such as proves the existence of God or the nature of the universe. Such systems, of course, constitute merely sets of self-satisfying statements and arguments based either on accumulated opinions, as in the case of religious metaphysics or on magnified analyses from science and practical life.⁵

III. *Attaining Truth and Certainty.* The ambition to achieve absolute truth and incontestable certainty is closely related to the aim of ultratranscendence. Hence all these ideals may constitute the pillars of single logical systems. This eternal quest for truth and certainty has motivated many logicians to establish the "universal validity of logical conclusions."⁶ If this cannot be accomplished in general or particular cases by reasoning, resort is had to the alternatives of verbal manipulation or intuition.

IV. *Rigor.* Prominent among the aims of logicians, especially those working under mathematical influences, is the forging of tools for securing rigor of knowledge and technique. This laudable ambition when confined to specialized fields, for example, mathematical symbolization, has much to recommend it. Rigor is no mean quality of specific systems, no matter what the materials out of which they are constructed. But unfortunately the universalizing traditions of logic do not allow logicians to be satisfied with such localized aims. As a consequence what begins as an attempt at clarity, discrimination, stability, and relevance ends as a futile crashing of the boundaries between particular jobs and cosmic attainment. Although mathematical logicians to a degree withdraw from traditional metaphysical activity they still inject such expansive dreams into their specialized work and products as finally to end up with either the empty sack of absolute rigor⁷ or the dense nebulae of Platonic reals,⁸ both exchangeable for verbal compatibilities.

V. *Consistency.* Since, despite the logician's belief that he is

⁵ Dubs, *Rational Induction*.

⁶ Ushenko, *Problems*, p. 43.

⁷ Hilbert und Ackermann, *Grundzüge*.

⁸ Russell, *Principles*.

concerned with thought and reasoning, he actually is a system constructor, it would be a miracle indeed if he did not aim effectively toward consistency, toward tight integration of the items he systematizes. In some sense, of course, consistency is a *sine qua non* of system. Especially is this true when consistency is taken to be some sort of coherence, an integrative binding to hold things together in more or less intimate connection. But seldom is consistency in logic achieved on the basis of concrete fittingness and belongingness of actual things. Less frequently still is coherence and consistency dealt with from the standpoint of interbehavior with natural and cultural events. The logician's urge to ultimacy drives him to the finality of identity, to the absolute of indiscernibility where actuality and qualities are so lacking as to leave nothing but the strings without anything to be tied, except, perhaps, the words substituting for the bound and the binding.

Logical consistency, then, is the second barrel of the extreme formalist's gun, the other being, appropriately enough, his muchly vaunted and wanted rigor. The necessary and laudable consistency or coherence to bring items together is carried so far, as in the case of rigor, that the original values are lost. To avoid friction altogether one has no machine at all beyond a blueprint. Admittedly it is well to pursue consistency wherever and however it will be appropriate for system building. In mathematics, for instance, and other fields where abstractness is desirable more consistency can be sought for than in other fields. But to be so fearful of contradiction that system is achieved without regard to anything serving as materials of systematization makes a hollow mockery both of the principle and the aim. The history of logic amply demonstrates that to push consistency to extremes results inevitably in the stress of words and sentences. Even in mathematics consistency is valuable only in the degree that it brings together proper relationships as mathematical things.

VI. *Irrefragable Demonstration.* One of the most ambitious though disastrous aims of logicians has been the development of absolute processes or activities transcending the range of concrete events. For the most part such processes are denominated reasoning, though upon occasion various forms of mystic intuition are drawn upon. Since such a logical theory runs counter to any actual achievement, to any establishment of fact, it is reduced to the

depth of obtaining personal and nonobjective conviction. To a considerable extent, then, demonstration and proof logic constitutes a theory and mechanism of argument.

That such theory answers well to certain systems cannot be denied. To establish belief in supernatural entities as the Scholastic systems do or to develop absolute proofs as other tautological formal systems strive for, constitute fine adjustments in the general system-building domain. What one may well object to, however, is the restriction of logical work to the interrelationship of empty and trivial sentences and the limitation of system building to such highly specialized situations and techniques.

VII. *Specific System Building*. In contrast to the logical theory based upon such exclusive aims and ambitions as we have surveyed stands the theory that logic consists merely of system-building operations. According to this theory the distinctive work of logic is the interconnection of items derived from any field of interest. Whether the systems are closed or partially open are contingencies depending upon factors localizable in the particular field of action. Moreover, the question whether certainty, truth or conviction are desired is a matter of the interests and aims of the particular logician and the circumstances under which he works.

B. *The Powers of the Logician*

Reflected in the mirror of universal and exclusive logic is the principle that the logician operates on the basis of one kind of process or power. It is hardly in accord with conventional logical theory to allow for combinations of processes. Logical theorists differ, however, concerning the nature of this power. Some regard it as reason; others as intuition, whereas the instrumental logicians rely upon a sort of general problem-solving capacity. On the whole, too, these powers are considered as inherent in the individual logician, though, of course, all logicians are similarly endowed. The nature of these powers is made to depend on the individual theorist's idea concerning the nature of mind. Some look upon mind as a unified entity; others envisage it as a coordination of discrete states. The instrumental logician who eschews the notion of a psychic power or factor regards mind as the activity of biological processes.

Even to the most biassed observer it is plain that a vicious circle

marks the theorist's attitude toward the relation of logic and the logician. On one arc we see that the notion of universal and exclusive logic is based on the idea of unique psychological powers of logicians, whereas, on the other, it is the acceptance of such powers that leads to the view of universal and exclusive logic. As we have already pointed out, only an interbehavioral type of psychology, one in which psychological data are events involving the interaction of individuals with particular kinds of stimulus objects, allows for every variety of system building. According to this view the logical capacities of system builders are derived from succeeding contacts with the particular materials forming the basis of the system. What are called reasoning, intuiting, and problem-solving capacities comprise various degrees of expertness in dealing with particular materials.

We must not be misunderstood to say that logical theorists display an intense interest in psychological matters. That is hardly compatible with the wide separation usually made between logic and psychology. But since a psychological view is inescapable the logician simply absorbs psychological notions from the intellectual atmosphere. Infrequently he may attempt to substantiate his belief in psychic entities or "directly inspected contents of consciousness" by logical considerations.⁹ Because of the importance of the psychological sources of logic we consider some of the outstanding borrowed theories.

I. *Reason and Reasoning.* Of the many notions concerning reason (harmony—intelligibility of the universe, power or faculty of mind, etc.,) logicians on the whole adopt the idea of some power of the individual to discover truth or reality or to draw necessary or proper conclusions. When we attempt to differentiate between the logician's formulae about reasoning and his practice it may fairly be said of the latter that reasoning becomes system building in the form of proper interrelations of propositions whether regarded as implied or involved. It is inescapable that the criteria for propriety (called validity) are deeply seated in the cultural matrix.

II. *Intuition.* What intuition is as the power or process of logic, is a problem too ample for handling in the present work. So numer-

⁹ Lewis, *Some logical considerations.*

ous in the history of thought are the constructions concerning this faculty that a large volume would be needed to do justice to the main ones. Furthermore, we know that the attempt to analyze these constructions would reveal at once their similarity to those concerned in dealing with reasoning.

Limiting our considerations to technical system builders, for example logicians and mathematicians, rather than to purely verbal creators of arguments and proofs for the existence of a deity or some other supernatural essence or entity, we may regard intuition as a reaction sensitive to things reacted on. In other words, intuition consists of activities not limited to verbal or symbolic construction. Pertinent here is the view that while intuition is the power to know the basic truths of logic no single intuition is sufficient. Rather, there must be a series of self-connecting intuitions.¹⁰

Beginning with the Greek mathematical system builders intuition has implied a type of interbehavior in which some sort of natural or cultural events loomed large to the system builder. Another way of saying this is that the degree of abstraction from things was definitely limited. Even the Cartesian clear and distinct ideas merely suggested a criterion derivable from culturally accepted appreciations of the nature of things. In modern mathematics intuition implies a stress of construction, so that contact is maintained with the things abstracted from. The intuitional logician or mathematician does not want to overemphasize the arbitrariness in postulation. Actually he wants to build on the basis of the arithmetic of natural numbers. He regards the pure formalist and logician as system builders who crave oblivescence concerning the abstracted things which justify their procedures and structures.

What the logical theorist may learn from the arguments about the powers of the logician and the irreconcilability of views concerning such powers is in brief that there are no such powers. Concerning the controversies engendered by the opposition between sponsors of different powers and the resulting systems one may conclude that they are based upon the false ideal of one exclusive method and system along with the failure to take into

¹⁰ Ushenko, *Problems*.

account the interbehavioral basis of all system building. As to the one and only conception of mathematics, in this connection we can do no better than to quote Bell.

There may be no necessity to have but one kind of mathematics, if the objective is scientific applications; and there are no reasons for supposing that a single, all-inclusive mathematics can embrace all the useful kinds without internal contradictions. Again, in the same direction, but going deeper, it has been very hesitantly suggested that complete self-consistency is more of a luxury than a necessity in a usable mathematics.¹¹

C. *The Logician's Background or Orientation*

The historical development of logical systems is clearly a reflection of the cultural background of those who construct them. The same may be said of any particular current system, although it is more difficult to analyze because any present logician through the accumulation of cultural records and institutions has innumerable possibilities of orientation. Now since the primary orientation of logicians whether general or mathematical is philosophical we need only point out that all sorts of details are available. The various idealisms, realisms, and pragmatisms give scope and form to logical work and theory. For example, logicians favoring a nominalistic view of universals or categories construct their systems with close regard to linguistic factors. By the same token the realist builds within a framework of essences, presumed to be organizable. Again, conceptualists are more inclined toward the manipulative organization of thoughts or ideas.

As we have already indicated, to tie logic to some philosophical system inevitably results in a notion of universality and exclusiveness of logic. Indeed we need not think of philosophy itself as leading to a fixed and complete cosmos howsoever described. As used here the terms *fixity* and *completeness* include dynamic and evolving as well as static and finished cosmoses.

Instead of regarding philosophy as a process of asserting ultimate principles or ultimate structures we may look upon it as an enterprise of general orientation beyond that afforded by any single science. This allows for an actual adaptation of the individual to the world of nature and of culture without assuming

¹¹ Development, p. 531.

he can discover and describe its ultimate character. Santayana declared in a heat of criticism of German philosophy, which he took to be predominantly idealistic, that it was essentially an egotism to comprehend the universe in one's own thought.¹² A similar statement might be made of any type of cosmic organization.

In place of such egotistic motives we may substitute an inter-behavioral view of philosophy, based upon actual observation of the cultural origin and development of reflection and speculation. One is not obliged on this ground to assume a unique kind of knowledge called philosophical which goes beyond that of science or even everyday observation. The philosopher, rather, is looked upon as one who attempts to check one set of results as over against another. For example, he refuses to accept the claims of any single science to be ultimate and basic or to reduce the findings of one to the basic principles of another, as is implied in the notion that psychology, for example, is based on biology, and biology on chemistry and physics, and these two on mathematics.

To be more specific, a philosopher as contrasted with a physicist is not satisfied to reduce events to numbers, quantity or numerical relations, howsoever valid and valuable these reductions may be for handling the original events. Implied here is the assumption that the primary difference between a philosophic and a scientific view is that of breadth of interest dictated by proximity to specific achievement. For one interested in the number of people in a population or intensity of some quality, the separation of quality and quantity is not only admissible but necessary. However, a place must be allowed for other interests and isolation of additional factors. The philosopher as compared with the technical scientist wants to take account of the findings of biological and psychological observations as well as those of the quantitative physicist. As to speculation, that means merely going beyond the attainable knowledge by judicious extrapolation.

If, then, philosophy does not involve any supercosmic system, there is no philosophical justification for any universal and inclusive logic. System building may be regarded as specific and non-philosophical. Delivered from the yoke of egotistic system and from the shackles of age-old tradition the system-building enter-

¹² Egotism.

prise can be envisaged and evaluated in accordance with its actual occurrence.

THE LOGICIAN AT WORK

Whether the logical theorist regards logic as: (1) the principles governing reasoning and thinking, (2) the basis for the structure and validity of mathematics or (3) the rules for properly conducting scientific investigation, in the final analysis he is dealing with the methods and techniques of system building. For the most part, of course, logical theorists are concerned with proper methods, since the convention prevails that logic involves directive or normative guidance. Now each conventional logical theorist proposes what in his estimation are the best methods and techniques. In the meantime this emphasis of proper procedures makes him more or less insensitive to what system builders actually do. By contrast, the theorist who regards logic simply as system building may profit from the observation of how systems are constructed and analyze the relationships between the methods employed and the characterization of successful or desirable system structure.

Keeping close to the actual work of the system builder we find that in each instance a specific task is performed with conditions more or less unique to that task. To follow certain fixed lines of action, therefore, would in no way allow for differentiation between different enterprises. If the problem can be handled precisely as others have been handled, then clearly there is no room for ingenuity or reasoning. Particular classes of situations certainly require only routine treatment. In others the building process requires a great amount of constructive power because of the novelty and importance of the material and problem. To treat all the different system-building situations as the same kind invites an inevitable and improper analogizing procedure. We cannot substitute generalized techniques for particular conditions and requirements. This point may be summarized by the proposition that system building must be idiosyncratic—that is, it requires unique expertness in dealing with particular kinds of concrete or abstract materials.

So different are system-building enterprises that they cannot even be grouped together on the basis that they are all instrumental to certain biological or survival values or human purposes.

It is in this way that the instrumental theorist sets up a generalizing principle against both the idealistic logician, who makes logic a process for encompassing the universe, and the symbolic logician, who attempts to find a basis for mathematics whether with or without the implied Platonic or medieval realism. These differences between technical system builders or logicians afford ample evidences of the place that must be allowed for different methods or types of system-building work.

Concerning the technical system builder, he can hardly escape differences in ways of working since he differs from others in aim, tradition, and school. Then, of course, there is the particular job on hand, a circumstance bringing to the fore the kind of materials with which he works and the particular kind of method necessary. Historically this fact is illustrated by the so-called inductive and deductive methods. In the latter case the systems erected are conditioned by the need to attain to argumentative conviction or the organization of terms or essences. In the inductive method the systems are different because the materials are regarded as things or events from which laws and ultimate structural orders are derived. All the more is the problem of differences in methods and techniques intensified when we take into account nontechnical system building as well as technical logic.

Once more we invoke our distinction between logic as organonic and energonic. Here we contrast the theory that logic consists of large-scale traditional systems with the theory that logic is concerned with innumerable, specific enterprises carried on under particular circumstances. To throw this distinction into relief it is well to consider the specific operational techniques fostered by the adherents of the opposed theories. What attitudes do they assume toward such techniques as formalization, hypostaticized description, analysis, synthesis, etc.?

I. *Formalizing.* To study actual logical work is to discover at once that the conventional logician's stress of the formalization technique is only partially justified. It is true, of course, that as system building logical work must be described as structuring, ordering, organizing, in other words, a process of interrelating units or factors. This is not, however, to agree that logical work is primarily or inherently a matter of discovering or creating forms. There is a vast difference between the technique of ordering

specific things into systems and the inherent concern with forms and structures without regard to purposes, materials, and situations.

The overemphasis of formalization as a logical technique not only conceals the specific system-building processes in particular situations, making logic into a single and simple enterprise, but also magnifies some particular sort of system building into an exclusive form, for example, mathematical or sentential structuralization. On the latter basis logic becomes entirely an enterprise of manipulating abstractions.

The opposed evaluations of the formalizing techniques may be well illustrated by two examples. First, consider the statement of the extreme formalist that logical truth depends only upon a particular organization in a statement of the basic particles "is," "not," "and," "or," "unless," "if," "then," "neither," "nor," "some," "all," etc., independently of its other ingredients. Quine asserts that the statement "If every man is mortal and Socrates is a man then Socrates is mortal" is true no matter what is substituted for "man" or "Socrates." The truth inheres in the following structure: "If every——is——and——is a——then——is——" provided the first and fourth, the second and last and the third and fifth blanks are filled alike." Certainly we have here a tightly knit system, but can this type of formalization be regarded as universal logic?

The second example is the remark of Lewis that logic is circular.¹⁴ When he implies that logic is system there is no objection. But it does not follow that logic must be exclusive systems the elements of which are points on a circle rather than on a straight line, or that the systems must always be formal.

II. *Symbolizing*. In all human enterprises, of course, it is necessary to make use of such aids as marks, records, and symbols. It has been the unsavory record of logical history that such aids have come to dominate the system builder. Accordingly, the symbolizing process which originally constituted a logical technique has become so predominant a feature of logic that the method has replaced the original enterprise; the scaffolding which formerly was designed only as a constructional aid has become the building itself.

¹⁴ Quine, *Mathematical Logic*, p. 2.

¹⁵ *Mind*, pp. 207 ff.

As a consequence the logician's essential work, namely, to build systems, has become misinterpreted as simply the task of organizing symbol systems with as little regard as possible to the designata. This so-called rational organization of terms, as exemplified by (1) the work of Scholastic and neo-Scholastic logicians who attempted to refine the arguments of classical logic, and (2) the mathematical logicians who build systems of symbolic representations of abstract propositions, hardly suggests the tremendous scope and frequency of system building. Such limitation of logical work has been often enough pointed out in the conflicts between those who favor intension and those who argue for purely material systems.

To point out the objection to an exclusive preoccupation with symbols and symbol systems is in no wise to overlook an important and valuable type of system building. To organize linguistic and symbol systems and to develop methods for transforming such systems is work of great importance when dealing with materials in which abstracted relations constitute the materials worked upon. Moreover, it is inadvisable to minimize the accuracy and precision attained by the symbolizing procedure. It is another matter, however, to regard the work of symbolization and symbol-system transformations as the general and exclusive activities of logic.

III. *Totalizing and Generalizing.* Obviously system building involves bringing together elements to form a totality, as comprehensive a general unit as possible. On the other hand, there is hardly a theorist who has seriously taken the central interbehavioral fact sufficiently into account to refrain from regarding the totalizing technique as a cosmic, ontological or universal methodological procedure.

With respect to the present technique we find the greatest divergence between the theory and practice of logic. The study of logicians at work plainly reveals that in their preoccupation, choice of, and attack upon materials and construction of final products they differ enormously. Yet the view wittingly or unwittingly entertained that logic is one and exclusive leads to a revered if not revealed *Summa*. As between the ontologists and methodologists or epistemologists each regards his type of logic as the only proper and valid system. A similar competition prevails even among the

members of particular groups. Among the methodologists each looks upon his own version as the best example of proper logic. The mathematical methodologists condemn the concrete scientific methodologist, whereas the latter are disdainful of the formalists. Similarly, the various ontological logicians each regards his own system as the only path toward reality or truth.

To the neutral observer it is thoroughly convincing that many good and comprehensive systems can be constructed by an appropriate selection of postulates and materials. Accordingly, all totalizing and generalizing activities have their definite bounds. Generalizing is, therefore, a specific procedure, and this despite the contempt of all theoretical logicians for system building occurring outside the sacred precincts of the schools.

IV. *Hypostatizing Descriptions.* Next to the totalizing technique the hypostatization of descriptions is probably the most pervasive. Incidentally, too, there is a close connection between these two processes. Essentially hypostatization consists of the conversion of acts and things into linguistic elements. For ease of manipulation and system building, acts of judgment and inference and the manipulation of things are converted into statements or sentences. To illustrate, a process of inference is converted into a set of sentences organized into a syllogism. Hypostatization in this manner becomes a system of logic.

On the whole the technique of conversion transforms quality to quantity, counting to number, individuals to classes, and in general, events become the statements of events. In whatever field this occurs and for whatever purpose the essential procedure is the solidification of the fluid references to things into verbal fixity. Accordingly, we find a continuity in the dialectic conversion of events into the categories of being, not being, etc., and the transformation by Brown, Herschel, and Mill of their descriptions of scientific method into inductive logic. Proper and useful descriptions are hypostatized into a set of fixed and exclusive items of a system.

V. *Tautologizing.* Several techniques employed by professional logicians are at most applicable only to particular types of systems. These techniques may be named in a descending hierarchical order as absolutizing sentences, necessitating statements, vali-

dizing relations, and identicizing elements. Those who hold that logic must be absolute in method and necessary in product reduce logic to a system of tautologous statements more or less symmetrical in organization.

Tautological systems naturally must be composed of symbolic or linguistic materials. It is only by withdrawing from all problems of nature and expelling all eventual components that such systems and techniques are possible. When natural relations are dealt with, selection is rigorous, so that the organization of identities stands out prominently.

Despite the extremely specialized character of the tautological technique and the fact that it is so predominantly autistic in character a comprehensive logical theory must include it among the others. We need only observe that the system building requiring this technique generally is motivated by interests such as absolute necessity, validity, etc., far removed from the ordinary affairs of scientific work or everyday events.

VI. *Analogizing*. An important technique used by logicians who seek ultimate rigor and certainty for their universal systems is the analogous adoption of some established method. A type or model good enough in a given situation is selected and made to serve universally or in specific though not necessarily related circumstances. Obviously the selection is influenced by already accepted ideas concerning the nature and purpose of logic.

When the dominant idea of the logician is rigid deduction the model chosen is a presumably closed and complete system such as Euclidean geometry. Those more inclined toward invariable implication adopt the analogy of some form of hypothetico-deductive system brought into prominence through the development of non-Euclidean postulation.

Similarly, the theorist who is more alive to dynamic than static processes, to growth and change, attempts to draw his analogy from the complex organizations of things in nature and society. To such a logician it is impressive that cells grow to be organic individuals, who are united into societies, and societies into states, states into empires, that atoms comprise molecules and molecules compounds, that planets comprise worlds and worlds galaxies. Moreover, comprehensive laws can be devised to organize uni-

verses and totalities by the integration of facts. Such analogical techniques stimulate the development of inductive and inquiry logics.

Analogical techniques are likewise the bases for all sorts of fictional and conventional systems. With a set of linguistic elements to work with one can simulate any desired structure or organization of factors. A telling illustration of the extreme use of analogy is the development of a logic by one who asserts that, like science, it operates with the methods of hypothesis and verification, but has the possibility of transcending all experience and supporting any sort of belief.¹⁸

VII. *Analyzing*. Because logic is after all system building, the relating and binding of factors, it is inevitable that analyzing should be one of the most prominent of logical techniques. The operations of separating out factors or items from larger complexes in order to identify and fixate them are most essential for careful and critical system building. Yet as every observer of logical work soon discovers there are a great many forms of behavior subsumed under the caption analysis.

When we take into account actual work performed we may well differentiate between the investigative reductions carried out by the qualitative and quantitative chemist, the studies of the mathematician interested in the continuities of his domain, and the analysis of the components of given situations by experts in those particular domains. Each type of analysis is useful and commendable for avowed, specialized system building.

As is usual, however, the conventional logician overlooks the essentially specific character of analysis not only because of the particular situations in which it is employed, but also because analysis in logic must be considered from the standpoint of specific systems to be built. On the whole, then, analysis is regarded as rigorous reduction and not genuine analysis. This is exemplified by classical deductive logic. When content logicians (ontologists) employ analysis the process is a dialectic; the ultimate material of the system is verbally created rather than concretely resolved. The Being, Nothing, and Becoming brought to light by this holistic analysis can only be accepted as components of a spe-

¹⁸ Duba, Rational Induction.

cialized verbal system and not as significant factors available for significant building.

When the logician is more formally inclined and stresses statements, his analysis is primarily a technique of simplification. He uses this procedure to empty his statements of content and for the most part orders terms by subsumption and superstructure without regard to designation. Generally this amounts to a turning away from the obvious and important constructional processes which are so securely enconced in particular circumstances.

VIII. *Synthesizing*. Just as deduction is considered as the analytic process in conventional logic, so induction is thought to be synthetic. Conventional inductive logic is presumed to add similar to similar with the expectation that a general, rigorous principle or structure will emerge. The sort of synthesizing process implies that the materials of logic consist of completed and permanent essences which can be put together or formally classified on the basis of inherent characteristics. From this angle we find strong justification for the view of such logicians as Lotze, Bradley, Bosanquet, Russell, and others that there is no difference between induction and deduction or that the former is reducible to the latter.¹⁶ Unfortunately, this view leaves entirely out of account actual system-building operations based upon concrete events and leading to the necessity or desire of a particular systematic structure and the actual means by which the materials are discovered and organized.

Another possible view concerning the synthesizing technique is that it is coincident with all system building. On this basis the logical worker could depart from the formal organization of judgments or inferences and concern himself with concrete details. But this view is equally unacceptable. Is there any special point in distinguishing this process as coordinate with the other techniques? Rather, why not take the term in an interbehavioral sense to refer to the actual operations of any system builder who synthesizes materials of whatever sort as a subordinate though important activity in the construction of a systematic structure. In this sense, the work of synthesis may include the building up of chemical

¹⁶ Lotze, *Logic*, p. 288; Bradley, *Principles*, p. 342; Bosanquet, *Logic*, II, p. 119, Russell, *Principles*, p. 11n.

compounds as a basis for constructing a chemical system, for instance.

Up to this point in our discussion of the logician's work we have indicated techniques contrasting with those actually employed, but only by way of criticism and negation. Now we add a more positive series of characterizations of logical techniques when specific system-building enterprises are under consideration, but continue, however, the oppositional form of presentation.

IX. *Relative versus Absolute Techniques.* To take seriously the principle that logic constitutes a field or concrete human enterprise removes the need for any sort of absolute or universal technique. Since there is no absolute subject matter there is no place for absolute processes to deal with it. All logical or system-building activity is relative to the problems or materials at hand. The latter are found, of course, in particular circumstances. Naturally the philosophical logician may build an absolute and universal system by verbal creation, but here no less than in all other cases the structure is of a particular sort and the operations and techniques relative to a special circumstance. The traditional logician's dream of a universal and absolute technique such as ancient or modern dialectics is a fantasy which confuses the autistic generalizing of a method with the operation with general principles. It is impossible for the logician to discover rather than assume general premises or universal judgments for dealing with actual and pertinent problems.

As every student of logic knows, all conceptions concerning absolute techniques are based upon constructions concerning the nature of Reason or Mind or the structure of the Universe. All notions of absolute processes somehow imply *a priori* or transcendent knowledge based upon immutable categories that make Reality, or into which elements of Reality must fit to make things. The crude forms of such *a priori* ideas show fair promise of liquidation. We find frequent assertions that *a priori* knowledge is analytic. That the life of the *a priori* is guaranteed only by convention, definition, and postulation is also growing in appreciation. Still discernible, however, among the relative *a priorists* is the belief that there is a mind which makes the *a priori*" or that there are absolutely in-

" Lewis, Mind.

variant relations, despite the fact that propositions are only *a priori* for certain fields.¹⁸

Perhaps it is better to revise altogether the conception of the *a priori*. If the term refers only to established propositions or principles developed in concrete interactions with specific things it departs entirely from traditional notions. It implies then no absolute knowledge or domains. On the other hand, it does presuppose that within the range of actual events the observer who is in interaction with them can construct laws and principles which can carry over from one point or factor to another.

X. *Specific versus General Techniques*. The obvious opposition between logic as (1) an unusual and inclusive discipline of some type and (2) a specific system-building enterprise suggests at once a polarity between simple holistic and multiple techniques and processes. An outstanding difference is, of course, the essential adaptiveness of the latter to the problems and materials at hand. But something more must be strongly stressed—namely, the inclusion among specific techniques of the manipulative and investigative procedures which are ruled out of propositional (verbal) and ontological logics.

XI. *Situational versus Autonomous Techniques*. In consonance with the idea that logic deals with the formal characteristics of things or the formal organization of sentences or propositions, logic has been lifted out of the situation in which it occurs, and logical processes have been regarded as independent or autonomous. From the ergonomic standpoint logical procedures can never be detached from the situations with which they are environed.

The entire notion of autonomous techniques has arisen, of course, from the metaphysical ambitions and occupations of logicians. To build cosmic systems it is requisite to abstract from actual things and to organize processes of combining propositions on the largest scale without regard to concrete objects and operations. As we have already implied, however, this merely makes the technique autonomous of concrete situations, but leaves it thoroughly imbedded in an imagined (constructed) and willful matrix. Moreover, that such construction can take place at all points to a generalization process broken loose from objective situa-

¹⁸ Cohen, Reason.

tions. To deal with an autonomous process of science, inquiry, being or essence is simply to build in the void instead of in the field or laboratory.

XII. *Progressive versus Final Techniques.* Naturally it is the aim of all system builders to round out whatever system they are building and to attain a measure of completion. The question arises, then, are there any finalistic and definitive techniques? The answer depends upon the kind of system to be built. Certainly for local and circumscribed systems designed for practical purposes a technique may be not only adequate but final. Nevertheless, even in such situations practical needs may change; the systems may require reconstruction, and hence the techniques become obsolete altogether and lose not only their definitiveness but their relevancy and adequacy. Though for certain practical purposes one may be satisfied with the foundation premises and the structure erected upon them, such systems are always subject to increase of information and improvement in *aperçu*; thus the finalistic character of the techniques gives way to a progressive mode of operation.

Only the most arbitrary of logical systems can be built with final techniques. Now since all logicians would agree in rejecting the implied imputation of triviality for completely arbitrary systems we may at once exclude finalistic techniques from logic altogether.

The progressive character of logical techniques can be no better illustrated than by the mathematical logics. Though the mathematical and symbolic logicians claim an origin reaching back to Leibniz or even Aristotle, they are constantly insisting upon the progress of their operations as manifested in the improved symbols employed as well as in the increasing precision and accuracy of the methods. The reduction of conjunction, alternation, and denial to joint denial is one of many illustrations of this point.

XIII. *Temporal versus Timeless Techniques.* Since logical work and logical systems are always localized in definite spatio-temporal frames and subject to particular conditions it is paradoxical to assume that there are techniques for dealing with timeless materials and systems. Essentially such techniques have at their roots the setting up of timeless systems and various sorts of

eternal truths by manipulating sentences representing or expressing nonspatiotemporal propositions. Timeless techniques, for instance, have been employed by theological logicians to establish eternal and perfect beings, by mathematicians to inaugurate various infinities and infinitesimals.

Such timeless techniques, however, illustrate the fallacy of illimitable construction by verbal fiat, rather than the ability to achieve timeless essences. In other words, they begin and end as linguistic or verbal behavior. All logical techniques are operations designed by logicians whose powers of extrapolation are strictly limited by the danger of stepping off the solid ground of contact with things to fall into the abyss of autistic creation.

CHAPTER XII
LOGICAL THEORY AS INTERBEHAVIORAL
ANALYSIS (II)

THE PROBLEM OF LOGICAL MATERIALS

THOUGH the problem of materials is of paramount importance for logical theory, it is often overlooked by logicians because their interest has been centered upon instruments for accomplishing various purposes, such as proof, demonstration or the achievement of scientific results. Even those who stress actions instead of systems are concerned with mental powers and the description of methods, and have not, therefore, to any appreciable extent considered the things operated upon.

Precisely as logical theory shifts from organon or instruments of proof or discovery to system building the materials out of which systems are built take on their proper importance. Obviously the kind of technique to be used and the kind of product achieved depend in a very intimate way upon the types of materials.

The logical theorist cannot fail to notice that the various traditional notions of logic reflect a constructive preoccupation with certain kinds of material. In many cases the materials have seriously prejudiced the theory. Surely, then, the task of system building presupposes that account be taken of the materials employed.

PSYCHOLOGICAL BASIS OF OVERLOOKING MATERIALS

Basic to the neglect of system-building materials by traditional logicians and their concentration upon judgment and inference or thought and reasoning is undoubtedly the espousal of a mentalistic psychology. When thinking and reasoning are looked upon as acts of mind they can well be autonomous and independent of things. Even those logicians who define logic as the theory of thought about things overstress the thoughts and simply consider the materials as objects of thought; hence things interacted with recede to the vanishing point.

When, on the other hand, psychological activities are described as interbehavior, it is impossible to overlook logical materials as

stimulus objects. As we have sufficiently indicated, thinking and reasoning constitute interbehavior with things just as much as any other kind of psychological activity. Thus even if logic is taken to be activities of thinking and reasoning, the materials or stimulus objects would still loom large. When we define logic as system building, it not only follows that logical operations can not go on without materials but also that these materials influence the type of system constructed.

TYPES OF LOGICAL MATERIALS

Under the aegis of universal or organonic theory the materials of logic were all abstract and subject to treatment only by means of sentences. To a great extent, then, the materials constituted elements in games or puzzles. The illustrations of logical processes were for the most part trivial, with the mortality of man as perhaps the closest approach to any serious and factual interest.

When system building is the enterprise under consideration any sort of stimulus objects out of which systems can be built constitutes the logical material. The system-building view is certainly tolerant toward all the trivial things involved in the construction of puzzles and games, but these comprise only the less important logical materials.

1. *Thing Materials.* Things as materials for system building demand special emphasis. These objects are actually manipulated in a structuring process and not handled through judgments or propositions. When verbal substitutes such as marks, tokens or records are employed they must be sharply separated from sentences representing judgments. Acts of judging and their results constitute specialized items or factors for system building. To do justice to thing materials in system building it is only necessary to discard the tradition that thinking and reasoning constitute inevitable features of logic (see chap. 7). Even if it be granted that the most important system building does involve thinking processes they must still be taken merely as particular kinds of processes among many others.

2. *Language Materials.* Because of the pervasive character of linguistic behavior and linguistic institutions (documents, records, treatises, etc.,) a considerable amount of system building consists of the organization of linguistic materials. In such items we dis-

cover hierarchies of remoteness and abstractness. Language things of various sorts are organized or systematized for diverse purposes, or the emphasis of the things is subordinate to what they symbolize, refer to or substitute for. The following classification of materials illustrates the varying emphases and varieties of language systems.

(1) *Intrinsic Verbal Materials.* Verbal actions and things serve as material for systems because of the systematizer's interest in their intrinsic worth or desirability as verbal tools. The system builder may be further interested in: (a) simply collecting instances of verbal usage or types of alphabet, or (b) indicating preferences or prescriptions for usage. The resulting systems consist of grammars, dictionaries, and glossaries of various sorts.

(2) *Registration Language Materials.* Materials of this sort are of importance not for themselves but for their functional value in recording and preserving nonlinguistic events or transactions. Such materials make up historical, narrative, and log systems. Here is involved no question concerning the adequacy of representational or symbolic value, though the emphasis is not placed upon the marks or symbols but upon the events or things registered.

(3) *Descriptive or Reference Language Materials.* Another type of linguistic material employed in carrying on organizing and systematizing activities consists of verbal and symbolic elements when the workers are primarily interested in the things referred to or symbolized. Words and symbols as linguistic description or reference are the only possible means of organizing the things referred to. Frequently the structuring procedure is made necessary because the question arises whether words are adequate to refer to or represent the events. Lawyers construct elaborate systems of words to achieve exactness and precision of description no matter how stilted and artificial the language material may be. In addition, there is the meticulous structuring of articles, paragraphs, sections, and clauses. Similarly the Outline, Compend, Handbook, and Index makers construct elaborate organizations of descriptive and indicator units to bring system into things referred to for purposes of memorizing, easy reference guidance, and comprehensive aggregation. Resulting products consist of Legal Codes,

Handbooks of Medical Systems, and Encyclopedias, especially of the Hegelian sort, and Scholastic Summas.

(4) *Intrinsic Symbol Materials*. A highly specialized type of language material comprises technical symbols and propositions (symbol structures) which are of importance to the logician or system builder because the symbols are themselves identified with form, structure, and system. In other words, they represent both the material or content and the theory or essence of systems. Obviously such materials are highly abstruse and specialized and constitute the basis for symbolic and mathematical logics. It is characteristic of such language materials that they involve a minimum of reference or description. Thus the individual units are regarded as sustaining the most limited and invariable relations to each other.

Another and less abstruse form of intrinsic symbol material is that in which descriptions and categories are presumed to be identical with the things described. Intrinsic language of this type, which constitutes phases or factors at least of concrete things whether perceptual or intellectual, comprises the stock in trade of rationalistic philosophers¹ who identify their logical constructions with the things and events with which the logician is presumed to concern himself.

3. *Scientific Materials*. Before we begin the consideration of the tremendous mass of scientific material available for system-building enterprises we must reject the notion that the sole task of logic consists of the momentous work of providing scientists with methods of discovery and investigation. Instead we simply indicate the various features of the scientific domain which provide stimulus and occasion for building systems.

As a matter of choice and illustration we may divide into three types the scientific sources for logical work. (a) First, the system builder can isolate, classify, and order the general field of investigated events; so he can divide off the various sciences from each other and indicate the variations and interrelations between them. In this way systems are built up by separating off inorganic, organic, and anthropic materials, for example.

Within each of these distinguished domains the system builder

¹ Cassirer, *Le Langage*. Also, *The influence*.

finds ample scope for his activities. For instance, he constructs periodic tables of elements, charts of organic constituents and substituents, geologic hierarchies, tables of boiling and freezing points, tables of every sort of physical constants, and organizations of units (individuals) and compounds (families, gens) among human aggregations, etc.

(b) Next, logicians can work on the various formulae, laws, and principles derived from the investigation of the events in particular domains. Here, of course, the emphasis is centered upon materials constructed upon the basis of contacts with the original or discovered events. Naturally the logician finds freer and more spontaneous auspices for his work on this level.

(c) Finally, there is the material for system building composed of the systems already constructed on the two lower levels. On this third level logicians are occupied in organizing and ordering systems of scientific systems. Moved by various cosmic interests the system builders hope to order data and principles in a total coherent structure.

Examples of logical systems using such materials are provided by Humboldt in his *Cosmos*, Comte in his *Philosophie Positive*, and Spencer in his *Synthetic Philosophy*. Many philosophers, logicians, historians, librarians, and others have exercised their talents in constructing such systems.³ Whether the criterion be degree of abstractness, importance, fundamentalness, utility or some other, the sciences as units, their interrelation and resulting systems, stand out for the most casual observer. Sciences as building blocks fully reveal themselves in the perennial assumption that mathematics is basic to all others.

4. *Relations as Materials*. Especially to be mentioned as materials for system building are relations of all types since they are all too frequently overlooked. Often they are taken, rather, as positions or matrices whether simply discovered or organized.

To correct this view it is only necessary to consider the frequent instances in which certain items which at first were regarded as purely formal symbols, such as -1 , $\sqrt{-1}$ or the whole series of complex numbers, later became recognized as models or equa-

³ Benjamin, *Logical Structure*; Bliss, *System, and Organization*; Flint, *Philosophy*; Goblot, *Le Système*; Peirce, *Collected Papers*; Richardson, *Classifications*; Tennant, *Philosophy*; Thomson, *Introduction*.

tional descriptions of actual relations. Similarly, simple integers as parts of calculative or ordering systems can be such potent factors in scientific work (even though they abstract from concrete qualities or properties); also relations as parts of relation systems can be dealt with by means of symbols for system-building purposes. This is equally the case whether the resulting systems are regarded as everyday facts or as epistemological or ontological structures.

AUSPICES AND CONDITIONS OF LOGIC

Traditional logic has not only neglected but positively rejected problems concerning auspices and conditions. This is, of course, an inevitable consequence of the attribution of ultimacy and universality to logic. In all logical folklore the most firmly established item is that logic and its laws are presupposed by and basic not only to all science but all rational investigation.³ When, on the other hand, logic is taken to be an enterprise of specific system building the innumerable opportunities and occasions for logical activity bespeak the importance of its auspices and conditions. On the whole, logical auspices determine primarily whether or not system building takes place at all, what is systematized, and the scope and freedom of systems, whereas the conditions affect the particular techniques and procedures employed.

1. *System-Building Auspices.* As our chapter on Doctrinal Succession in Logic has indicated the entire tradition of technical logic illustrates the process in which specific auspices of a general cultural sort have facilitated its development. In brief, the whole doctrinal succession shows how specific auspices direct the development and modification of technical system-building theory. Needless to say, similar logical auspices account for the processes of non-technical system building.

(1) *Philosophical Auspices.* Naturally the philosophical auspices provide the most comprehensive scope for system building. In fact, to such auspices may be attributed the general notion of one logic only, as well as the construction of such a system to take in the entire cosmos. Incidentally, too, it is such auspices with their subtle effect upon system builders which determine the notions of absoluteness and totality of coverage and inclusion.⁴

³ Cf. Lewis, Structure.

⁴ This is the common element that runs through Scholastic and numerous other types of *a priori* or philosophic logic.

Since these philosophical auspices inevitably result in a tremendous network of presupposition, systems built under such ascendancy provide the greatest freedom of system construction and lead to the creation of various autistic structures more or less dissociated from concrete events. Hence the overemphasis of form and the neglect of the fact that logical construction is continuous with system-building practice.

As regards the material of construction the philosophical auspices provoke special inclination toward linguistic elements. Such elements can be most readily detached from actual thing materials, so that anything can be referred to or symbolized. In this way by devious abstractional methods it is easy to attain notions of pure logic, that is, logic uncontaminated with anything but the processes and results of pure reasoning.

(2) *Scientific Auspices.* Systems built with some attention to scientific materials tend toward a more contentful structure. Still, the dominance of universalistic traditions leads to comprehensive systems stressing abstractions. Accordingly methodological logics are still inclined toward the one-system idea and all the variants are concerned with presumed improvements in such single systems. To a great extent methodological logics stress epistemological problems in contrast to the philosophical issues which center around ontological inquiries. In general, logics built under scientific auspices stress the methods of building instead of results.

(3) *Mathematical Auspices.* As the entire range of intellectual history shows, mathematical ways of thinking have exerted a powerful influence upon logical theorists as well as all other intellectual workers. The logical result of work performed under mathematical auspices indicates the paradox that a close adherence to material still results in the most formal systems. The reason, of course, is that mathematics is concerned with relations, and relations have always been regarded as formal and empty or even as attributes of relata. Thus the mathematical auspices of logic have not tended to divert logicians from the notion of a single logic, though it has restricted somewhat the freedom with which systems are constructed. This restriction is closely related to the presumed rigid validity arising from preoccupation with the organization of relations.

To illustrate what is meant by logical auspices we may follow

through some steps in the development of the logicization of arithmetic. With a developing background of postulational theory originating in the domain of geometry logicians attempted to follow up the same procedure in the field of numbers. The result was a system which at best can be called a description of some basic processes of arithmetic. Not satisfied with the building of a satisfactory descriptive system these logicians like to look upon their work as the reduction of arithmetic or mathematics as a whole to logic. In adopting this view they had to overlook all the actual developments of processes and products step by step with all sorts of trials and errors and with a full compliment of intuition so antipathetic to logicians.

(4) *Practical Auspices.* Though it seems evident that the study of system building in practical situations should have led to a very different type of logical theory from any represented by the prominent logical systems this has not proved to be the case. Actual instances of system building have had no influence in the court of logic as against the ideas that there is something ultimate and absolute about logic. Were the actual practical auspices under which system building occurs taken into account logical theory would certainly become more relativistic and specific and allow fully for problems of relevancy and temporary utility. In line with our general viewpoint this procedure would not lead to any liquidation of the other types of absolutistic, permanent, and comprehensive universalistic systems, but rather give to each its proper evaluation with regard to the auspices under which it is constructed.

2. *System-Building Conditions.* As we have indicated, system-building conditions constitute primarily the circumstances affecting the procedures and general techniques employed. An outstanding question is whether the logician builds for the sake of system or for some other purpose. Logical techniques differ if the work is pure in the above sense or if it is applied.

(1) *Pure and Applied Logic.* The work of pure system is conditioned by an effort toward completeness and comprehensiveness. On the whole this involves rigorous elimination of materials, and the attainment of simple and mechanical techniques, for example, equations, squares of opposition, syllogistic organization. In general, the technique tends toward tautology.

If logic is applied, as when the system is secondary, the tech-

nique or procedure tends toward a lesser completeness and simplicity. In this case the watchword is organization, bringing together relevant or required items. Under such circumstances the system builder strives toward order and arrangement or inclusion and filling of gaps. Applied logic also includes trial and error instead of so-called self-evident processes favoring exhibition and demonstration.

(2) *Deliberate and Casual Logic.* When system building is deliberate the builder sets up a definite pattern which he attempts to follow, whereas in casual system building he is guided by the materials. In other words, some accepted logical theory insinuates itself into deliberate system-building technique in contrast to the more naturalistic and incidental procedure in casual systematizing. Though professional logicians may not like to include the casual type in the august domain of logic, the similarities in process and result cannot be gainsaid. Such similarities, moreover, simply reinforce the contention that a definite continuity exists between the everyday activities of system building and the most technical and elaborate logical enterprise.

CANONS OF LOGIC

That logic has always been intimately interrelated with canons, with rules and laws, with criteria of validity may be taken as evidence that logicians have always worked as system builders. Their beliefs, of course, have been contrary to their practice. Traditionally they have regarded canons as revelations of the ultimate character of reasoning or the inviolable laws of thought. Whether logical canons are limited to Identity, Contradiction, and Excluded Middle^a or expanded to include others they are inseparably bound up with ultimate, transcending, and universal systems. One can hardly doubt that traditional canons constitute products derived by generalizing assumptions or postulates for particular systems into fixed universal rules. From the standpoint of relative and specific logic traditional canons must give way to particular rules for the construction of specific systems.

Specificity canons, for example, imply that system building should be performed with due regard to particular kinds of materials operated upon in given frames of reference. Above all,

^a These laws are discussed in a later chapter.

specificity canons constitute injunctions against the belief that logic consists of some autonomous type of activity prior to other activities either in everyday affairs or in science. In short, there are no "logical" canons or principles governing all other kinds of intellectual work. When one builds a system, obviously there are rules, but instead of being rules of reason they are, rather, directions for carrying out particular enterprises. Since there is no reasoning except as specific interbehavior with particular things, there can be no isolated canons of reasoning. Reasoning is an operational manipulation of materials reasoned about.

Logical canons envisaged as guides and injunctions concerned with particular system-building enterprises apply to particular details of such enterprises. Accordingly, we may differentiate between rules concerned with materials, their selection, and differentiation, and rules which have to do primarily with processes or operations of building systems out of them. Again, we must not overlook the difference between rules involving materials and ordering procedure and rules concerning the validity of the system or its desirability and usefulness for particular purposes. At this point we must forestall the objection that such canons depart completely from the absolute rules of traditional logic by vigorously asserting that if they serve as useful functions in the organization of systems they are not to be regarded as arbitrary or capricious, but as achieving full power of canons in every valid sense.

Canons and Postulates. When dealing with concrete system-building interbehavior an exposition of logical canons overlaps the discussion of postulates. It is not surprising that the statements of background and relationship between logic and other disciplines forecast the rules to be followed in system building. We may, however, draw satisfactory dividing lines between postulates and canons.

Setting up postulates constitutes a process of marking out a field of work with definite indications as to the interrelationship between logic in this case and other types of activity. By comparison, canons result from observations concerning what sorts of operations help or hinder in the particular task undertaken. This fact is illustrated by the suggestion that the traditional "laws-of-thought" canons of identity, contradiction, and excluded middle

may be best interpreted as rules discovered to be useful in argument. Similarly, by rejecting all notions of absolute mind or reason canons may be regarded as selected suggestions useful in furthering prospective system building.

Positive and Negative Canons. By negative or avoidance canons we refer to the suggestions concerning what errors and ineptitudes to shun in building systems. On the other hand, there are also positive suggestions concerning the proper way to erect a system, as illustrated by the procedural rules for setting up a proper mathematical system.

To a great extent the negative canons may be related to entangling cultural or traditional beliefs and attitudes. We must avoid, for instance, carrying over an argument from one field in which it may be entirely satisfactory to another in which it does not fit so well. Such avoidance leads directly to the adherence canons, the primary aim of which is to cling closely to the materials and problems at hand.

(A) *Negative Canons.*

(a) *Canons of Nonuniversality.* Our first series of negative canons enjoins all occupation with cosmic systems. The logical theorist must realize that logical work is not only specific to particular situations and subject matter but also to the activity of particular individuals. Unless we confuse our formulae with the universes they are presumed to symbolize, our interbehavior can never be cosmic in character, nor can we ever arrive at systems embracing the universe or nature as a whole, unless the terms *universe* or *nature* signify for us simply the outer boundaries of a localized and particular situation. Likewise we can never pass to such cosmic entities as a uniform nature. Expansive behavior of this kind we learn from intellectual history always encloses us in the cell of our private or autistic ideas. Cosmic logics with their universal systems have never gone beyond the creations produced from the materials of private sensations or reasons, and in the end become reduced to mere verbal formulae.

Nonuniversalistic canons, therefore, warn us against traditional philosophical systems, unless, of course, we are willing to occupy ourselves with systems having no existential import and no fundamental use or significance. The history of logic indicates that

specificity, uniqueness, and limited character are the main attributes of all traditional philosophical logics. Those who erect such systems simply assume certain traditional and conventional beliefs and proceed to build systems without regard to the principles effective in significant logical work.

Logic, then, must not only be free of traditional philosophy, but must strive for a place of its own among the intellectual disciplines. Certainly there is a place for a science of logic which will concern itself with system-building procedures as definite and actual inter-behavior in specific situations.

The nonuniversalistic canons are not merely admonitions against comprehensive philosophical systems, but scientific ones as well. Eminent scientists frequently proclaim the work of science to be the construction of all-inclusive systems of fact. For example, Einstein⁹ writes: "Science is the attempt to make the chaotic diversity of one's sense experience correspond to a logically uniform system of thought." Such a view overlooks the enormity of scientific subject matter and the limited capacities and knowledges of scientists.

At most this view signifies that no fundamental clashes of attitude or technique should occur between various scientists. Actually, Einstein goes on to limit his remarks to physics. And even there he writes of the various branches and their specialization in which the workers strive to achieve a theoretical understanding of more or less restricted fields of experience. True, as he says:

There has always been present the attempt to find a unifying theoretical basis for all these single sciences, consisting of a minimum of concepts and fundamental relationships, from which all the concepts and relationships of the single disciplines might be derived by logical processes.

Such a system, if it could be achieved even in a single scientific domain, would, of course, mark a static equilibrium and would be far from the actual processes of science with their specificity of subject matter, problems, and systematic organization.

(b) *Canons of Nontranscendence.* The injunction to avoid all ultimate and transcendent criteria is aimed at every sort of *a priori* factor implying absolute materials or techniques of knowledge.

⁹ Considerations.

Obviously such factors cannot find a place in human interbehavior which is perforce limited to specific operations upon particular things and conditions.

There are two primary sources for *a priori* or transcendent principles: (1) the historical philosophical belief in ultimate reasoning powers, which can grasp absolute and permanent self-existing entities, or in creative psychic powers which produce its objects in whole or part, and (2) the transformation of concrete hypotheses, postulates, and assumptions into permanent transcendent principles.

The importance of the nontranscendence criteria lies precisely in the fact that while there is no dearth of critics of absolute *a prioris*' there is still need to bring this criticism down to actual psychanthropic situations. On this level it is plain that all logic consists of system building performed with concrete materials of various sorts and in which all principles and criteria are developed in the process. Otherwise the logician arbitrarily constructs transcendent criteria. Again, on this level, the rational or other processes are themselves derived from contact with things, whereas the materials are likewise developed from their ingredients in the interbehavior. On this basis the *a priori* relation to a particular system or situation can be observed in its stage by stage evolution, along with the fact that no logical processes or principles have any transcendence or any power of carrying over except the process of system building itself. The development of a logic based upon a preexisting mind or the absoluteness of certain discovered relations among elements as well as upon the independent power of terms, words, and sentences can be easily traced to various cultural circumstances.

It is a patent fallacy to regard a set of conclusions or working principles developed in one situation as applicable to another except when the two situations have common factors. Principles as carry-over factors are indispensable tools in building systems in everyday life and in scientific situations, but such principles are specific with respect to particular sectors of system-building activity. At most, such principles have to do with interpolations and extrapolations from concrete situations. To avoid transcendent principles is to place a definite check upon system construction.

¹ Cohen, Reason, Lewis, Mind.

(c) *Canons of Canon Avoidance.* The purport of these rules is to avoid all ultimate criteria for the employment of system-building techniques. The fundamental point is to eliminate the traditionally fixed techniques and operational goals designed for the organization of argument, proof, as well as scientific investigation and the general systematization of scientific results.

Of the two forms of technique imposed by conventional logical canons one consists of construction and the other of reduction methods. Both are designed to achieve absolute and certain results, despite the fact that the materials worked upon and the techniques involved preclude any such fixity of procedure.

The reduction principles imply formalization, the attenuation of materials to fit into prescribed forms. Because symbols or formal propositions are the goals set for all systematizing activity, psychological happenings are reduced to the more fundamental biological events, while the biological field is reduced to the physical and finally to a mathematical level. Nothing is presumed to be more fundamental and certain than equations unless it be the still more tenuous pattern of formal logics themselves. This procedure in science, of course, always results in a stultification and impoverishment of the field. Omitted as unimportant are the actual data and concrete problems of science. Likewise overlooked is the fact that the systematic organization of formulae and propositions is no more an ultimate and absolutely certain process than concrete investigations.

The constructional phase of traditional canons is primarily concerned with setting up finalistic systems—for example, integrated systems of tautologies. The result is that system-building techniques become ends in themselves and not features of systematizing enterprises in particular situations.

(d) *Canons of Tradition Avoidance.* While all negative canons are designed to avoid system-building traditions, especial vigilance must be exercised against traditions which place limits upon system-building activities. Since all system-building rules must be derived from actual operations, no rule can be effective which circumscribes operations or materials because of traditional logical work. Scientific and logical experience indicates that any established line of work or technique, no matter how excellent, exerts a conservative influence and certainly interferes with the search for new

techniques and processes. In view of the fact that novelty of materials and problems is inevitable the avoidance of traditions is advisable. System building should be sensitive to specific problems, cultural settings, and the possibility of new postulate systems.

(B) *Positive Canons.*

(a) *Canons of Identification.* The primary significance of identification canons for logical theory is their mnemonic value in indicating the scope of system building and variability of the specific enterprises. Keeping before us such a table of logical definitions as the following is a useful device in avoiding fallacies.

- a. Logic as Rules of Argument
- b. Logic as Rules of Proof or Evidence
- c. Logic as Correct Thinking
- d. Logic as Scientific Method
- e. Logic as Rules of Validity
- f. Logic as Theory of Inquiry
- g. Logic as the Morphology of Knowledge
- h. Logic as System Building

To consider that different logical theories have been constructed on the basis of diverging interests and materials is to avoid confusion of techniques, criteria, and problems. Canons of identification may thus be of use in determining the relationship between configurations, contents, and purposes of systems. In other words, these canons suggest that at times we must determine the suitability of materials going into our structures and not confine ourselves to simple processes of structuralization. A fallacy following from the latter procedure is setting up structures comprised exclusively of abstractions and forgetting that system-building is for the purpose of carrying on some interbehavioral operation, hence simply forcing through a structure without regard to its purpose or use.

(b) *Canons of Subject Matter.* A fundamental canon of specificity logic reminds us that any kind of systematized material constitutes the subject matter of logic. Thus it is fallacious to confine logical materials to ideas, relations, symbols or to propositions and propositional systems. As previously pointed out, basic to this devastating fallacy is the confusion of referential or symbolic descriptions employed in systems and system building with

the materials or processes described. Witness those logicians who stress the verbal fallacies involved in improper distribution of terms in formal syllogisms instead of interbehavioral fallacies of improperly handling materials in system building.

To avoid making logic into a specialized domain invariably transcending concrete facts of everyday life the present canons call attention to the following variations of view concerning the subject matter found in logical literature.

- a. The Logical as the Ideal
- b. The Logical as Conceptual
- c. The Logical as Conventional
- d. The Logical as Systematic

(c) *Canons of Procedure.* Procedural canons are concerned both with the attitudes involved in system building and with the adaptation of the procedure to the materials systematized.

An attitudinal fallacy to be avoided is that logic is some sort of infallible process invariably attaining absolute existence or truth. On the contrary, the procedural canon makes clear that system building constitutes a trial and error procedure whenever the worker has difficult or unavailable materials with which to cope. Logical work is human and concerns human problems; consequently the actual procedures comprise concrete interbehavior with definite things. Whenever systems are built out of absolutely controlled materials, either by avoiding probable events, tenuous objects, or by using only arbitrarily constructed things, the logical situation is arbitrary and the procedure atypical.

Following our procedural canons we adapt the procedure to the materials worked upon, the procedure consequently being specific to those materials. For example, if we are dealing with mathematical relations the procedure is different from the processes employed in systematizing concrete data of science. To reduce logic to a single type of procedure with fixed rules violates all good principles of system building.

(d) *Canons of Validity.* Instead of constituting fixed formulae statable in abstruse sentences validity canons suggest that system-building criteria are determined by the problems at hand, the materials dealt with, and the goal to be reached. This is a lesson mathematicians learned when quaternions were developed despite

the commutative law of multiplication. In actual system building we may employ all sorts of validating criteria varying from absolute noncontradiction in mathematics to indefinite criteria of serviceability in the organization of rules of conduct. The following table suggests variations in validating criteria with concomitant variations in system-building situations.

- | | | |
|-----------------|-------|----------------------------|
| a. Mathematics | | Noncontradiction |
| b. Science | | Agreement with facts |
| c. Argument | | Conviction |
| d. Law | | Achievement of social ends |
| e. Technology | | Workability |
| f. Formal Logic | | Completeness |

Unless one is committed to some abstruse tautological system or some chosen invariant subject matter these criteria may vary somewhat during the system-building process.

(e) *Canons of Simplification*. All system building constitutes a simplification of the materials worked upon. To fit any sort of material into a system involves classification, formulation, and organization, operations which reduce the number of elements on the basis of similarity, difference, and analogy. This simplification of materials correlates with simplifications in operation. In other words, particular sorts of techniques are arbitrarily preferred or prove to be more effective in some manner. Similarly, there is simplification in the product in the sense that masses of material are reduced to simple records with respect to number, quality, etc. As a rule, too, system organization eventuates in formulae, diagrams, and propositions referring to achieved results. Statistical reports indicate such reduction of constructions to simple form.

The advantages derived from such simplification both in handling or manipulating events and in understanding them have led to a number of fallacies concerning simplifying processes and results. Naturally, canons of simplification are designed to avoid misinterpretation of simplification processes and products. Simplified items are no more genuine or fundamental than the original complicated materials. Simplifications of procedure or construction are similarly only pragmatic elements for some purpose. To be avoided here consequently is the notion that a mathematical formula or some symbolic representation of events is more valu-

able or more fundamental than the original complex from which it is derived.

The merits and demerits of simplification and facile organization may be estimated from the constant references found in mathematical literature to elegance and symmetry of mathematical proofs and systems. Of two mathematical proofs when they *are* proofs, neither one possesses more validity or systemic value than the other, but one may be more symmetrical, compact, and direct, or in general what mathematicians call more elegant or beautiful. This fact is informing concerning the comparison between simplicity and complexity or even between smoothness of organization and confusedness. Although clearness and distinctness may be appealing and in some cases exceedingly useful, we must not overemphasize the systematic if we are interested in events.

(f) *Canons of Fittingness or Relevance.* Among the serious problems in system building are those concerning the coherence and relevance of the materials constituting the system. It is possible to assert that certain materials fit into a system and to give that system the appearance of cohesion and stability, whereas the materials do not actually belong together.

The fallacies involved here concern improper correlations of various sorts. The question is whether certain analogies fit, whether certain implications are genuine or simply forced. To a great extent the problem is solved on the basis of the expertness of the system builder who is presumed to be intellectually equipped to judge the fitness of the materials for systematic organization. On the other hand, the question arises whether the materials fit together from the standpoint of the original specifications set up for the system.

Some concrete illustrations of inapplicable materials for systems are found in the use of the hypothesis dodge. Various writers argue correctly that all scientific systems are built upon the basis of hypotheses. Accordingly, they erect systems out of pure assumptions, which may be neither necessary nor applicable. In this manner psychologists fall back upon neural traces and psychic states to develop an explanatory system for psychological action. Another instance is the assertion that one's scientific opponent is afraid of names. For example, it is said that it is simply a word phobia to object to the use of mind in psychology, and then on

this premise are introduced all sorts of mystic factors into a psychological system.

LOGICAL PRODUCTS

Ingrained as logical theory is with ideas concerning: (1) universal rather than specific norms, (2) the basic and governing character of logic, and (3) the epistemological and ontological relevance of logical processes and rules, it is quite remote from actual system building. Probably not too harsh a charge to make against logical theorists is that they do not take sufficiently into account the evolution of logic and mathematics.

On the whole, then, conventional logical theory does not adequately deal with logical products. For the most part it is concerned with specialized structures which not only are taken to be exclusive types of systems, but because of the normative ideal are not regarded actually as products. Conventional logical theory overlooks the fact that logical products like any other constructed system are functions of many factors, intention or goal, criteria of manipulation enterprises, and the general situations in which the construction takes place. But instead of envisaging logical systems as definite products of particular enterprises theorists typify logical systems on the basis of preferred constructional processes or some special kind of material, as in the mathematical logics.

Taking into consideration the continuity of operations between the various systems of technical logic and those of other scientific, intellectual, mathematical, and everyday life situations, we may well conclude that logical products constitute specific systems emerging from particular interests, materials, and enterprises.

With respect to any sort of system or logical product we may ask concerning its need, use, and significance. Systems are built for expository and critical purposes, for the development and demonstration of new relations, for self-satisfaction as well as for all the other reasons usually mentioned. In addition, when we include the more practical aspects and types of system building we are able to relate the processes of system construction to the other enterprises and situations with which they are interrelated. To make such connections is to throw considerable light on the character and occurrence of logical processes and products, and not infrequently to recognize the subordinate and auxiliary nature of

logical processes as compared with their conventional exaltation.

An interesting problem concerning logical products is their basic shape and composition. Bosanquet⁸ has severely criticized the competitors of his ontological type of logic because for them inference is linear—there is presumed to be a precise beginning and a terminal point in the process. Other logicians who make a sharper distinction between process and product assert that logic either as specific definitional or general structure system is circular.⁹

Now whether logic is regarded as linear or circular, the implied recognition of its systematic character is clear. At the same time there is also indicated the notion of completeness. When actual operations and products are under consideration, however, the completeness of a logical product is a special case depending upon various conditions. By virtue of the materials and status of techniques logical processes and systems may be partial, tentative, and even incompletable. When completable they may also take on any shape as determined by material and technique among other factors. Symmetry, circularity, linearity, and diameter are in no sense *a priori* specifications for logical products. Whatever shape a system assumes depends basically upon the interbehavioral operations which initiate it and tend toward its completion.

Not only do logical systems and products have only so much completeness as the circumstances of the enterprise make possible, but this is likewise true of their validity. To take this notion of enterprise in its proper sense thoroughly excludes any absolute or universally and permanently valid system. Deriving our notion of enterprise from the actual records of logical interbehavior we realize how futile is the quest for a single, normative system basic to all science and other system making.

Is it necessary to repeat that the logical theory here expressed in no wise minimizes the status or value of validity? Logical products are in some cases absolutely valid, as in tautological systems, but this validity, of course, is conditioned by the coordinates of the system.

⁸ Bosanquet, *Implication*, chap. 2, *passim*.

⁹ Lewis, *Mind*, pp. 82, 109f.

BIBLIOGRAPHY

- Angell, J. R. The relations of structural and functional psychology to philosophy, *Univ. of Chicago Decennial Publications, First Series*, vol. III, part II.
- Angus, S. The Religious Quests of the Graeco-Roman World, A Study in the Historical Background of Early Christianity. London, Murray, 1929.
- Aristotle, The Works of. Translated into English. (J. A. Smith and W. D. Ross, eds.) Oxford, Oxford Univ. Press, 1928.
- Augustine, De Trinitate. Libri XV, in *Patrologiae, Series Prima, Tomus XLII*. (J. P. Migne, ed.) Paris, 1845.
- Bell, E. T. The Development of Mathematics. New York, McGraw-Hill, 1940.
- Benjamin, A. C. The Logical Structure of Science. London, Kegan Paul, 1936.
- Bills, A. G. Changing views of psychology as science. *Psychological Review*, 1938, 45, 377-394.
- Black, M. Conventionalism in geometry and the interpretation of necessary statements. *Philosophy of Science*, 1942, 9, 335-349.
- Introduction to his translation of Carnap, *The Unity of Science*. London, Kegan Paul, 1934.
- The analysis of a simple necessary statement. *Journal of Philosophy*, 1943, 40, 39-46.
- Bliss, H. E. The Organization of Knowledge. New York, Holt, 1929.
- A System of Bibliographic Classification. New York, Wilson, 1935.
- Bolzano, B. Wissenschaftslehre: Versuch einer ausführlichen Darstellung der Logik. Leipzig, Meiner, 1914 (originally published Sulzbach, 1837), 4 vols.
- Boole, G. The Laws of Thought. London, Walton and Maberly, 1854.
- The Mathematical Analysis of Logic. London, Bell, 1847.
- Boring, E. G. Titchener on meaning. *Psychological Review*, 1938, 45, 92-95.
- Bosanquet, B. Logic. Oxford, Clarendon Press (2nd ed.), 1911, 2 vols.
- Implication and Linear Inference. London, Macmillan, 1920.
- Boyer, C. B. The Concepts of the Calculus: A Critical and Historical Discussion of the Derivative and the Integral. New York, Columbia Univ. Press, 1939.
- Bradley, F. H. The Principles of Logic. Oxford, Oxford Univ. Press (2nd ed.), 1922, 2 vols.
- Braithwaite, R. B. The relevance of psychology to logic. Symposium: *Aristotelian Society*, Supplementary vol. XVII, 1938.
- Bridgman, P. W. The Logic of Modern Physics. New York, Macmillan, 1927.

- The Nature of Physical Theory. Princeton, Princeton Univ. Press, 1936.
- Brill, A. Antrittsrede in Tübingen, 1884, quoted from Fink, K. *Geschichte der Elementar-Mathematik*. (Beman and Smith trans.) Chicago, Open Court, 1910.
- Buchler, J. Charles Peirce's Empiricism. New York, Harcourt, Brace, 1939.
- The accidents of Peirce's system. *Journal of Philosophy*, 1940, 37, 264-269.
- Bush, W. T. Superstition and Logic. *Journal of Philosophy*, 1932, 29, 232-236.
- Burtt, E. A. The Metaphysical Foundations of Modern Physical Science. New York, Harcourt, Brace, 1925.
- Cajori, F. History of Mathematics. (2nd ed.) New York, Macmillan, 1917.
- Campbell, J. W. Units in mechanics. *Science*, 1937, 86, 441.
- Cantor, G. Grundlagen einer allgemeinen Mannichfaltigkeitslehre, Ein mathematisch-philosophischer Versuch in der Lehre des Unendlichen. Leipzig, Teubner, 1883.
- Contributions to the Founding of the Theory of Transfinite Numbers. (P. E. B. Jourdain, trans.) Chicago, Open Court, 1915
- Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. (E. Zermelo, ed.) Berlin, Springer, 1932.
- Carmichael, R. C. The Logic of Discovery. Chicago, Open Court, 1930.
- Carnap, R. Philosophy and Logical Syntax. London, Kegan Paul, 1935.
- Introduction to Semantics. Cambridge, Harvard Univ. Press, 1942.
- The Logical Syntax of Language. New York, Harcourt, Brace, 1937.
- Testability and meaning. *Philosophy of Science*, 1936, 3, 419-471; 1937, 4, 1-40.
- Cassirer, E. Le Langage et construction du monde des objects. *Journal de Psychologie normale et pathologique*, 1933, 30, 16-44.
- The influence of language on scientific thought. *Journal of Philosophy*, 1942, 39, 309-327.
- Castro, M. The respective standpoints of psychology and logic. In *Univ. of Chicago Philosophical Studies*, Chicago, Univ. of Chicago Press, 1913, no. 4.
- Church, A. An unsolvable problem of elementary number theory. *American Journal of Mathematics*, 1936, 58, 345-363.
- Cohen, M. R. Reason and Nature. New York, Harcourt, Brace, 1931.
- Cohen, M. R. and Nagel, E. An Introduction to Logic and Scientific Method. Harcourt, Brace, New York, 1934.
- Condillac, E. B. Oeuvres complètes de Condillac. Paris, Houel, 1798.
- Courant, R. and Robbins, H. What is Mathematics? New York, Oxford, 1941.

- Couturat, L. The Principles of Logic. In *Encyclopaedia of the Philosophical Sciences*, vol. I. *Logic*, London, Macmillan, 1913.
- Crane, E. J. Words and sentences in science and industry. *Science*, 1937, 86, 549-553.
- Croce, B. The Philosophy of Giambattista Vico. (R. G. Collingwood, trans.) New York, Macmillan, 1913.
- Dantzig, T. Number: The Language of Science. New York, Macmillan, 1939.
- DeMorgan, A. Formal Logic. London, Taylor and Walton, 1847.
- Dewey, J. The Development of American pragmatism. In *Studies in the History of Ideas*, New York, Columbia Univ. Press, 1925, vol. II.
- Logic: The Theory of Inquiry. New York, Holt, 1938.
- The sphere of application of the excluded middle. *Journal of Philosophy*, 1929, 26, 701-705.
- The applicability of logic to existence. *Journal of Philosophy*, 1930, 27, 175-179.
- Essays in Experimental Logic. Chicago, Univ. of Chicago Press, 1916.
- Dubs, H. H. Rational Induction. Chicago, Univ. of Chicago Press, 1930.
- Ducasse, C. J. Some comments on C. W. Morris's "Foundations of the Theory of Signs." *Philosophy and Phenomenological Research*, 1942, 3, 43-52.
- Symbols, signs and signals. *Journal of Symbolic Logic*, 1939, 4, 41-52.
- Propositions, opinions, sentences, and facts. *Journal of Philosophy*, 1940, 37, 701-711.
- Concerning the status of pseudo-object sentences. *Journal of Philosophy*, 1940, 37, 309-324.
- Is a fact a true proposition? A Reply. *Journal of Philosophy*, 1942, 39, 132-136.
- Eaton, R. M. *General Logic: An Introductory Survey*. New York, Scribners, 1931.
- Eddington, A. S. The Nature of the Physical World. New York, Macmillan, 1929.
- The Philosophy of Physical Science. New York, Macmillan, 1939.
- Einstein, A. Considerations concerning the fundamentals of theoretical physics. *Science*, 1940, 91, 487-492.
- Enriques, F. The Historic Development of Logic. (J. Rosenthal trans.) New York, Holt, 1929.
- Fairbanks, A. The First Philosophers of Greece. New York, Scribners, 1898.
- Flint, R. Philosophy as Scientia Scientiarum. London, Blackwood, 1904.
- Frege, F. L. G. Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet. Vol. I, Jena, Pohle, 1893; vol. II, Jena, Pohle, 1903.
- Die Grundlagen der Arithmetik. Eine logisch-mathematische Untersuchung über den Begriff der Zahl. Breslau, Koebner, 1884.

- Galilei, G. *Dialogues Concerning Two New Sciences*. (H. Crew and A. De Salvo, trans.) New York, Macmillan, 1933.
- Goblot, E. *Le Système des Sciences*. Paris, Colin, 1922.
- Gödel, K. *On Undecidable Propositions of Formal Mathematical Systems*. (Mimeographed lecture notes by J. B. Rosser and S. C. Kleene.) Princeton, Institute for Advanced Study, 1934.
- *Ueber formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*. *Monatshefte für Mathematik und Physik*, 1931, 38, 173-198.
- Gödel, K. *Ueber die Länge von Beweisen*. *Ergebnisse eines mathematischen Kolloquiums*, 1936, 7, 23-24.
- Hegel, G. W. F. *The Science of Logic*. (W. H. Johnston and E. G. Struthers trans.) New York, Macmillan, 1929.
- *Wissenschaft der Logik*. (G. Lasson ed.) Leipzig, Meiner, 1923.
- Helmer, C. The significance of undecidable sentences. *Journal of Philosophy*, 1937, 34, 490-494.
- Languages with expressions of infinite length. *Erkenntnis*, 1937-38, 7, 138-141.
- Hilbert, D. and Ackermann, W. *Grundzüge der theoretischen Logik*. Leipzig, Berlin, Springer, 1928, (2nd ed.) 1938.
- Hilbert, D. *Foundations of Geometry*. Chicago, Open Court, 1902.
- Hofstadter, A. Logical form and epistemic function. *Journal of Philosophy*, 1938, 35, 712-717.
- On semantic problems. *Journal of Philosophy*, 1938, 35, 225-232.
- Hume, D. *Treatise on Human Nature*. (L. A. Selby-Begge, ed.) Oxford, Clarendon, 1896.
- Huntington, E. V. A set of postulates for abstract geometry. *Mathematische Annalen*, 1912-13, 73, 522-559.
- The Fundamental Propositions of Algebra. In *Monographs on Topics of Modern Mathematics relevant to the Elementary Field* (J. W. A. Young ed.) New York, Longmans, Green, 1911.
- The Continuum and other Types of Serial Order. Cambridge, Harvard Univ. Press (2nd ed.), 1921.
- Husserl, E. *Logische Untersuchungen*. Halle, Niemeyer, 1913-1921 (2nd ed.), 2 vols.
- *Formale und transzendente Logik: Versuch einer Kritik der logischen Vernunft*. Halle, Meiner, 1929.
- Jäsche, G. B. *Immanuel Kant's Logik: Ein Handbuch zu Vorlesungen*. Königsberg, Nicolavious, 1800.
- Jones, J. *The Mysterious Universe*. New York, Macmillan, 1934.
- *The New Background of Science*. Cambridge, Cambridge Univ. Press, 1933.
- Jeffries, H. *Scientific Inference*. Cambridge, Cambridge Univ. Press, 1937.
- Jevons, W. S. *Elementary Lessons in Logic*. New York, Macmillan, 1912.
- *Principles of Science*. London, Macmillan, 1924.
- Johnson, W. E. *Logic*. Cambridge, Cambridge Univ. Press, 1921.

- Joseph, H. W. B. *An Introduction to Logic*. Oxford, Clarendon, 1916.
- Kant, I. *Inaugural Dissertation. De Mundi Sensibilis atque Intelligibilis Forma et Principiis*. (W. J. Eckoff trans.) New York, Columbia College, 1894.
- *Critique of Pure Reason*. (M. Müller trans.) New York, Macmillan, 1911.
- *Sämmtliche Werke*. (G. Hartenstein ed.) Leipzig, Voss, 1868.
- Kantor, J. R. *An Outline of Social Psychology*. Chicago, Follet, 1929.
- *Principles of Psychology*. New York, Knopf, 1924-26, 2 vols.
- *An Objective Psychology of Grammar*. Bloomington, Indiana, Indiana Univ. Publications, Science Series, no. 1, 1936.
- *Logic and superstition*. *Journal of Philosophy*, 1932, 29, 232-236.
- *Philosophical implications of organismic (interbehavioral) psychology*. In *Essays in Philosophy* (T. V. Smith and W. K. Wright eds.) Chicago, Univ. of Chicago Press, 1929.
- *Can the psychophysical experiment reconcile introspectionists and objectivists?* *American Journal of Psychology*, 1922, 33, 481-510.
- *The operational principle in the physical and psychological sciences*. *Psychological Record*, 1938, 2, 1-32.
- *An interbehavioral analysis of propositions*. *Psychological Record*, 1943, 5, 309-339.
- *The nature of psychology as a natural science*. *Acta Psychologica*, 1938, 4, 1-61.
- Kapp, E. *Greek Foundations of Traditional Logic*. New York, Columbia Univ. Press, 1942.
- Keeton, M. T. *On defining the term fact*. *Journal of Philosophy*, 1942, 39, 123-132.
- Keynes, J. N. *Studies and Exercises in Formal Logic*. (4th ed.) London, Macmillan, 1906.
- Keyser, C. J. *Mathematical Philosophy*. New York, Dutton, 1932.
- Klein, F. *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Berlin, Springer, Part I, 1926; Part II, 1927.
- Leibniz, G. W. *Dialog über die Verknüpfung zwischen Dingen und Worten*. In Buchenau-Cassirer, *Hauptschriften zur Grundlegung der Philosophie*, Band I, Leipzig, Meiner, 1903.
- *Zur allgemeinen Charakteristik*. In Buchenau-Cassirer, *Hauptschriften zur Grundlegung der Philosophie*, Band I, Leipzig, Meiner, 1903.
- *New Essays Concerning Human Understanding*. (A. G. Langley trans.) Chicago, Open Court, 1916.
- Lesser, A. *Superstition*. *Journal of Philosophy*, 1931, 28, 617-628.
- Lewis, C. I. *A Survey of Symbolic Logic*. Berkeley, Univ. of California Press, 1918.
- *Alternative systems of logic*. *Monist*, 1932, 42, 481-507.
- *Mind and the World Order*. New York, Scribner, 1932.

- Some logical considerations concerning the mental. *Journal of Philosophy*, 1941, 38, 225-233.
- The structure of logic. *Journal of Philosophy*, 1921, 18, 505-516.
- Lewis, C. I. and Langford, C. H. *Symbolic Logic*. New York, Century, 1932.
- Linke, P. F. Logic and epistemology. In *Philosophy Today* (E. L. Schaub ed.) Chicago, Open Court, 1928.
- Lipps, T. *Grundzüge der Logik*. Hamburg, Leipzig, Voss, 1893.
- Locke, J. *An Essay Concerning Human Understanding*. London, Johnson, 1801.
- Love, A. E. H. *Theoretical Mechanics*. Cambridge, Cambridge Univ. Press, 1906.
- Lotze, R. H. *Logic*. Oxford, Clarendon (B. Bosanquet ed.) (2nd ed.) 1888.
- McGilvary, E. B. Professor Dewey, Logician-Ontologist. In *A Symposium of Reviews of John Dewey's Logic: The Theory of Inquiry*. *Journal of Philosophy*, 1936, 36, 561-581.
- The Lorentz transformation and space-time. *Journal of Philosophy*, 1941, 38, 337-349.
- Mach, E. *The Science of Mechanics*. Chicago, Open Court, 1907.
- Mead, G. H. A behavioristic account of the significant symbol. *Journal of Philosophy*, 1922, 19, 157-163.
- *The Philosophy of the Act*. Chicago, Univ. of Chicago Press, 1938.
- Mill, J. S. *A System of Logic, Ratiocinative and Inductive*. London, Parker, 1862.
- *An Examination of Sir William Hamilton's Philosophy*. New York, Holt, 1884, 2 vols.
- Moore, W. Indexical and presentative functions of signs. *Philosophy of Science*, 1942, 9, 367-371.
- Morris, C. W. *Foundations of the Theory of Signs*. Chicago, Univ. of Chicago Press, 1938.
- Murphy, A. E. Concerning Mead's philosophy of the act. *Journal of Philosophy*, 1939, 36, 85-103.
- Nagel, E. Can logic be divorced from ontology? *Journal of Philosophy*, 1929, 26, 705-712.
- Charles Peirce's guesses at the riddle. *Journal of Philosophy*, 1933, 30, 365-386.
- Osgood, W. F. *Mechanics*, New York, Macmillan, 1937.
- Peano, G. *Formulaire de Mathématiques*. Turin, Bocca, 1894-1908, 5 vols.
- Peirce, B. *Linear Associative Algebra*. (C. S. Peirce ed.) New York, Van Nostrand, 1882.
- Peirce, C. S. *Studies in Logic*. By Members of Johns Hopkins University, Boston, Little Brown, 1883.

- Collected Papers. (C. S. Hartshorne and P. Weiss eds.) Cambridge, Harvard Univ. Press, 1931-35, 6 vols.
- Pieri, M. Della geometria elementare come sistema ipotetico deduttivo. Monografia del punto e del moto, *Memorie della Reale Accademia delle Scienze di Torino*, Serie II, 1899, 49, 173-223.
- Sur la geometrie envisagée comme un systeme purement logique. *Bibliothèque du Congrès International de Philosophie*, 1901, 3, 268-377.
- Plato. Dialogues. (B. Jowett trans.) Oxford, Oxford Univ. Press, 1892, 5 vols.
- Poincaré, H. The Foundations of Science. New York, Science Press, 1921.
- Quine, W. V. Designation and existence. *Journal of Philosophy*, 1939, 36, 701-709.
- Reply to Professor Ushenko, *Journal of Philosophy*, 1942, 39, 68-71.
- Mathematical Logic. New York, Norton, 1940.
- Ramsey, F. P. The Foundations of Mathematics and other Logical Essays. New York, Harcourt, Brace, 1931.
- Richardson, E. C. Classifications, Theoretical and Practical. New York, Scribners, 1901.
- Rignano, E. The Psychology of Reasoning. New York, Harcourt, Brace, 1923.
- Ritchie, A. D. Whitehead's defence of speculative reason. In *The Philosophy of Alfred Whitehead*. Evanston, Northwestern University, 1941.
- Rosinger, K. E. Material truth in rational thinking. *Journal of Philosophy*, 1933, 30, 293-302.
- Ross, W. D. Aristotle, London, Methuen, 1923.
- Rosser, J. B. Extensions of some theorems of Gödel and Church. *Journal of Symbolic Logic*, 1936, 1, 87-91.
- An informal exposition of proofs of Gödel's Theorems and Church's theorem. *Journal of Symbolic Logic*, 1939, 4, 53-60.
- Royce, J. The Principles of Logic. In *Encyclopaedia of the Philosophical Sciences*, vol. I. (A. Ruge ed.) London, Macmillan, 1913.
- The Spirit of Modern Philosophy. New York, Houghton-Mifflin, 1892.
- Russell, B. Introduction to Mathematical Philosophy. New York, Macmillan, 1924.
- Principles of Mathematics. Cambridge, Cambridge Univ. Press, 1903; (Reissue) New York, Norton, 1938.
- Discussion on the axiom of infinity. *Hilbert Journal*, 1904-05, 2, 809-812.
- Symposium, The relevance of psychology to logic. Symposium, *Aristotelian Society*, Supplementary vol. no. XVII. London, Harrison, 1938.
- The theory of implication. *American Journal of Mathematics*, 1906, 28, 164.

- Santayana, G. *Egotism in German Philosophy*. New York, Scribners, 1916.
- Schiller, F. C. S. *Formal Logic*. London, Macmillan, 1912.
- *Logic for Use*. London, Bell, 1929.
- Sheffer, H. M. Review of *Principia Mathematica*, vol. 1 (2nd ed.) *Isis*, 1926, 8, 226-231.
- Spencer, H. *Principles of Psychology*. New York, Appleton, 1883, 2 vols.
- Stebbing, L. A. *A Modern Introduction to Logic*. London, Methuen, 1942 (3rd ed.).
- *Sounds, shapes, and words*. *Aristotelian Society*, Supplementary vol. XIV, London, Harrison, 1935.
- Stevens, S. S. *Psychology and the science of science*. *Psychological Bulletin*, 1939, 36, 221-263.
- Stevens, S. S. and Davis, H. *Hearing: Its Psychology and Physiology*. New York, Wiley, 1938.
- Strong, E. W. *Procedures and metaphysics: A Study in the Philosophy of Mathematical-Physical Sciences in the 16th and 17th Centuries*. Berkeley, Univ. of California Press, 1936.
- Struik, D. J. Kepler as mathematician. In *Johann Kepler, 1571-1630, A Tercentenary Commemoration of his Life and Work*. Baltimore, Williams and Wilkins, 1931.
- Tarski, A. Einige Betrachtungen ueber die Begriffe der ω -Widerspruchsfreiheit und der ω -Vollständigkeit. *Monatshefte für Mathematik und Physik*, 1933, 40, 97, 112.
- Der Wahrheitsbegriff in den formalisierten Sprachen. *Studia Philosophia*, 1936, 1, 261-405.
- Grundlegung der wissenschaftlichen Semantik. *Acts du Congrès International de Philosophie Scientifique*, Paris, 1936.
- On undecidable statements in enlarged systems of logic and the concept of truth. *Journal of Symbolic Logic*, 1939, 4, 105-112.
- *Introduction to Logic and to the Methodology of the Deductive Sciences*. New York, Oxford, 1941.
- Tennant, F. R. *Philosophy of the Sciences*. Cambridge, Cambridge Univ. Press, 1932.
- Thomson, J. A. *Introduction to Sciences*. New York, Holt, 1911.
- Ueberweg, F. *System der Logik und Geschichte der logischen Lehren*. Bonn, Marcus, 1882.
- Ushenko, A. P. *The Problems of Logic*. Princeton, Princeton Univ. Press, 1941.
- Dr. Quine's theory of truth-functions. *Journal of Philosophy*, 1942, 39, 60-67.
- Veblen, O. A System of Axioms for Geometry. *Trans. Mathematical Society*, 1904, 5, 343-384.
- Venn, J. *Symbolic Logic*. London, Macmillan, 1881, (2nd ed.) 1894.
- *Empirical Logic*. London, Macmillan, 1889.
- Wallace, W. *The Logic of Hegel*. (Translated from the *Encyclopaedia of the Philosophical Sciences*.) Oxford, Clarendon, 1892.

- Weiss, P. The essence of Peirce's system. *Journal of Philosophy*, 1940, 37, 253-264.
- Weyl, H. Ueber die neue Grundlagenkrise der Mathematik. *Mathematische Zeitschrift*, 1921, 10, 39-79.
- Whitehead, A. N. Introduction to Mathematics. New York, Holt, 1911.
- Whitehead, A. N. and Russell, B. Principia Mathematica. Cambridge, Cambridge Univ. Press, 1910-12, 3 vols., (2nd ed. 1925-27).
- Windelband, W. History of Philosophy. New York, Macmillan, 1910.
- The principles of logic. In *Encyclopaedia of the Philosophical Sciences*. London, Macmillan, vol. I (A. Ruge ed.), 1913.
- Wittgenstein, L. Tractatus Logico-philosophicus. New York, Harcourt, Brace, 1922.
- Wolf, A. A History of Science, Technology, and Philosophy in the Sixteenth and Seventeenth Centuries. New York, Macmillan, 1935.
- Wundt, W. Logik. Stuttgart, Enke (3rd ed.), 1906-8, 3 vols.
- Young, J. W. Lectures on Fundamental Concepts of Algebra and Geometry. New York, Macmillan, 1911.

SUBJECT INDEX

- Absolutism, in logic, 10f., 123f., 155f., 168, 230f., 320f., 331.
- Abstractions, and logic, 13f.; as nonexistent, 163ff.; as subsistential, 163ff.; not inherent in things, 165.
- Act vs. product, 224.
- Algebraic logic, definition of, 90f.; vs. logistic logic, 90f.; and symbols, 92; Boole's system of, 91ff., 114; function of, 281f.; ultimacy and generality in, 93.
- Algorithmic logic (see Algebraic logic).
- Algorithms and calculi, 287.
- Analogistic inference, 191.
- Analogizing, 317f.
- Analytic vs. dialectic logic, 30, 57, 61.
- Analyzing, as a logical technique, 318f.
- A priori*, the, 29, 68, 70, 158, 166; and instrumental logic, 103; elimination of, from logic, 178; and *a posteriori* knowledge, 186; relative vs. absolute, 320f.; revised notion of, 321; sources of, 335f.
- Aristotelian logic, 23ff., 46ff.; not absolute, 34f.; transformation of, by Scholastics, 48f.
- Arithmetic, Frege's logical system of, 285; logicization of, 330f.
- Art, logic as, 128; mathematics as, 133.
- Associationistic logic, 115, 117f.
- Assuming interbehavior, in logical work, 175.
- Attitudes, intellectual, and logic, 173f.
- Auspices of logical work, 300f., 329ff.
- Axioms, development of, 43; and assuming interbehavior, 174; as constructional products, 258f.; in mathematical enterprises, 259.
- Behavior fields, as psychological events, 151ff.
- Believing interbehavior, in logic, 174.
- Biological processes, in logic, 7, 116f., 101; as participants in psychological events, 149f.; as explanatory principles, 150.
- Biological psychology, 116f., 135f.
- British empirical tradition, in logic, 112, 178.
- Cardinals, transfinite, 291.
- Calculating operations, 281.
- Calculus, the, 55, 87; as operational tool, 214, 286f.
- Canons, logical, 301, 332ff.
- Categories, for the Greeks, 70; and Reality, 320; Kantian, 57ff., 61, 68; in traditional logic, 29.
- Cause, as a semiotic construction, 243f.
- Certainty in logic, 68, 305.
- Classes, as constructional products, 256ff.
- Classification, and division, 38ff.
- Coherence, as logical or rational criterion, 187f., 341f.
- Comprehensiveness, in logic, 10, 107, 124f., 303f.
- Concepts, ideas, and terms, 172.
- Conceptualists, logical, 16, 37f.
- Connotation, in nominalism and neo-classicism, 271f.
- Consistency, as logical aim, 302, 305f.
- Construction, and system building, 9, 294; based on contact with stimulus objects, 16; based on the psychic, 144; analogical, vs. definite reference, 137; vs. data, 172; scientific, 139; and reasoning, 132; and dualism, 145; and the constructor, 224; materials and products of, 230ff., 246ff.
- Constructionistic mathematics, 218.
- Constructor, the, and system validity, 225; and system of products, 235.
- Continuum of nature, 150f., 153.
- Contradiction, law of, 38, 43f., 92, 332, 333f.
- Contraposition, as transpositional inference, 280.
- Conversion, as a logical technique, 280, 316.
- Conventional, the, as definition of logic, 159.
- Cosmic dialectics, 124f.
- Cosmic systems, in logic, 4, 303f., 305, 310, 315f., 321, 328, 329, 334.
- Counting, as nonreferential linguistic activity, 198f.
- Creational interbehavior, 152f.
- Crude data vs. construction, 147.
- Cultural evolution, in logical systems, 284ff.
- Cultural tradition, in absolutistic logic, 168; in propositions, 232; in psychology, 135; and the rational, 188; and the naming process, 216.

- Culture, and logic, 6, 11ff., 21ff., 24ff., 47ff., 51ff., 56, 60ff., 63f., 67ff., 73, 107; and reasoning, 188; as product of language, 18.
- Data transcendence, 304.
- Deduction, 22, 30, 33ff., 52, 81, 88, 93, 95f., 108, 135, 160, 313, 319; related to induction, 288; as agency of discovery, 182; as a type of inference, 190.
- Deduction-implication, 95f.
- Definition of logic, by methods, 157f.; by types, 158ff., 338.
- Demonstration and proof logic, 37, 306f.
- Denotation, in nominalism and neoclassicism, 271f.
- Descriptions, confused with objects described, 90, 123, 147, 172, 208, 327; in instrumental logic, 105; double role of, 261; hypostatization of, 316.
- Dialectics, in Greek logic, 49f.; in instrumental logic, 104, 125; Hegelian, 60ff., 124f., 166; Kantian, 57.
- Dichotomy problem in logic, 162.
- Doctrinal functions, 293.
- Dualism, as a cultural fact, 143; implemented by: experimentation, 144, constructionism, 145, operationism, 145; and verbal conceptualism, 146f.; biological principles in, 149.
- Dualistic psychology, 134ff.
- Epistemology, Kantian, 57f.; and logical comprehensiveness, 124f.
- Equations, as products of symbolization, 224; mathematical, 121.
- Essences, names of, 50; as logical material, 119; and psychologism, 119; in construction of classes, 256f.
- Evolutionary doctrine, influence upon logic, 69.
- Excluded middle, law of, 38, 92, 159, 161, 164, 220, 332, 333.
- Exclusiveness, in logic, 315.
- Experimentalism, vs. rationalism, 82.
- Experimentation, and dualism, 144; effect of, upon Kantian logic, 112.
- Expressions, and confusion of terms, 267f.
- Extension, for nominalist and neoclassicist, 150f.
- Extensional, the, in physics and psychology, 150f.
- Events, as statements of events, 316; linguistic abstractions from, 163f.
- Field character of linguistics, 206.
- Fields, interbehavioral, in logic, 5ff.
- Finality, in logic, a function of specific situations, 10.
- Form, vs. content or matter, 28f., 102f., 127f., 159f., 294; in instrumental logic, 102; in Kantian logic, 58f.
- Forms, vs. functions, 276f.; and meanings, 276f.
- Formal, the, as exclusive logical subject matter, 127f., 159f.; vs. the factual, 160.
- Formal logic, 17, 23, 38, 39f., 48ff., 73, 77, 159f., 272.
- Formalism, and words and symbols, 272; a technique of manipulating abstractions, 314.
- Formalistic mathematics, 95, 217f., 286.
- Formalizing, as logical technique, 43, 313f.; of language and symbols, 195f., 220.
- Frame of reference, cultural, influence upon logic, chap. 2, 111.
- Functions, their various connotations, 251ff.; as a product of interbehavioral fields, 252; mathematical usage of, 252.
- Generalizing, in logical work, 10, 315f.; in algebraic logic, 90f.; identified with logic, 97; in mathematics, 97.
- Geometry, 11, 23, 32, 54, 209; non-Euclidean, 97, 118, 208f.; Euclidean, 58, 61, 118, 208f., 231; connection of conventional logic with Greek, 172.
- Grammar, and logic, 75; segregation of, from things, 294.
- Grammatical syntax, 221.
- Greek logic, 31ff.
- Humanistic logic, 99f., 116.
- Hypostatization, 29, 35; of propositions, 135, 229; of descriptions, 316; as a system of logic, 316.
- Hypothesis, 1ff., 31f., 176, 318; proper and improper use of, 341f.
- Hypothesizing, in logic, 1, 171, 176, 316, 341f.
- Hypothetico-deductive principle, 33, 89, 98, 192, 317; and deduction, 35.
- Icon, Index, and Symbol, Peirce's, 237.
- Ideal, the, and the formal, 159f.
- Ideas, as logical subject matter, 164.
- Identity, law of, 38, 332, 333f.
- "If then" proposition, 160.
- Implication, and inference, 185f.; as construction, 288f.

- Inclusiveness, in logic, 303f.
- Indefinables, as logical elements, 218, 230f.
- Induction, 22, 29f., 33, 35, 52, 63ff., 135, 190f., 288, 313, 319.
- Inference, 9, 33, 81, 88, 96; definition of, 180; not a fixed process, 180f.; and discovery, 182; and deductive reasoning, 182f.; and implication, 184; effectiveness of, 186f.; in mathematics, 187; types of, 189ff.; linear vs. circular, 343; no general forms of, 298.
- Infinite, the, 9f., 47, 51, 60, 98f., 137f., 156, 218f.; and limits problem, 260; as mathematical proof, 290f.
- Infinitesimals, place of, in the limits problem, 260.
- Inquiry (see Theory of inquiry); logic as inquiry into, 125f.; concerns events, not propositions, 276; forms vs. functions in, 276f.; no general forms of, 298.
- Institutions, in logic, 21; intellectual, 23ff.; and propositions, 36ff.
- Instrumental logic, 12; characteristics of, 101ff.; technological, 102f.; critical phase of, 103; as a form of dialectics, 104; linguistic factors, in, 105; psychological implications of, 105f.; independent of psychology, 125f.; and dialectical logic, 166f.; unique handling of terms in, 273.
- Integers, as products, 234.
- Intellectual interbehavior, as logical operations, 173ff.
- Intellectualism, in logic, 101.
- Intension, in nominalism and neo-classicism, 271f.; vs. extension, 271f.
- Intentional psychology, 119.
- Interbehavioral fields, 5ff., 151ff.
- Interbehavioral linguistics, 205ff.
- Interbehavioral psychology, contribution of, to logical theory, 136f.; and dichotomy problem, 162; and specificity logic, chap. 6; and logical capacity, 308; regards ideas as actions, 164.
- Interbehavioral principle, the, tolerates all systems, 169.
- Intuition, and meaning, 206; as power of logician, 307ff.; construction concerning, 308f.; in modern rationalism, 55.
- Inversion, 280.
- Judging, in logical operations, 173; language acts concerning, 277.
- Judgments, and propositions, 277f.
- Kantian logic, 56ff.; opposition to, 118.
- Knowing, and creation of objects, 106; confused with linguistic interbehavior, 207.
- Knowledge, and instrumental logic, 106; as logical subject matter, 124f.; objects of, and things, 104; reduced to signs, 240.
- Language, misuse of, in logic, 18f.; distinct from logic, 16, 18, 45f.; and Aristotelian syllogism, 25f.; naïve for Aristotle, 47; Scholastic treatment of, 50; Locke's use of, 56; in Mill's system, 66; career of, in logical evolution, 70f.; identified with things, 196; and symbolic logic, 74ff.; division of study of, 76; of physics, 81; as treated by symbolinguists, 86; as an instrument of reasoning, 92; in instrumental logic, 105; from interbehavioral standpoint, 137, 205ff.; close affiliation of logic with, 194ff.; referential and nonreferential, 196ff.; meaning in, 206; analysis of, 202ff.; use of, in system building, 226f., 294f.; fixity of, 262f.; and the timeless, 263; as logical materials, 325f.; ultimate reality achieved through, 54f.
- Language and symbols, 211ff.; made identical, 195f.; according to interbehavioral linguistics, 205.
- Language things, in instrumental logic, 105; as materials of system building, 325f.
- Laws and Canons, in logic, 41ff., 301, 332ff.
- Limits, as constructional and mathematical products, 295f.; and the infinite, 260.
- Linguistics (see Language).
- Linguistic events, and system building, 17, 75; and operational contact with things, 164f.; and symbolinguism, 75ff., 85f.; as activities vs. things, 196ff.; and language things, 200f.; field character of, 206; confusion of, with symbolic interbehavior, 207f.
- Logic, not one thing, 10, 30, 31, 70, 79, 100, 108, 329f.; as operational, 2ff., 212; practice and theory in, 4f.; and interbehavioral fields, 5ff.; transcendence of, 3f., 6, 212, 220, 261, 299, 303, 305, 311, 331; and biological processes, 7; as system building, 7ff.,

- 66f., 96, 161, 168, 296ff., 307; as specific, 6, 9ff., 168, 294; as absolute and universal, 10f., 48ff., 63, 65f., 124f., 168, 189, 230, 294, 305, 317, 320, 329, 331, 335f.; simplification in, 11; vs. language and symbols, 16ff., 261ff., 327; as a human enterprise, 6, 19; and science, 19, 229f.; influences operating upon, 6, 20f., 299ff.; cultural basis of, 21f., 273f., 300f.; institutional history of, 22f.; origins of, 24f.; as Organon, 26, 313; formalization procedures in, 43, 127, 195; doctrinal succession in, chap. 3, 329; metaphysical factors in, 46; transformation of Aristotelian to Scholastic, 48f.; career of, 67ff.; separation of, from philosophy, 69f., 334f.; symbolic, 77ff.; syntactic, 79ff.; semantic, 83ff.; and mathematics, 8, 77, 88ff., 93f., 96f., 122f., 254f., 295; and the interbehavioral principle, 96f.; humanistic or nonformal, 99ff., 116; instrumental, 101, 116, 125f., 276f., 304; many kinds of, 108, 166f.; validity problem in, 8, 123, 343; as science and as art, 128; as inquiry into inquiry, 125f.; sociological activities in, 154f.; scientific vs. absolute, 155; and ontology, 161ff.; can logic deal with nonexistents? 163; identified with problem solving, 171, 178ff.; as public and private, 194; and signs, 201f., 212f.; operations and products of, chap. 9, 342f.; not language, 18, 228, 235f.; as timeless, 262, 322f.; forms and functions in, 276f.; pure and applied, 128, 331f.; deliberate and casual, 332; completeness of, 98, 343.
- Logic and Psychology, interrelations of, 13ff., chap. 5; regarded as incompatible, 109; coordinated, 133f.; reduction of L. to P., 127; L. and inadequate P., 134f.; L. and interbehavioral P., 166, 308.
- Logic of order, 98f., 236f.; rationality factor in, 108.
- Logic of science, 65, 79f., 81.
- Logical algebra, Boole's, 91ff., 114.
- Logical auspices, 300f.; philosophical, 329f.; scientific, 330; mathematical, 330; practical, 331.
- Logical canons, 332ff.
- Logical comprehensiveness, 10.
- Logical consistency, 305f., 341.
- Logical definition, 22, 28ff., 89; symbolinguist's view of, 217.
- Logical evolution, and language, 70f.
- Logical exclusiveness, 307f.
- Logical institutions, 22ff.
- Logical materials, 324; language as, 325ff.; relations as, 328f.; overlooked, 324; symbols as, 327; scientific, 327; things as, 325.
- Logical necessity, 10.
- Logical, nonlogical, and prelogical existence, 165.
- Logical operations, specificity of, 293; pragmatic continuum of, 5.
- Logical origins, 22ff.
- Logical periods, Aristotelian, 46f.; Scholastic, 47ff.; Renaissance, 51ff.; Enlightenment, 56ff.; Kantian, 56ff.; Hegelian, 60ff.; technological revolution, 63ff.; Mill's, 65f.; contemporary scene, 66f.
- Logical positivists, 79f.
- Logical products, 222, chap. 9, 284, 342f.; specificity of, 293ff.
- Logical techniques, 300, 313ff.
- Logical theory, in demonstration and proof logic, 306f.; vs. thing theorized, 105; contribution of interbehavioral psychology to, 136f.; genetic and validity aspects of, 293; as interbehavioral enterprise, 296ff.; continuous with practice, 296, 315; interbehavioral vs. conventional, 298; as analysis, 298f.; based on interests and materials, 338; ignores logical products, 342f.
- Logician, the, vs. cultural trend, 107; as elementary vs. technical system builder, 67ff.; contemporary influences upon, 72; accepts psychology, 112ff.; rejects psychology, 117ff.; neglects psychology, 128ff.; cooperation of, with psychologist, 133f.; place of, in logical situation, 134f.; as theorist, 296, 342; interests, aims, motives of, 299f., 302ff.; psychological notions of, 308; cultural background of, 310f.; at work, 312ff.
- Logistic logic, as aspect of mathematical logic, 88; as a type of philosophy, 89f.
- Logistic mathematics, 95, 217.
- Logocentric predicament, 230f.
- Logos conception, the, 4, 18, 48.
- Mathematician, the, builds upon logic, 93f.; as system builder, 8; creativity of, 292; and logical techniques, 113f.
- Mathematical logic, 67, 88ff., 93, 132, 167, 238f., 285, 322.

- Mathematical symbols, in Kant and Hegel, 71; and interbehavioral psychology, 137; in modern logic, 201f.
- Mathematical systems and proofs, 289f.
- Mathematics, axioms in, 259; and specificity logic, 9, 10; and system-building operations, 18, 295, 230f.; in Greek logic, 31ff.; relation of, and logic, 55, 93ff., 175, 331; Kantian ideas of, 59f.; and symbolic logic, 77f., 86, 99; pure, 80, 89, 97, 289; and symbolinguistic logic, 86; three interpretations of, 95, 217f.; two general theories of, 95f.; formality of, 97; as logic, 88f., 96f., 119; as art, 133; symbols in, 120f.; construction and product in, 234f., 285; arbitrary creation vs. concrete action in, 156f.; Peirce's definition of, 160; inference in, 187; as a language, 86, 195, 202, 219, 235; signs in, 201f.; logistic, formalistic, and constructionistic, 218; mysticism in, 216; and science, 235; as verbo-verbal interpretation, 245; and relations, 253f.; limits construction in, 259f.; intuition in, 308f.; validating criteria of, 341; and syntactic propositions, 279; doctrinal functions in, 293; rationality in, 188.
- Meaning, in symbolinguistic logic, 130; in symbols and language, 206; in semiotics, 242, 245; of words and symbols, 263ff.
- Meaningless sentences, 19, 131.
- Medievalists, the, logic of, 26, 35, 43.
- Mentalistic logician, the, and the nominalistic tradition, 16.
- Mentality (see Mind); logicians' notion of, 20; not autonomous, 164.
- Metaphysics, disease of speech, 137; and psychology, chap. 6, 57, 142f.; and operationism, 147f.; in unifying semiotics, 241f.
- Methodological vs. philosophical logics, 330.
- Methodology of science, 9, 10, 52, 69f.
- Mind, conception of, by logicians, 20, 156, 307f., 320; and Aristotle's reasoning faculty, 26f.; Scholastic conception of, 50; for Kant, 59, 112f., for Mill, 66, 115; Boole's treatment of, 133f.; for neo-Platonists, 50; from instrumental standpoint, 116f.; and reasoning, 132f.; in dualistic psychology, 134; from interbehavioral standpoint, 136f.; excluded from psychology, 142; as preexisting, 336; rational vs. empirical view of, 53.
- Mysticism, and symbology, 81f.; in mathematics, 216.
- Naming, as nonreferential language, 198; operations involved in, 216.
- Names, identified with essences, 50; and terms, 273ff.
- Natural science, vs. human science, 54; inferences in, 188.
- Nature, and knowledge of, 147; continuum of, 150f.; confused with science, 235.
- Necessity, as logical factor, 10, 160; no *a priori* source of, 289.
- Neo-Platonism, and logic, 49, 50.
- Nominalism, 14, 16, 37f., 274; vs. realism, 50, 310.
- Noncontradiction, criterion of, 8, 188; as formula, 67f.
- Nonexistents, 163.
- Nontemporal, the, 160.
- Numbers, nature and relationship of, 132; interdependence of, and things, 214; derivation of, 234; as constructions, 246ff.; Cantor's new, 291; evolution of, 248f.; as Platonic entities, 249.
- Objective psychology, development of, 15; and Aristotelian logic, 46f.; need of, in logical doctrine, 70.
- Objects, confused with objects known, 106.
- Obversion, 280.
- One, the, and the Many, 28.
- Ontology, and system building, 161ff.; 315f.
- Operational principle, the, deviation of mathematicians and logicians from, 93f.; as support for dualism, 145f.; vs. symbolinguistic treatment of symbols and syntax, 212ff.
- Operations, in logic, 2ff.; upon logic as an object, 27; confused with products, 233ff.; always interactions with things, 234; specificity of, 293ff.; of the theorist, 297.
- Order, logic as science of, 98f.
- Organon, logic as, 4, 26f., 52, 63, 65, 300, 303, 313, 324, 325; the Aristotelian, 27.
- Patristics, the, words and symbols for, 71.
- Perceiving, in logical operations, 171.
- Philosophy, separation of, from logic, 69f., 334f.; as logic of science, 79f.; and the logician, 303, 310f.; as orientation enterprise, 310f.; interbehavior-

- al view of, 311; *vs.* science, 311; auspices of, for logic, 329f.
- Physics, system building in, 168.
- Platonic realism, 94, 118, 160, 205.
- Postulates, nature of, chap. 1; principles underlying, 31ff.; and assuming interbehavior, 175; *vs.* canons, 333f.; evolutionary influence upon theory of, 69; mathematical influence upon theory of, 2, 32f.
- Pragmatic logic, 12.
- Predicate, the, 28, 34, 36, 91.
- Probability, 10, 12.
- Problem solving, logic interpreted as, 171, 178ff., 233, 307f.
- Process made into system, 316.
- Product Construction, in logic, 210, 284; in mathematics, 289f.
- Proof, as used by mathematician, 160; in logical theory, 306f.; in mathematics, 289f.; no general forms of, 298.
- Propositional functions, 38, 55, 88, 96f., 215, 232, 257f.; syntactic structures as, 279.
- Propositions, Aristotelian, 36ff.; development of constructions concerning, 36ff.; equated with facts, 196, 223; Post Aristotelian, 37f.; implicational character of, 38; true-false criteria of, 38, 232, 281; distinguished from sentences, 70f., 222f., 282f.; in instrumental logic, 104; and dualistic psychology, 134; "if then" type of, 160; from interbehavioral standpoint, 223; and antecedent existence, 165; *vs.* judgment, 172, 277; and syllogisms, 172; as behavioral products, 172, 184, 223, 278ff.; generic *vs.* universal, 275; not necessarily subject matter of logic, 276; meaning of, 276f.; as syntactic structures, 278f.; as transpositional inference, 280; as referential, 280f.; as calculative, 281f.
- Psychologism, attacked by: Husserl, 119; Frege, 121f.
- Psychology, and logic, 13ff., 111; as a natural science, 15, 140, 161; Aristotelian objective, 46f.; excluded from logic, 109, 117ff.; neglected by logicians, 128ff.; cooperation of, with logic, 133f.; as science of interbehavioral fields, 140f.; philosophical *vs.* scientific, 142f.; and dualistic metaphysics, 146f.; and the nonspatio-temporal, 148; mind-body problem in, 148f.; and biological events, 149f.
- Pythagorean number system, and mathematical logic, 93.
- Questions, with and without sense, 80.
- Quotation, and statement, 245.
- Ratiocination, 181.
- Rational, the, as definition of logic, 158f.; criteria of, 187ff.; and specificity principle, 188; and rationalization, 189.
- Rational calculus, the, 55.
- Rationalism, *vs.* empiricism, 53, 54; *vs.* experimentalism, 82; in logic, 108; Kant's connection of, with psychology, 112; and symbology, 215f.
- Realism *vs.* idealism, 208.
- Realism, Greek naïve, 46.
- Realists, the, and logic, 16, 37f., 274.
- Reality, and absolute processes in logic, 320.
- Reasoning, as adjustment, 178; cosmic, 3, 112, 132; logician's idea of, 20f., 308; and absolutism in logic, 320; and logical operations, 3ff., chap. 7; transcending, 6; and logic as Organon, 26f.; in medieval period, 35; in Aristotle's time, 37; Scholastic, 51; in Kantian logic, 57, 59, 112f.; as a universal power, 51; as objective, 121; and mathematical processes, 132; place of, in system building, 170ff.; logical and psychological, 121; not invariably related to logical operations, 170f.; reduced to problem solving, 171; as specific interbehavioral fields, 172f., 209; confused with syllogistic systems, 172; as inferential, 180; *vs.* thinking, 181; deductive, 182; efficacy of, 186; criteria of, 187ff.; cultural influences upon, 188; as power of logician, 307f.; no isolated canons of, 333; no ultimate power of, 298, 336; logic as process of, 131f., 329f.; as relativistic, 187.
- Recording, as nonreferential language, 199.
- Reference frame, influence upon logic, chap. 2, 111.
- References, and symbols in logic, 226.
- Referential interbehavior, in logical operations, 164f.; as linguistic, 196f.; *vs.* naming, 198.
- Relations, mathematical, 167, 330; as interbehavioral products, 253ff.; and traditional logic, 254; as things *vs.* descriptions, 255; sentences concerning,

- 281; as materials for system building, 328f.; in order logic, 98.
 Relative *a priori*, 320f.
 Rigor, as a logical aim, 305; changed notion of, 293.
- Scholastic logic, 48ff.; Renaissance of, 118.
 Scholastics, the, 19, 37, 47ff., 67f., 71; Aristotle transformed by, 48ff.
 Science, and logic, 19, 133f., 299; and the dualistic dogma, 150f.; Greek, 47; and Kantian logic, 112; in Mill's logic, 65; foundations of, 139f.; deduction in, 183; and mathematics, 235; vs. nature, 235; operations in, 229f.; and semiotics, as unification of, 240; and philosophy, 311.
 Science of order, as logic, 98f.
 Scientific methodology, 4, 9, 10, 44, 52, 69, 71.
 Semantic logic, development of, 83ff.
 Semantics, 76, 219f.
 Semiotics, relation of, to linguistics and logic, 205; Peirce's, 237ff.; Morris's unifying, 239ff.; Ducasse's, 243ff.; interpretative feature of, 244; interbehavioral approach to, 244f.
 Sentences, for Aristotle, 37; and symbolinguism, 55f., 129ff.; for the Greeks, 70f.; and semantic logic, 84f.; metalogical and logical, 84; logical activities reduced to, 136; and symbol configuration, 224; and propositions, 222f., 267f., 275, 282; and statements, 223f.; type and token forms of, 267; as act products, 268; disagreement concerning, 275; as materials of inquiry, 275f.; as judgments, 277; as syntactic structures, 278f.; as referential, 280; as calculative, 281; as products of construction, 278; organization of, 129, 136.
 Sentential functions, 232, 279f., 293.
 Series, as constructive products, 255f., 279f.
 Signs, and symbol interbehavior, 199f.; as nonlinguistic things, 201f.; in symbolinguism, 212f.; not behavior, 242; from interbehavioral standpoint, 199f., 244; in some typical semiotic systems, 237ff.; and their significant, 209f.
 Simplification, canons of, 240f.; in logic, 11.
 Spatiotemporal frame of reference, 6, 46, 134.
- Specificity, in logical work, 9ff.; vs. generalization, 10; of logical operations and products, 293ff.; canons of, 332f.; in system types, 166f.
 Speculation interbehavior, 153, 311.
 Subject—Predicate, 26, 34, 36, 78, 88.
 Sufficient reason, principle of, 38.
 Syllogisms, Aristotelian, 25, 33f.; medieval, 35; rules of, 42f.; emptiness vs. fullness of, 120f.; designed for a purpose, 288.
 Syllogistic inferences, 182f., 190.
 Syllogistic logic, and logistic logic, 90.
 Symbolic logic, 40, 71, 74, 77ff., 99, 167, 233, 245.
 Symbolinguism, 81f.; and scientific linguistics, 85f.; as a technique of neglecting psychology, 128ff.
 Symbolinguistic logic, 73ff.; 85f.; three phases of, 76f.; and mathematics, 86; implies essentiality, 212f.; distinct contribution of, 79, 88.
 Symbolism, and mysticism, 81f.
 Symbolizing interbehavior, 137, 199ff., 269, 314f.
 Symbols, in logic, 16ff., 45f., 71, chap. 8, 211ff., 221ff., 261ff.; lend reality to referents, 18; and propositions, 37f.; Scholastic treatment of, 50, 71, 314; in symbolinguistic logic, 73ff., 85ff., 233; as used by deductionists and inductionists, 82; in algebraic logic, 91f.; from interbehavioral standpoint, 137, 167, 213; as substitute stimuli, 167; and language, chap. 8; meaning in, 206; and essences, 212f.; characteristics of, 213ff.; in logistic, formalistic, and constructionistic mathematics, 217f.; configurations of, 219ff., 224f.; origin of, 224f.; fixity achieved by, through personal declaration, 262; emptiness of, 195.
 Syntactic logic, 79ff.; moves over to semantics, 220f.; sentential functions in, 279f.; transpositional inference in, 280.
 Syntax, 76, 219ff.; construction of logic by, 221; empirical vs. pure, 221.
 Synthesizing, as logical technique, 319f.; from interbehavioral standpoint, 319f.
 System building, and logical operations, 7f., 296ff.; as construction, 9, 284, 294; as organization of symbols and propositions, 11, 314f., 278f.; as specific interbehavior, 8, 16; and language, 17f., 226f.; and logical institu-

- tions, 22; Greek pragmatic, 46f.; and mathematics, 67, 96f.; as order logic, 98; and voluntaristic logic, 101; non-inclusiveness of, 8f., 104f., 133f.; types of, 166ff.; specificity of, 8, 168, 293, 311f.; thinking and reasoning in, 170ff., 178; and problem solving, 179f.; validity of, 8, 225, 339f.; continuity of process and product in, 236f.; scientific vs. unscientific, 236; terms as products of, 294f.; strata of, 284ff.; auspices and conditions of, 6, 300f., 329ff.; criteria and canons of, 301, 332ff.; actual work of, 312; as idiosyncratic, 312; logical techniques in, 300, 313ff.; logical and scientific materials of, 324ff.; use of hypothesis dodge in, 341f.
- System construction**, 66f.; in logic and science, 299f.; based on grammatical universe of discourse, 278f.; avoidance of transcendent principles in, 336.
- Systematic**, the, as definition of logic, 160f.
- Systems of logic**, validity and equality of, 99, 107f., 225, 315f.; vs. scientific systems, 229f.; many kinds of, 10, 108, 166f.; as products, 284; on order logic plan, 98f.; as organization of terms, 315.
- Tautological systems**, 10, 13, 212, 297f., 307, 316f., 331, 337.
- Terms**, logical 27f.; and development of semantic logic, 83; for the nominalist, 269ff.; Mill's classification of, 270; for the neoclassicist, 271f.; for the instrumentalist, 273; dependent upon sentences, 274; as products of system building, 274.
- Theory** (see Logical theory); vs. practice in logic, 4f., 296ff., 315, 342; identification of, with thing theorized about, 105; considered as logic, 159; in interbehavioral vs. conventional logic, 298.
- Theory of inquiry**, 19, 101ff., 126f., 233.
- Thing language**, vs. activity language, 17.
- Things**, transformed into objects of knowledge, 104; confused with described, 240.
- Thinking**, in Aristotle's time, 37; personal and impersonal, 121; logical and psychological, 121; reduced to problem solving, 171; actual interbehavior, 170, 172ff.; no general forms of, 101, 298; varying views concerning, 176f.; according to specificity principle, 178; logical vs. nonlogical, 177.
- Thought**, science of, 27; as process and product, 124; laws of, 43; as form and content, 127; identified with things, 161f., 239.
- Totalitarian logic**, 111f., 93, 111, 118, 126, 233, 315f., 329.
- Transcendence**, of logic, 3f., 6, 11f., 13f., 170, 212, 299, 304f., 306, 320ff.
- Trial and error**, a system-building procedure, 339.
- True-false**, criteria of, 38, 70f., 99, 185, 232, 281.
- Truth and certainty**, in logic, 305.
- Ultimacy**, vs. specificity in traditional logic, 10, 109, 332; in instrumental logic, 104.
- Unitarianism**, in logic, 107.
- Universality**, in logic, 10, 48f., 159, 179, 220f., 299, 303, 305, 307, 310f., 320, 331.
- Universals**, not inherent in things, 165; as constructions, 299; in Mill's nominalism, 269f.; and the intension-extension features of terms, 272; as words, 16.
- Use and mention** (see Quotation), 83f., 223, 235.
- Validity**, in logic, 8, 123, 339f.
- Variable**, the, definition of, 210, 249; locus and range of, 249f.; as an interbehavioral object, 250f.
- Voluntaristic logic**, 100 (see Humanistic logic).
- Words**, manipulation of, 18; Scholastic treatment of, 50f.; for the Greeks, 70f.; became symbols, 71; not fixed, 262; as sounds and shapes, 265f.; and meaning, 264ff.; functions of, 269; treated interbehaviorally, 263f.; do not exist independently, 265.

NAME INDEX

- Ackermann, 80n., 305n.
 Agricola, 53.
 d'Alembert, 295n.
 Angell, 110.
 Angus, 50.
 Antisthenes, 28.
 Archimedes, 260.
 Argand, 248.
 Aristotle, 17, chap. 2, 46ff., 64ff., 73ff.,
 85, 88, 91, 99, 194, 234, 285, 322.
 Arius, 51.
 Augustine, 50.

 Bacon, 52f., 64, 66, 71.
 Bain, 116.
 Bell, 254n., 255, 292n., 310.
 Benjamin, 328n.
 Bentham, 75, 242.
 Berkeley, 106.
 Bills, 146n.
 Black, 88n., 289n.
 Bliss, 328n.
 Boethius, 48.
 Bolzano, 94, 119, 291.
 Boole, 78, 91f., 93, 113ff., 175.
 Boring, 146n., 147.
 Bosanquet, 4, 162, 274, 303, 319, 343.
 Boyle, 148, 235.
 Bradley, 4, 162, 303, 319.
 Braithwaite, 262n.
 Brentano, 118f.
 Bridgman, 145, 215.
 Brill, 35.
 Brouwer, 95, 292.
 Brown, 316.
 Buchler, 238n., 251f.
 Burt, 53n.
 Bush, 187n.

 Cajori, 87n.
 Campbell, 214.
 Cantor, 94, 254n., 290f.
 Carmichael, 132, 192n.
 Carnap, 80, 81n., 84, 86n., 110, 129n.,
 130n., 170n., 219n., 220n.
 Cassirer, 327n.
 Castro, 116n., 125n.
 Church, 220n.
 Comte, 65, 328.
 Cohen, 17, 93n., 109n., 121n., 155n.,
 184, 194n., 321n., 336n.
 Condillac, 19.
 Copernicus, 53.
 Courant, 252.

 Couturat, 74, 230f.
 Croce, 53.
 Crane, 216n.

 Dantzig, 292n.
 Darwin, 116, 178.
 Democritus, 28, 144.
 Davis, 144.
 DeMorgan, 17, 78, 91, 114, 117.
 Descartes, 53, 55, 149, 309.
 Dewey, 18, 101f., 105f., 122, 125ff.,
 136, 160ff., 165, 170., 173n., 176,
 181n., 186, 215, 223n., 238n., 255n.,
 262, 273ff., 304.
 Ducasse, 130n., 131, 196n., 223, 242n.,
 243ff., 245.
 Dubs, 305n., 318n.
 Duns Scotus, 17, 194.

 Eaton, 93n., 121n., 124n., 185, 258n.,
 272.
 Eddington, 148.
 Einstein, 165, 208, 335.
 Enriques, 25n., 32n., 33, 35, 46f.
 Euclid, 32, 35, 43f., 68, 82, 84, 208,
 218, 283, 286f.
 Euler, 78, 254.

 Fairbanks, 234n.
 Faraday, 181.
 Fechner, 144f.
 Fermat, 205.
 Fibonacci, 255.
 Flint, 328n.
 Frege, 88, 121f., 285.
 Fries, 117.

 Galileo, 52f., 56, 65, 68, 148, 235, 253,
 290f.
 Gauss, 248, 253f.
 Goblott, 328n.
 Gödel, 220.
 Grassmann, 248.

 Hamilton, 12, 91, 113, 248.
 Harley, 175.
 Hegel, 4, 60f., 71, 125, 158, 162, 166,
 207, 238, 240, 274, 303.
 Helmer, 19n., 214.
 Heraclitus, 48.
 Herchel, 23, 65, 71, 179, 316.
 Hilbert, 80f., 95, 231, 305n.
 Hobbes, 19, 75.
 Helmholtz, 118, 143.
 Hofstadter, 84n., 277n.

- Humboldt, 328.
 Hume, 6, 57, 118.
 Huntington, 231, 290.
 Husserl, 111, 119f.
- James, 117, 178.
 Jäsche, 23n., 56n., 58.
 Jeans, 148.
 Jeffries, 32, 304n.
 Jevons, 6, 39f., 91f., 114, 175, 270n.
 Johnson, 231.
 Joseph, 26, 30, 34, 99n., 127n., 190, 271, 277n.
- Kant, 23, 56f., 60, 68, 71, 80, 112f., 114f., 117f., 234, 238.
 Kantor, 146n., 155n., 177n., 187n., 195n., 197n., 223n., 242n., 282n.
 Kapp, 33n.
 Keeton, 196n.
 Kepler, 52f., 235.
 Keynes, 43, 123f.
 Keyser, 293.
 Klein, 94n.
 Kroneker, 95, 234, 292.
- Ladd-Franklin, 92.
 Langford, 78n., 87n., 114n., 280n.
 Lambert, 78.
 Leibniz, 18, 38, 54f., 57, 78, 82, 87, 112, 158, 213, 215, 252, 322.
 Lesser, 187.
 Lewis, 74, 78, 87n., 114, 211n., 280n., 308n., 314, 320n., 329n., 336n., 343.
 Lipps, 110.
 Linke, 117.
 Locke, 19, 54, 56, 57, 66, 71, 237n.
 Lotze, 143, 319.
 Love, 214n.
- Mach, 253.
 Mauthner, 75.
 McGilvary, 104, 165, 261n.
 Mead, 241.
 Meinong, 7, 119.
 Mill, 12, 15, 23, 63f., 71, 75, 109, 115, 179, 269ff., 316.
 Minkowski, 208.
 Mitchell, 93.
 Moore, 238.
 Morris, 240ff.
 Moseley, 183.
 Müller, 143.
 Murphy, 242n.
- Nagel, 7, 93n., 109n., 121n., 155n., 162n., 184, 194n., 238n.
 Nelson, 117.
- Newton, 54, 56, 87, 112, 148, 213.
- Osgood, 214.
- Peano, 80, 88f., 89, 91.
 Peirce, 17, 91f., 93n., 114, 117, 131, 194, 237ff., 250f., 263ff., 328n.
 Peirce, B., 160.
 Pieri, 231f.
 Plato, 55, 75, 87, 94, 205, 249, 252, 256, 360.
 Plotinus, 62.
 Poincaré, 19n., 75, 91, 167n.
- Quine, 87, 93n., 223n., 232n., 235, 245, 314.
- Ramsey, 95.
 Richardson, 328n.
 Rignano, 173n.
 Ritchie, 148n.
 Robbins, 252.
 Roscellinus, 75.
 Rosinger, 81n.
 Ross, 33, 34.
 Rosser, 220n.
 Rousseau, 61.
 Royce, 62, 94, 98, 109.
 Russell, 7, 13f., 80, 89f., 95, 109, 119, 162, 249n., 257n., 258f., 274, 305n., 319.
- Saccheri, 41, 159.
 Santayana, 311.
 Schiller, 99n., 100, 116.
 Schroeder, 92, 114.
 Sheffer, 230.
 Shelley, 25.
 Socrates, 28f.
 Spencer, 88, 116, 137, 328.
 Stahl, 56.
 Stebbing, 264ff.
 Stevens, 144, 147.
 Struik, 143.
 Swift, 213.
- Tarski, 76n., 84, 87, 93n., 220n.
 Tennant, 328n.
 Tertullian, 51.
 Theophrastus, 48.
 Thompson, 113.
 Thomson, 328n.
 Torricelli, 56.
- Ueberweg, 24n.
 Ushenko, 84, 245, 305n., 309n.
- Valla, 53.

- Veblen, 231.
Venn, 40, 77f., 114, 175.
Vico, 53, 66.
Vieta, 213.
Vivés, 53.
- Wallace, 60n.
Weiss, 238.
Wessell, 248.
Weyl, 95, 292.
Whately, 113.
- Whewell, 65.
Whitehead, 80, 90, 119, 148n., 251n.,
258n.
Windelband, 53n., 58n.
Wittgenstein, 79f., 216, 220n.
Wolff, 52n., 57.
Wundt, 117f., 143.
- Xenophanes, 137, 234.
- Young, 231, 295n.

**THE UNIVERSITY OF MICHIGAN
GRADUATE LIBRARY**

DATE DUE



UNIVERSITY OF MICHIGAN



3 9015 02643 0689

Generated on 2018-03-05 17:58 GMT / <http://hdl.handle.net/2027/mdp.39015026430689>
Public Domain, Google-digitized / http://www.hathitrust.org/access_use#pd-google



